9/20/99

Three Examples of the Lévy-Nádai Solution Technique

I will work through the single series solution for x = 0, x= a simply supported, and y = 0, y = b (1) simply supported, (2) clamped, and (3) free. In each case I take a series solution in x and find the associated y eigenfunctions for each case.

Select M = 3 as a now tried and true choice

```
STUDENT > M:=3:
```

They will all have the same x dependence, f[m] = sin(mπx/a). I'll have different functions for the various y variations. Let's let Y0 denote the particular solution to the problem, and Y1 be the simply-supported correction, Y2 the clamped correction and Y3 the free boundary correction. The corresponding displacement functions will be w0, w1, w2 and w3, respectively.

```
STUDENT > f:=array(1..M):
          Y0:=array(1..M):Y1:=array(1..M):Y2:=array(1..M):Y3:=array(
          1..M):
STUDENT > w0:=0:w1:=0:w2:=0:w3:=0:
          for m from 1 to M do
          w0:=w0+Y0[m]*f[m]:
          w1:=w1+Y1[m]*f[m]:
          w2:=w2+Y2[m]*f[m]:
          w3:=w3+Y3[m]*f[m]:
          od:
STUDENT > for m from 1 to M do f[m]:=sin(m*Pi*x/a) od:
```

Q[m] denotes the coefficients of the particular solution, to be inserted later. A, B, C and D denote the cofficients of the various terms in the y eigenfunctions

```
STUDENT > Q:=array(1..M):
          A1:=array(1..M):A2:=array(1..M):A3:=array(1..M):
          B1:=array(1..M):B2:=array(1..M):B3:=array(1..M):
          C1:=array(1..M):C2:=array(1..M):C3:=array(1..M):
          D1:=array(1..M):D2:=array(1..M):D3:=array(1..M):
```

I will limit this analysis to the case where the surface loading is at most a function of x, so that the particular solution will be constant in y, given by Q[m], which can be found later. I can define Y1 through Y3 using the homogeneous solutions for the correction terms.

```
STUDENT > for m from 1 to M do
          Y0[m]:=Q[m]:
          Y1[m]:=A1[m]*sinh(m*Pi*y/a)+B1[m]*m*Pi*y/a*cosh(m*Pi*y/a)
          +C1[m]*cosh(m*Pi*y/a)+D1[m]*m*Pi*y/a*sinh(m*Pi*y/a):
          Y2[m]:=A2[m]*sinh(m*Pi*y/a)+B2[m]*m*Pi*y/a*cosh(m*Pi*y/a)
          +C2[m]*cosh(m*Pi*y/a)+D2[m]*m*Pi*y/a*sinh(m*Pi*y/a):
          Y3[m]:=A3[m]*sinh(m*Pi*y/a)+B3[m]*m*Pi*y/a*cosh(m*Pi*y/a)
          +C3[m]*cosh(m*Pi*y/a)+D3[m]*m*Pi*y/a*sinh(m*Pi*y/a):
          od:
```

<u>Start with the simply-supported case</u>

Boundary conditions

w0 + w1 must vanish on y = 0, b and so must its second derivative. (Of course, w0 is independent of y.) We denote the boundary conditions on y = 0 by bc1 and bc2, and those on y = b as bc3 and bc4. This notation continues throughout.

```
STUDENT > w1yy:=diff(diff(w1,y),y):
          y:=0:bc1:=w0+w1;bc2:=w1yy;
          y:=b:bc3:=w0+w1:bc4:=w1yy:
          y:='y':
```

$$bc1 := Q_1 \sin\left(\frac{\pi x}{a}\right) + Q_2 \sin\left(2\frac{\pi x}{a}\right) + Q_3 \sin\left(3\frac{\pi x}{a}\right) + C1_1 \sin\left(\frac{\pi x}{a}\right) + C1_2 \sin\left(2\frac{\pi x}{a}\right) + C1_3 \sin\left(3\frac{\pi x}{a}\right)$$

$$bc2 := \left(\frac{C1_1 \pi^2}{a^2} + 2\frac{D1_1 \pi^2}{a^2}\right)\sin\left(\frac{\pi x}{a}\right) + \left(4\frac{C1_2 \pi^2}{a^2} + 8\frac{D1_2 \pi^2}{a^2}\right)\sin\left(2\frac{\pi x}{a}\right) + \left(9\frac{C1_3 \pi^2}{a^2} + 18\frac{D1_3 \pi^2}{a^2}\right)\sin\left(3\frac{\pi x}{a}\right)$$

