Problem Set 2

The equilibrium equations for plane stress in cylindrical coordinates are

$$\frac{1}{r}\frac{\partial(r\sigma_r)}{\partial r}+\frac{1}{r}\frac{\partial\tau_{r\theta}}{\partial\theta} - \frac{\sigma_\theta}{r} = 0, \qquad \frac{1}{r^2}\frac{\partial(r^2\tau_{r\theta})}{\partial r}+\frac{1}{r}\frac{\partial\sigma_\theta}{\partial\theta} = 0$$

1. Construct a scaling for this assuming the thickness, h is small compared to the mean radius R, and that the inner surface of the layer has a pressure field, p, not necessarily uniform

$$\left(\text{Hint: put } r = R + h\zeta, \text{ where } -\frac{1}{2} < \zeta < \frac{1}{2}\right)$$

Consider the strain relations

$$e_r = \frac{1}{E}(\sigma_r - \nu\sigma_\theta), \quad e_\theta = \frac{1}{E}(-\nu\sigma_r + \sigma_\theta), \quad \gamma_{r\theta} = c_1\tau_{r\theta}$$

2. Find the approximate expressions for the strains, and using

$$e_r = \frac{\partial u}{\partial r}, \quad e_\theta = \frac{1}{r}\frac{\partial v}{\partial\theta} + \frac{u}{r}, \quad \gamma_{r\theta} = \frac{1}{r}\frac{\partial u}{\partial\theta} + \frac{\partial v}{\partial r} - \frac{v}{r}$$

3. Obtain an expression for the radial displacement, u, analogous to the moment-curvature relation.

4. Find the radial displacement if the beam has a total included angle of $3\pi/4$ and its ends are pinned (no moment and no displacement at each end).