9/17/99

On the Bending of a Simply-Supported Aluminum Plate Caused by a Central Load

Look at a "real" problem: a half inch thick aluminum plate simply-supported and loaded by a load that can be approximated by a sine squared in both directions. I'll need to read in the paradigm to save me some difficulty, which means that these results are based on the N = 3 = M truncation.

```
STUDENT > read `paradigm.m`:
```

Then I need to set up the stress so that I can impose a specific load (P) in pounds later.

```
STUDENT > stress:=Q*sin(Pi*x/a)^2*sin(Pi*y/b)^2:
          load:=int(int(stress,x=0..a),y=0..b);
```

$$load := \frac{1}{4}\, b\, a\, Q$$

```
STUDENT > Q:=solve(load=P,Q);
```

$$Q := 4\,\frac{P}{b\,a}$$

```
STUDENT > stress;
```

$$4\,\frac{P\,\sin\left(\frac{\pi\,x}{a}\right)^2\,\sin\left(\frac{\pi\,y}{b}\right)^2}{b\,a}$$

1. Work out the displacement

Then I have the usual calculation of (dimensionless) w, that will need to be corrected by division by the flexural rigidity before numbers can be put in
.

```
STUDENT > EI:='EI':
          s:=array(0..M,0..N):
          for m from 0 to M do for n from 0 to N do
          int(int(stress*f[m,n],x=0..a),y=0..b):
          s[m,n]:="/EI od:od:
```

```
STUDENT > for m from 0 to M do for n from  0 to N do
          A[m,n]:=s[m,n]/c[m,n] od:od:
```

For further analytical purpose I want to introduce physical stuff

Define the flexural rigidity in terms of E, h and nu. (The strange notation is because Maple uses D and E as reserved words and I don't want to undo that and try to remember that I have done so. Therefore I will use EI to denote the bending rigidity and Y to denote Young's modulus when I'm working in Maple.)

```
STUDENT > EI:=Y*h^3/12/(1-nu^2);
```

$$EI := \frac{1}{12}\,\frac{Y\,h^3}{1-\nu^2}$$

Put in the values for aluminum

```
STUDENT > Y:=10500000:nu:=334/1000:
```

and let the plate be 1/2 inch thick and write out the flexural rigidity (in pound-inch)