The boundary conditions at y = 0 eliminate these coefficients

```
STUDENT > for m from 1 to M do C1[m]:=-Q[m]:D1[m]:=-C1[m]/2: od:
          factor(bc1);factor(bc2);
```

$$0$$

$$0$$

Use bc3 to eliminate B1[m]

```
STUDENT > B1[1]:=solve(op(1,bc3)+op(4,bc3),B1[1]);
          B1[2]:=solve(op(2,bc3)+op(5,bc3),B1[2]);
          B1[3]:=solve(op(3,bc3)+op(6,bc3),B1[3]);
          simplify(bc3,symbolic);
```

$$B1_1 := -\frac{1}{2}\frac{2 Q_1 a + 2 A1_1 \sinh\left(\frac{\pi b}{a}\right) a - 2 Q_1 \cosh\left(\frac{\pi b}{a}\right) a + Q_1 \pi b \sinh\left(\frac{\pi b}{a}\right)}{\pi b \cosh\left(\frac{\pi b}{a}\right)}$$

$$B1_2 := \frac{1}{2}\frac{-Q_2 a - A1_2 \sinh\left(2\frac{\pi b}{a}\right) a + Q_2 \cosh\left(2\frac{\pi b}{a}\right) a - Q_2 \pi b \sinh\left(2\frac{\pi b}{a}\right)}{\pi b \cosh\left(2\frac{\pi b}{a}\right)}$$

$$B1_3 := \frac{1}{6}\frac{-2 Q_3 a - 2 A1_3 \sinh\left(3\frac{\pi b}{a}\right) a + 2 Q_3 \cosh\left(3\frac{\pi b}{a}\right) a - 3 Q_3 \pi b \sinh\left(3\frac{\pi b}{a}\right)}{\pi b \cosh\left(3\frac{\pi b}{a}\right)}$$

$$0$$

And finally eliminate A1[m] using bc4. Here I had to do some silly-seeming stuff because Maple wouldn't do what I want.

```
STUDENT > bc4;
STUDENT > collect(simplify(op(1,bc4),symbolic),A1[1]);
STUDENT > A1[1]:=-op(2,")/op(1,")*A1[1];
```

$$A1_1 := -\frac{-2\sinh\left(\frac{\pi b}{a}\right) a\, Q_1 - \pi b \cosh\left(\frac{\pi b}{a}\right) Q_1 + 2\sinh\left(\frac{\pi b}{a}\right) a\, Q_1 \cosh\left(\frac{\pi b}{a}\right) + \pi\, Q_1\, b}{2\, a - 2\, a \cosh\left(\frac{\pi b}{a}\right)^2}$$

```
STUDENT > simplify(op(1,bc4),symbolic);
```

$$0$$

```
STUDENT > collect(simplify(op(2,bc4),symbolic),A1[2]);
STUDENT > A1[2]:=-op(2,")/op(1,")*A1[2];simplify(op(2,bc4),symbolic)
          ;
```

$$A1_2 := -\frac{-\sinh\left(2\frac{\pi b}{a}\right) a\, Q_2 - \pi b \cosh\left(2\frac{\pi b}{a}\right) Q_2 + \sinh\left(2\frac{\pi b}{a}\right) a\, Q_2 \cosh\left(2\frac{\pi b}{a}\right) + \pi\, Q_2\, b}{a - a \cosh\left(2\frac{\pi b}{a}\right)^2}$$

$$0$$

```
STUDENT > collect(simplify(op(3,bc4),symbolic),A1[3]);
STUDENT > A1[3]:=-op(2,")/op(1,")*A1[3];simplify(op(3,bc4),symbolic)
          ;
```

$$A1_3 := -\frac{2\sinh\left(3\frac{\pi b}{a}\right) a\, Q_3 + 3\pi b \cosh\left(3\frac{\pi b}{a}\right) Q_3 - 2\sinh\left(3\frac{\pi b}{a}\right) a\, Q_3 \cosh\left(3\frac{\pi b}{a}\right) - 3\pi\, Q_3\, b}{-2\, a + 2\, a \cosh\left(3\frac{\pi b}{a}\right)^2}$$

$$0$$

```
STUDENT > simplify(bc4,symbolic);
```

$$0$$

```
STUDENT > w1;
STUDENT > factor(op(1,"))+factor(op(2,"))+factor(op(3,"));
```

Print out simpler versions of A1[m] and B1[m] and then write out the solution w1.

```
STUDENT > for m from 1 to M do
          A1[m]:=simplify(A1[m],symbolic);
          B1[m]:=simplify(B1[m],symbolic);
          od;
          eval(collect(w1,[Q[1],Q[2],Q[3]]));
STUDENT >
```

$$A1_1 := \frac{1}{2}\frac{\left(-\pi b + 2\sinh\left(\frac{\pi b}{a}\right) a\right) Q_1}{\left(\cosh\left(\frac{\pi b}{a}\right) + 1\right) a}$$

$$B1_1 := -\frac{1}{2}\frac{Q_1 \sinh\left(\frac{\pi b}{a}\right)}{\cosh\left(\frac{\pi b}{a}\right)+1}$$

$$A1_2 := \frac{\left(-\pi b + \sinh\left(2\frac{\pi b}{a}\right)a\right)Q_2}{\left(\cosh\left(2\frac{\pi b}{a}\right)+1\right)a}$$

$$B1_2 := -\frac{1}{2}\frac{Q_2 \sinh\left(2\frac{\pi b}{a}\right)}{\cosh\left(2\frac{\pi b}{a}\right)+1}$$

$$A1_3 := \frac{1}{2}\frac{\left(-3\pi b + 2\sinh\left(3\frac{\pi b}{a}\right)a\right)Q_3}{\left(\cosh\left(3\frac{\pi b}{a}\right)+1\right)a}$$

$$B1_3 := -\frac{1}{2}\frac{Q_3 \sinh\left(3\frac{\pi b}{a}\right)}{\cosh\left(3\frac{\pi b}{a}\right)+1}$$

Now I can work out the second, clamped, set of functions.

We have the following boundary conditions, again supposing w0 to be independent of y.

```
STUDENT > B2:=array(1..M):C2:=array(1..M):w2y:=diff(w2,y):
          y:=0:bc1:=w0+w2;bc2:=w2y;
          y:=b:bc3:=w0+w2:bc4:=w2y:
          y:='y':
```

$$bc1 := Q_1 \sin\left(\frac{\pi x}{a}\right) + Q_2 \sin\left(2\frac{\pi x}{a}\right) + Q_3 \sin\left(3\frac{\pi x}{a}\right) + C2_1 \sin\left(\frac{\pi x}{a}\right) + C2_2 \sin\left(2\frac{\pi x}{a}\right) + C2_3 \sin\left(3\frac{\pi x}{a}\right)$$

$$bc2 := \left(\frac{A2_1 \pi}{a} + \frac{B2_1 \pi}{a}\right)\sin\left(\frac{\pi x}{a}\right) + \left(2\frac{A2_2 \pi}{a} + 2\frac{B2_2 \pi}{a}\right)\sin\left(2\frac{\pi x}{a}\right) + \left(3\frac{A2_3 \pi}{a} + 3\frac{B2_3 \pi}{a}\right)\sin\left(3\frac{\pi x}{a}\right)$$

```
STUDENT > for m from 1 to M do
          C2[m]:=-Q[m]:
          B2[m]:=-A2[m]: od:
          simplify(bc1,symbolic);simplify(bc2,symbolic);
```

$$0$$

$$0$$

```
STUDENT > bc3;
STUDENT > D2[1]:=solve(op(1,bc3)+op(4,bc3),D2[1]);
          D2[2]:=solve(op(2,bc3)+op(5,bc3),D2[2]);
          D2[3]:=solve(op(3,bc3)+op(6,bc3),D2[3]);
          simplify(convert(bc3,trig),symbolic);
```

$$D2_1 := -\frac{2\,Q_1\,a\,e^{\left(\frac{\pi b}{a}\right)} + A2_1\,a\left(e^{\left(\frac{\pi b}{a}\right)}\right)^2 - A2_1\,a - A2_1\,\pi\,b\left(e^{\left(\frac{\pi b}{a}\right)}\right)^2 - A2_1\,\pi\,b - Q_1\,a\left(e^{\left(\frac{\pi b}{a}\right)}\right)^2 - Q_1\,a}{\pi\,b\left(\left(e^{\left(\frac{\pi b}{a}\right)}\right)^2 - 1\right)}$$

$$D2_2 := -\frac{1}{2}\left(2\,Q_2\,a\,e^{\left(2\frac{\pi b}{a}\right)} + A2_2\,a\left(e^{\left(2\frac{\pi b}{a}\right)}\right)^2 - A2_2\,a - 2\,A2_2\,\pi\,b\left(e^{\left(2\frac{\pi b}{a}\right)}\right)^2 - 2\,A2_2\,\pi\,b - Q_2\,a\left(e^{\left(2\frac{\pi b}{a}\right)}\right)^2 - Q_2\,a\right) \Big/ \left(\pi\,b\left(\left(e^{\left(2\frac{\pi b}{a}\right)}\right)^2 - 1\right)\right)$$

$$D2_3 := \frac{1}{3}\left(-2\,Q_3\,a\,e^{\left(3\frac{\pi b}{a}\right)} - A2_3\,a\left(e^{\left(3\frac{\pi b}{a}\right)}\right)^2 + A2_3\,a + 3\,A2_3\,\pi\,b\left(e^{\left(3\frac{\pi b}{a}\right)}\right)^2 + 3\,A2_3\,\pi\,b + Q_3\,a\left(e^{\left(3\frac{\pi b}{a}\right)}\right)^2 + Q_3\,a\right) \Big/ \left(\pi\,b\left(\left(e^{\left(3\frac{\pi b}{a}\right)}\right)^2 - 1\right)\right)$$

$$0$$

```
STUDENT > simplify(convert(bc3,trig),symbolic);
```

$$0$$

```
STUDENT > bc4:
STUDENT > A2:=array(1..M):
          A2[1]:=solve(convert(simplify(op(1,bc4),symbolic),trig),A2
          [1]);
          A2[2]:=solve(convert(simplify(op(2,bc4),symbolic),trig),A2
          [2]);
          A2[3]:=solve(convert(simplify(op(3,bc4),symbolic),trig),A2
          [3]);
```