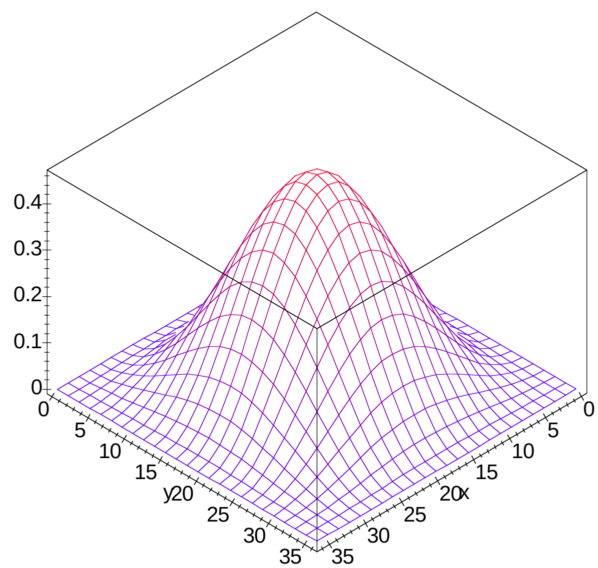
```
STUDENT > h:=1/2:evalf(EI);
```

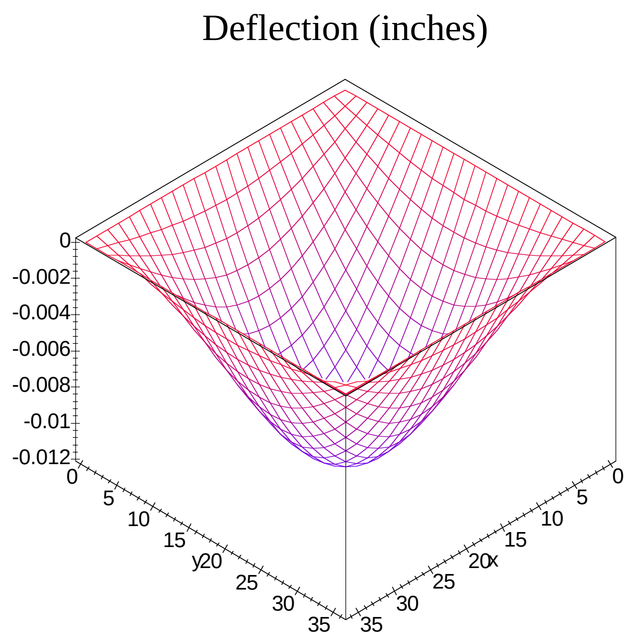
$$123108.4908$$

If I divide the displacement by the flexural rigidity I will have a correct expression for w. Length units need to be in inches for the way this is set up. I write (for a 150 pound load)

```
STUDENT > P:=-150:
          a:=36:b:=36:
          plot3d(-stress,x=0..a,y=0..b,axes=BOXED,shading=Z,scaling=
          UNCONSTRAINED,title=`Load (psi)`);
          plot3d(w,x=0..a,y=0..b,axes=BOXED,shading
          =Z,scaling=UNCONSTRAINED,title=`Deflection (inches)`);
          x:=a/2:y:=b/2:wMAX:=evalf(w);x:='x':y:='y':
          P:='P':a:='a':b:='b':
```

Load (psi)





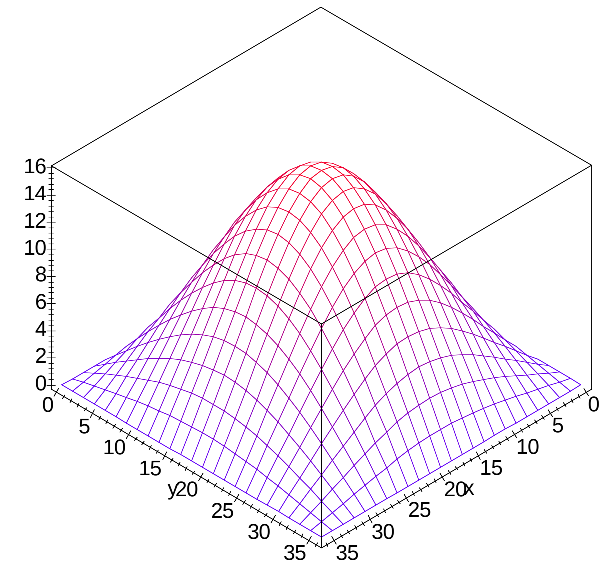
wMAX := -.01187274737

The maximum displacement is small enough that we can be reasonably confident in our prediction of displacement field.

2. Moment and shear eesultants

```
STUDENT > Mx:=-EI*(diff(diff(w,x),x)+nu*diff(diff(w,y),y)):
          My:=-EI*(nu*diff(diff(w,x),x)+diff(diff(w,y),y)):
          P:=150:a:=36:b:=36:
          plot3d(Mx,x=0..a,y=0..b,axes=BOXED,shading
          =Z,scaling=UNCONSTRAINED,title=`Bending Moment
          (lbs-in^2/in^2)`);
          x:=a/2:y:=b/2:momentMAX:=evalf(Mx);x:='x':y:='y':
          a:='a':b:='b':P:='P':
```

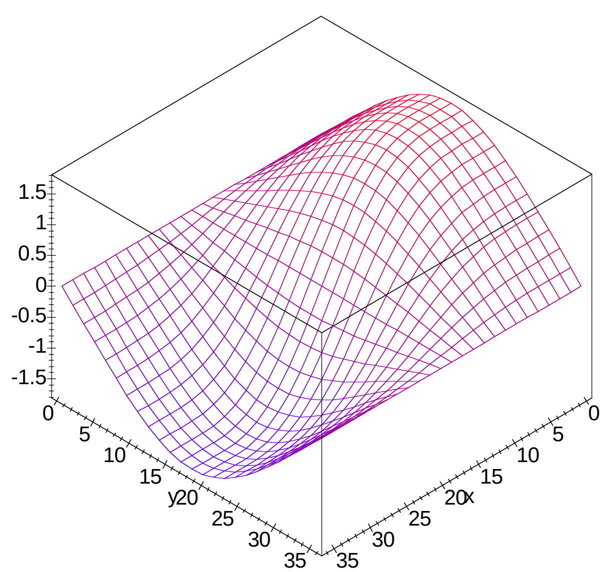
Bending Moment (lbs-in^2/in^2)



momentMAX := 15.84135659

```
STUDENT > Qx:=-EI*diff(Lw,x):
          Qy:=-EI*diff(Lw,y):
          P:=150:a:=36:b:=36:
          plot3d(Qx,x=0..a,y=0..b,axes=BOXED,shading
          =Z,scaling=UNCONSTRAINED,title=`Shear Force (Qx,lbs/in)`);
          x:=0:y:=b/2:shearMAX:=evalf(Qx);x:='x':y:='y':
          a:='a':b:='b':P:='P':
```