$$A2_1 := -Q_1\,a\Big(-\pi\,b\cosh\left(\frac{\pi b}{a}\right) - \pi\,b\sinh\left(\frac{\pi b}{a}\right) - \sinh\left(\frac{\pi b}{a}\right)a + \sinh\left(\frac{\pi b}{a}\right)\pi\,b\cosh\left(2\frac{\pi b}{a}\right)$$
$$+ 2\sinh\left(\frac{\pi b}{a}\right)a\cosh\left(\frac{\pi b}{a}\right) - \pi\cosh\left(\frac{\pi b}{a}\right)b\sinh\left(2\frac{\pi b}{a}\right) + 2\,\pi\cosh\left(\frac{\pi b}{a}\right)b\sinh\left(\frac{\pi b}{a}\right)$$
$$- \sinh\left(\frac{\pi b}{a}\right)a\cosh\left(2\frac{\pi b}{a}\right) - \sinh\left(\frac{\pi b}{a}\right)a\sinh\left(2\frac{\pi b}{a}\right) + 2\,\pi\cosh\left(\frac{\pi b}{a}\right)^2 b$$
$$- \pi\cosh\left(\frac{\pi b}{a}\right)b\cosh\left(2\frac{\pi b}{a}\right) + \sinh\left(\frac{\pi b}{a}\right)\pi\,b\sinh\left(2\frac{\pi b}{a}\right) + 2\sinh\left(\frac{\pi b}{a}\right)^2 a\Big) \Big/ \Big($$
$$-\pi^2\cosh\left(\frac{\pi b}{a}\right)b^2\sinh\left(2\frac{\pi b}{a}\right) - \pi^2\cosh\left(\frac{\pi b}{a}\right)b^2\cosh\left(2\frac{\pi b}{a}\right) + \sinh\left(\frac{\pi b}{a}\right)a^2\sinh\left(2\frac{\pi b}{a}\right)$$
$$+ \sinh\left(\frac{\pi b}{a}\right)a^2\cosh\left(2\frac{\pi b}{a}\right) + \pi^2\,b^2\sinh\left(\frac{\pi b}{a}\right)\sinh\left(2\frac{\pi b}{a}\right) + \pi^2\,b^2\sinh\left(\frac{\pi b}{a}\right)\cosh\left(2\frac{\pi b}{a}\right)$$
$$+ \pi\cosh\left(\frac{\pi b}{a}\right)b\,a\sinh\left(2\frac{\pi b}{a}\right) + \pi\cosh\left(\frac{\pi b}{a}\right)b\,a\cosh\left(2\frac{\pi b}{a}\right) - \sinh\left(\frac{\pi b}{a}\right)a^2$$

$$- \sinh\left(\frac{\pi b}{a}\right) a \pi b \sinh\left(2 \frac{\pi b}{a}\right) - \sinh\left(\frac{\pi b}{a}\right) a \pi b \cosh\left(2 \frac{\pi b}{a}\right) - \pi^2 \cosh\left(\frac{\pi b}{a}\right) b^2$$

$$- \pi \cosh\left(\frac{\pi b}{a}\right) b\, a - \sinh\left(\frac{\pi b}{a}\right) a \pi b - \pi^2 b^2 \sinh\left(\frac{\pi b}{a}\right)\Bigg)\Bigg)$$

$$A2_2 := Q_2\, a \Bigg(-2 \sinh\left(2 \frac{\pi b}{a}\right)^2 a - 2 \sinh\left(2 \frac{\pi b}{a}\right) \pi b \cosh\left(4 \frac{\pi b}{a}\right)$$

$$- 2 \sinh\left(2 \frac{\pi b}{a}\right) \pi b \sinh\left(4 \frac{\pi b}{a}\right) + 2 \pi b \cosh\left(2 \frac{\pi b}{a}\right) + 2 \pi b \sinh\left(2 \frac{\pi b}{a}\right) + \sinh\left(2 \frac{\pi b}{a}\right) a$$

$$+ 2 \pi \cosh\left(2 \frac{\pi b}{a}\right) b \cosh\left(4 \frac{\pi b}{a}\right) + 2 \pi \cosh\left(2 \frac{\pi b}{a}\right) b \sinh\left(4 \frac{\pi b}{a}\right)$$

$$- 4 \pi \cosh\left(2 \frac{\pi b}{a}\right) b \sinh\left(2 \frac{\pi b}{a}\right) + \sinh\left(2 \frac{\pi b}{a}\right) a \cosh\left(4 \frac{\pi b}{a}\right) + \sinh\left(2 \frac{\pi b}{a}\right) a \sinh\left(4 \frac{\pi b}{a}\right)$$

$$- 4 \pi \cosh\left(2 \frac{\pi b}{a}\right)^2 b - 2 \sinh\left(2 \frac{\pi b}{a}\right) a \cosh\left(2 \frac{\pi b}{a}\right)\Bigg) \Bigg/ \Bigg(-\sinh\left(2 \frac{\pi b}{a}\right) a^2$$

$$- 2 \sinh\left(2 \frac{\pi b}{a}\right) a \pi b \cosh\left(4 \frac{\pi b}{a}\right) - 2 \sinh\left(2 \frac{\pi b}{a}\right) a \pi b \sinh\left(4 \frac{\pi b}{a}\right)$$

$$+ 2 \pi \cosh\left(2 \frac{\pi b}{a}\right) b\, a \cosh\left(4 \frac{\pi b}{a}\right) + \sinh\left(2 \frac{\pi b}{a}\right) a^2 \sinh\left(4 \frac{\pi b}{a}\right)$$

$$+ \sinh\left(2 \frac{\pi b}{a}\right) a^2 \cosh\left(4 \frac{\pi b}{a}\right) - 4 \pi^2 \cosh\left(2 \frac{\pi b}{a}\right) b^2 \sinh\left(4 \frac{\pi b}{a}\right)$$

$$- 4 \pi^2 \cosh\left(2 \frac{\pi b}{a}\right) b^2 \cosh\left(4 \frac{\pi b}{a}\right) + 4 \pi^2 b^2 \sinh\left(2 \frac{\pi b}{a}\right) \sinh\left(4 \frac{\pi b}{a}\right)$$

$$+ 4 \pi^2 b^2 \sinh\left(2 \frac{\pi b}{a}\right) \cosh\left(4 \frac{\pi b}{a}\right) - 2 \pi \cosh\left(2 \frac{\pi b}{a}\right) b\, a - 4 \pi^2 \cosh\left(2 \frac{\pi b}{a}\right) b^2$$

$$- 2 \sinh\left(2 \frac{\pi b}{a}\right) a \pi b - 4 \pi^2 b^2 \sinh\left(2 \frac{\pi b}{a}\right) + 2 \pi \cosh\left(2 \frac{\pi b}{a}\right) b\, a \sinh\left(4 \frac{\pi b}{a}\right)\Bigg)\Bigg)$$

$$A2_3 := - Q_3\, a \Bigg(-\sinh\left(3 \frac{\pi b}{a}\right) a + 2 \sinh\left(3 \frac{\pi b}{a}\right)^2 a + 3 \sinh\left(3 \frac{\pi b}{a}\right) \pi b \cosh\left(6 \frac{\pi b}{a}\right)$$

$$+ 3 \sinh\left(3 \frac{\pi b}{a}\right) \pi b \sinh\left(6 \frac{\pi b}{a}\right) - 3 \pi \cosh\left(3 \frac{\pi b}{a}\right) b \cosh\left(6 \frac{\pi b}{a}\right)$$

$$- 3 \pi \cosh\left(3 \frac{\pi b}{a}\right) b \sinh\left(6 \frac{\pi b}{a}\right) + 2 \sinh\left(3 \frac{\pi b}{a}\right) a \cosh\left(3 \frac{\pi b}{a}\right) + 6 \pi \cosh\left(3 \frac{\pi b}{a}\right)^2 b$$

$$- \sinh\left(3 \frac{\pi b}{a}\right) a \cosh\left(6 \frac{\pi b}{a}\right) - \sinh\left(3 \frac{\pi b}{a}\right) a \sinh\left(6 \frac{\pi b}{a}\right) + 6 \pi \cosh\left(3 \frac{\pi b}{a}\right) b \sinh\left(3 \frac{\pi b}{a}\right)$$

$$- 3 \pi b \sinh\left(3 \frac{\pi b}{a}\right) - 3 \pi b \cosh\left(3 \frac{\pi b}{a}\right)\Bigg) \Bigg/ \Bigg(-\sinh\left(3 \frac{\pi b}{a}\right) a^2$$

$$- 3 \sinh\left(3 \frac{\pi b}{a}\right) a \pi b \sinh\left(6 \frac{\pi b}{a}\right) + \sinh\left(3 \frac{\pi b}{a}\right) a^2 \cosh\left(6 \frac{\pi b}{a}\right) + \sinh\left(3 \frac{\pi b}{a}\right) a^2 \sinh\left(6 \frac{\pi b}{a}\right)$$

$$- 3 \sinh\left(3 \frac{\pi b}{a}\right) a \pi b \cosh\left(6 \frac{\pi b}{a}\right) + 9 \pi^2 b^2 \sinh\left(3 \frac{\pi b}{a}\right) \cosh\left(6 \frac{\pi b}{a}\right)$$

$$+9\pi^2 b^2 \sinh\left(3\frac{\pi b}{a}\right)\sinh\left(6\frac{\pi b}{a}\right)+3\pi\cosh\left(3\frac{\pi b}{a}\right)b\,a\sinh\left(6\frac{\pi b}{a}\right)$$
$$-9\pi^2\cosh\left(3\frac{\pi b}{a}\right)b^2\cosh\left(6\frac{\pi b}{a}\right)-9\pi^2\cosh\left(3\frac{\pi b}{a}\right)b^2\sinh\left(6\frac{\pi b}{a}\right)$$
$$+3\pi\cosh\left(3\frac{\pi b}{a}\right)b\,a\cosh\left(6\frac{\pi b}{a}\right)-9\pi^2 b^2\sinh\left(3\frac{\pi b}{a}\right)-3\pi\cosh\left(3\frac{\pi b}{a}\right)b\,a$$
$$-3\sinh\left(3\frac{\pi b}{a}\right)a\,\pi\,b-9\pi^2\cosh\left(3\frac{\pi b}{a}\right)b^2\Bigg)$$

And I haven't been able to make this very much simpler using Maple

Plug in for uniform loading

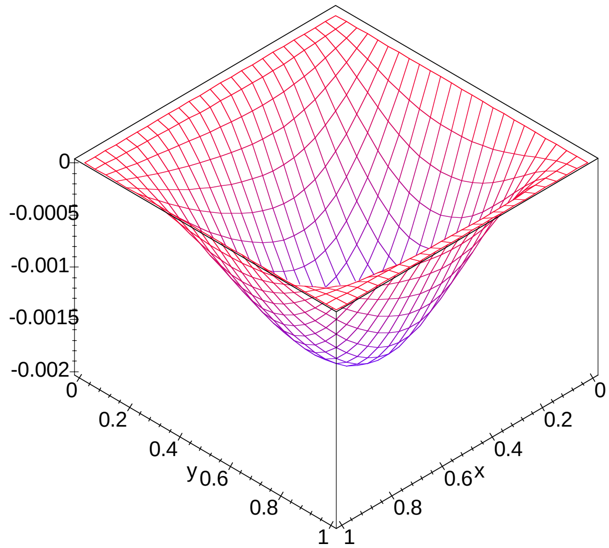
```
STUDENT > q:='q':a:='a':b:='b':
          for m from 1 to M do Q[m]:=-4*q/EI*a^4/Pi^5/m^5 od;
```

$$Q_1 := -4\frac{q\,a^4}{EI\,\pi^5}$$
$$Q_2 := -\frac{1}{8}\frac{q\,a^4}{EI\,\pi^5}$$
$$Q_3 := -\frac{4}{243}\frac{q\,a^4}{EI\,\pi^5}$$

Plot the two solutions to compare them

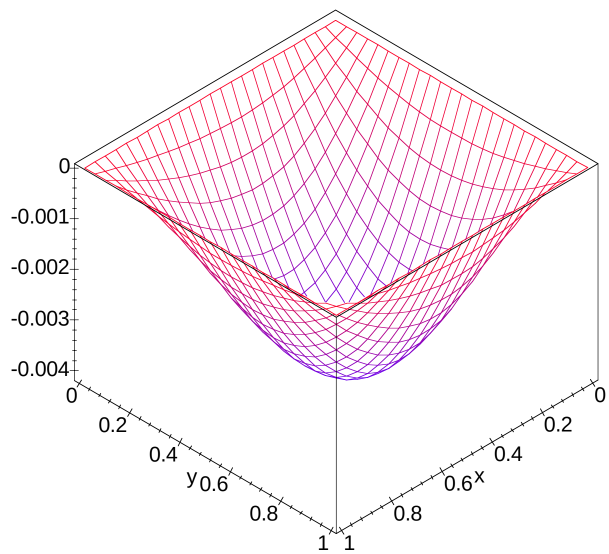
```
STUDENT > q:=1:EI:=1:a:=1:b:=1:
          plot3d(w0+w2,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`Cla
          mped Second Sides`);
          x:=a/2:y:=b/2:wMAX:=evalf(w0+w2);
          x:='x':y:='y':
          plot3d(w0+w1,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`Sim
          ply-Supported Second Sides`);
          x:=a/2:y:=b/2:wMAX:=evalf(w0+w1);
          x:='x':y:='y':
          q:='q':EI:='EI':a:='a':b:='b':
```

Clamped Second Sides



wMAX := -.00191359737

Simply-Supported Second Sides



wMAX := -.004058799394

<u>Now the third case -- free boundaries</u>

Again we set up the boundary conditions. We have x derivatives here, and we need to look at w0 when calculating all four boundary conditions

```
STUDENT > w0xx:=diff(diff(w0,x),x):
          w3xx:=diff(diff(w3,x),x):w3yy:=diff(diff(w3,y),y):
          moment:=nu*(w3xx+w0xx)+w3yy:
          freedom:=diff(w3xx+w0xx+w3yy,y):
```

```
             y:=0:bc1:=moment;bc2:=freedom;
             y:=b:bc3:=moment:bc4:=freedom:
             y:='y':
STUDENT > collect(bc1,[D3[1],D3[2],D3[3],C3[1],C3[2],C3[3]]);
```

$$2\frac{\pi^2\sin\left(\frac{\pi x}{a}\right)D3_1}{a^2}+8\frac{\pi^2\sin\left(2\frac{\pi x}{a}\right)D3_2}{a^2}+18\frac{\pi^2\sin\left(3\frac{\pi x}{a}\right)D3_3}{a^2}$$
$$+\left(-\frac{\nu\pi^2\sin\left(\frac{\pi x}{a}\right)}{a^2}+\frac{\pi^2\sin\left(\frac{\pi x}{a}\right)}{a^2}\right)C3_1+\left(-4\frac{\nu\pi^2\sin\left(2\frac{\pi x}{a}\right)}{a^2}+4\frac{\pi^2\sin\left(2\frac{\pi x}{a}\right)}{a^2}\right)C3_2$$
$$+\left(-9\frac{\nu\pi^2\sin\left(3\frac{\pi x}{a}\right)}{a^2}+9\frac{\pi^2\sin\left(3\frac{\pi x}{a}\right)}{a^2}\right)C3_3$$
$$+\nu\left(4\frac{q\,a^2\sin\left(\frac{\pi x}{a}\right)}{EI\,\pi^3}+\frac{1}{2}\frac{q\,a^2\sin\left(2\frac{\pi x}{a}\right)}{EI\,\pi^3}+\frac{4}{27}\frac{q\,a^2\sin\left(3\frac{\pi x}{a}\right)}{EI\,\pi^3}\right)$$

```
STUDENT > for m from 1 to M do
          D3[m]:=-1/2*(1-nu)*C3[m]-2*nu*q*a^4/EI/m^5/Pi^5
          od:
          factor(bc1);
```

$$0$$

```
STUDENT > bc2;
STUDENT > collect(bc2,[A3[1],A3[2],A3[3],B3[1],B3[2],B3[3]]);
```

$$2\frac{B3_1\,\pi^3\sin\left(\frac{\pi x}{a}\right)}{a^3}+16\frac{B3_2\,\pi^3\sin\left(2\frac{\pi x}{a}\right)}{a^3}+54\frac{B3_3\,\pi^3\sin\left(3\frac{\pi x}{a}\right)}{a^3}$$

```
STUDENT > for m from 1 to M do B3[m]:=0 od:
          factor(bc2);
```

$$0$$

```
STUDENT > bc3:=collect(bc3,[C3[1],C3[2],C3[3],A3[1],A3[2],A3[3]]);
STUDENT > factor(bc3);
STUDENT > collect(",[A3[1],A3[2],A3[3],C3[1],C3[2],C3[3]]);
STUDENT > nops(");
STUDENT > expand(op(7,""));
STUDENT > convert(",exp):expand("):convert(",trig);
STUDENT > subs(sin(Pi*x/a)=s1,sin(2*Pi*x/a)=s2,sin(3*Pi*x/a)=s3,bc3)
          ;
STUDENT > collect(",[s1,s2,s3]);
STUDENT > tbc3:=";
STUDENT > C3[1]:=solve(coeff(tbc3,s1),C3[1]):
STUDENT > C3[1]:=collect(convert(C3[1],trig),A3[1]):factor(op(1,"))+
          factor(op(2,"));
STUDENT > collect(convert(solve(coeff(tbc3,s2),C3[2]),trig),A3[2]):C
          3[2]:=factor(op(1,"))+factor(op(2,"));
```

$$C3_2:=-\frac{A3_2\,a}{\pi b\,(-1+\nu)}-\frac{1}{8}\nu\,q\,a^4\left(\pi b\cosh\left(2\frac{\pi b}{a}\right)+\pi b\sinh\left(2\frac{\pi b}{a}\right)-a+a\cosh\left(2\frac{\pi b}{a}\right)\right)$$

$$+\sinh\left(2\frac{\pi b}{a}\right)a-\nu\pi b\cosh\left(2\frac{\pi b}{a}\right)-\nu\pi b\sinh\left(2\frac{\pi b}{a}\right)-\nu\pi b+\pi b\right) \Big/ \left((-1+\nu)^2\right.$$

$$\left.\left(1+\cosh\left(2\frac{\pi b}{a}\right)+\sinh\left(2\frac{\pi b}{a}\right)\right)\pi^6\, b\, EI\right)$$

```
STUDENT > collect(convert(solve(coeff(tbc3,s3),C3[3]),trig),A3[3]):C
          3[3]:=factor(op(1,"))+factor(op(2,"));
```

$$C3_3 := -\frac{2}{3}\frac{A3_3\, a}{\pi b(-1+\nu)}-\frac{4}{729}\nu q a^4\left(3\pi b\cosh\left(3\frac{\pi b}{a}\right)+3\pi b\sinh\left(3\frac{\pi b}{a}\right)+2a\cosh\left(3\frac{\pi b}{a}\right)\right.$$

$$+2\sinh\left(3\frac{\pi b}{a}\right)a-3\nu\pi b\cosh\left(3\frac{\pi b}{a}\right)-3\nu\pi b\sinh\left(3\frac{\pi b}{a}\right)-3\nu\pi b-2a+3\pi b\Big) \Big/ \Big($$

$$(-1+\nu)^2\left(\cosh\left(3\frac{\pi b}{a}\right)+\sinh\left(3\frac{\pi b}{a}\right)+1\right)\pi^6\, b\, EI\Big)$$

```
STUDENT > simplify(bc3,symbolic);
STUDENT > convert(",exp):expand(");
```

$$0$$

```
STUDENT > tbc4:=subs(sin(Pi*x/a)=s1,sin(2*Pi*x/a)=s2,sin(3*Pi*x/a)=s
          3,bc4);
STUDENT > collect(",[s1,s2,s3]);
STUDENT > tbc4:=":
STUDENT > convert(factor(op(1,")),trig)
          +convert(factor(op(2,")),trig)
          +convert(factor(op(3,")),trig);
STUDENT > tbc4:=":
STUDENT > A3:=array(1..M):
          coeff(tbc4,s1);
STUDENT > collect(op(1,tbc4),A3[1]);
STUDENT > -factor(op(2,")/op(1,")*A3[1]);
```

$$-4\frac{\nu q a^4\left(\cosh\left(\frac{\pi b}{a}\right)+\sinh\left(\frac{\pi b}{a}\right)-1\right)}{\pi^5 EI(-1+\nu)\left(\cosh\left(\frac{\pi b}{a}\right)+\sinh\left(\frac{\pi b}{a}\right)+1\right)}$$

```
STUDENT > A3[1]:=":
STUDENT > collect(op(2,tbc4),A3[2]);
STUDENT > -factor(op(2,")/op(1,")*A3[2]);
```

$$-\frac{1}{8}\frac{\nu q a^4\left(\sinh\left(2\frac{\pi b}{a}\right)-1+\cosh\left(2\frac{\pi b}{a}\right)\right)}{\pi^5 EI(-1+\nu)\left(1+\cosh\left(2\frac{\pi b}{a}\right)+\sinh\left(2\frac{\pi b}{a}\right)\right)}$$

```
STUDENT > A3[2]:=":
STUDENT > collect(op(3,tbc4),A3[3]);
STUDENT > -factor(op(2,")/op(1,")*A3[3]);
```

$$-\frac{4}{243}\frac{\nu q a^4\left(-1+\cosh\left(3\frac{\pi b}{a}\right)+\sinh\left(3\frac{\pi b}{a}\right)\right)}{\pi^5 EI(-1+\nu)\left(\cosh\left(3\frac{\pi b}{a}\right)+\sinh\left(3\frac{\pi b}{a}\right)+1\right)}$$

```
STUDENT > A3[3]:=":factor(bc4);
STUDENT > simplify(",symbolic);
```

STUDENT > convert(",exp):expand(");

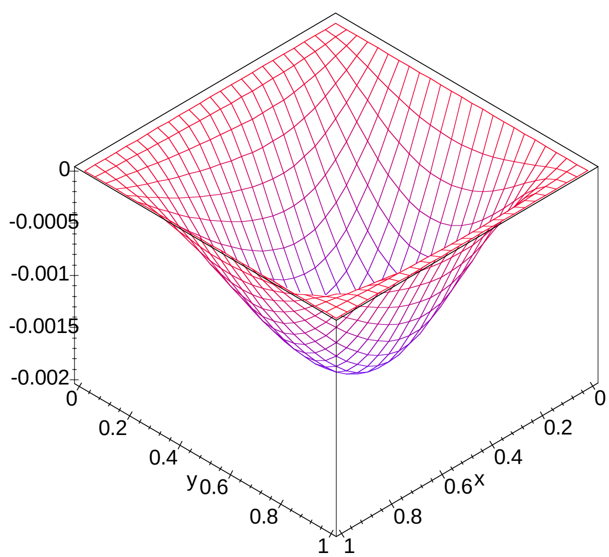
0

STUDENT > w3;

<u>**Now we can plot all three pictures**</u>

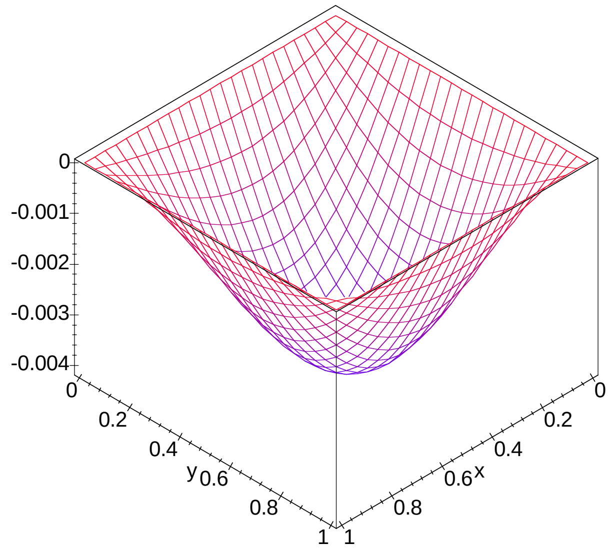
```
STUDENT > q:=1:EI:=1:a:=1:b:=1:
          nu:=1/3:plot3d(w0+w2,x=0..a,y=0..b,axes=BOXED,shading=Z,ti
          tle=`Clamped Second Sides`);
          x:=a/2:y:=b/2:wMAX:=evalf(w0+w2);
          x:='x':y:='y':
          plot3d(w0+w1,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`Sim
          ply-Supported Second Sides`);
          x:=a/2:y:=b/2:wMAX:=evalf(w0+w1);
          x:='x':y:='y':
          plot3d(w0+w3,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`Fre
          e Second Sides`);
          x:=a/2:y:=b:wMAX:=evalf(w0+w3);
          x:='x':y:='y':
          q:='q':EI:='EI':a:='a':b:='b':nu:='nu':
```

Clamped Second Sides



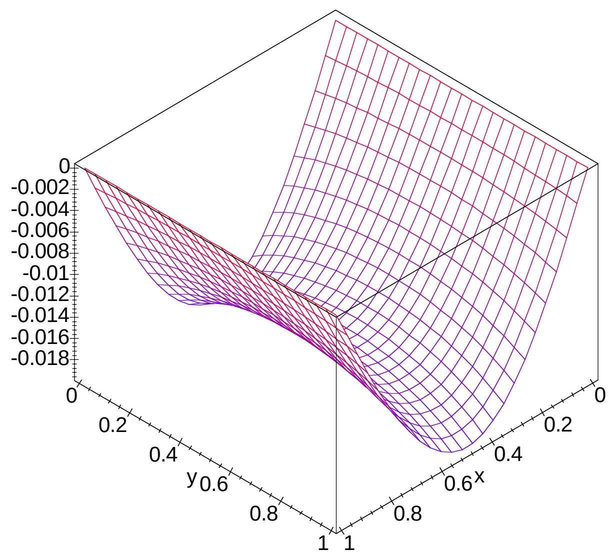
wMAX := -.00191359737

Simply-Supported Second Sides



wMAX := -.004058799394

Free Second Sides



wMAX := -.01952589665

Note that the maximum displacement for the free ends takes place at the outside edge, not the center. This maximum is less than the maximum for the simply-supported system, but greater than the simply supported-clamped combination. The total deformation is bigger, though.