Shear Force (Qx,lbs/in)

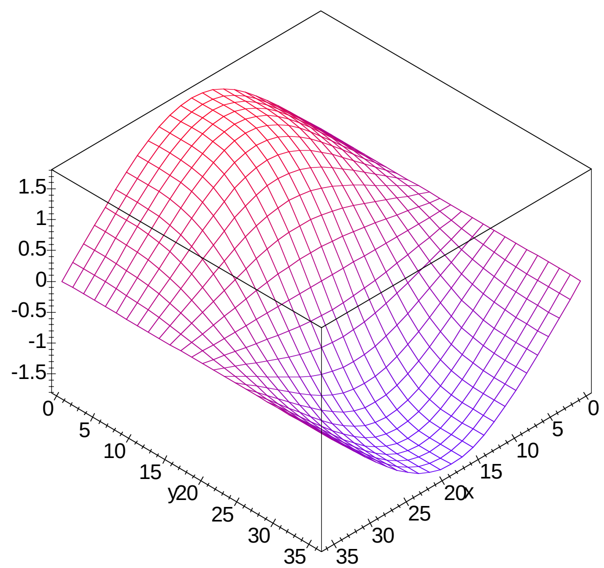


shearMAX := 1.732823182

```
STUDENT > P:=150:a:=36:b:=36:
          plot3d(Qy,x=0..a,y=0..b,axes=BOXED,shading
          =Z,scaling=UNCONSTRAINED,title=`Shear Force (Qy,lbs/in)`);
```

```
a:='a':b:='b':P:='P':
```

Shear Force (Qy,lbs/in)



Reset the parameters of the problem so that we can write out stresses in general terms

```
STUDENT > Y:='Y':h:='h':nu:='nu':
```

3. The model stresses

The in-plane stresses

```
STUDENT > sigma1:=-Y/(1-nu^2)*(diff(diff(DZ(x,y),x),x)+nu*diff(diff(
          DZ(x,y),y),y))*z;
          sigma2:=-Y/(1-nu^2)*(nu*diff(diff(DZ(x,y),x),x)+diff(diff(
          DZ(x,y),y),y))*z;
          tau:=-Y/(1+nu)*diff(diff(DZ(x,y),x),y)*z;
```

$$\sigma1 := -\frac{Y\left(\left(\frac{\partial^2}{\partial x^2}\mathrm{DZ}(x,y)\right)+\nu\left(\frac{\partial^2}{\partial y^2}\mathrm{DZ}(x,y)\right)\right)z}{1-\nu^2}$$

$$\sigma2 := -\frac{Y\left(\nu\left(\frac{\partial^2}{\partial x^2}\mathrm{DZ}(x,y)\right)+\left(\frac{\partial^2}{\partial y^2}\mathrm{DZ}(x,y)\right)\right)z}{1-\nu^2}$$

$$\tau := -\frac{Y\left(\frac{\partial^2}{\partial y\,\partial x}\mathrm{DZ}(x,y)\right)z}{1+\nu}$$

General expressions needed for the out of plane stresses

```
STUDENT > LW:=diff(diff(DZ(x,y),x),x)+diff(diff(DZ(x,y),y),y);
          LLW:=diff(diff(LW,x),x)+diff(diff(LW,y),y);
```

$$LW := \left(\frac{\partial^2}{\partial x^2}\mathrm{DZ}(x,y)\right) + \left(\frac{\partial^2}{\partial y^2}\mathrm{DZ}(x,y)\right)$$

$$LLW := \left(\frac{\partial^4}{\partial x^4}\mathrm{DZ}(x,y)\right) + 2\left(\frac{\partial^4}{\partial y^2\,\partial x^2}\mathrm{DZ}(x,y)\right) + \left(\frac{\partial^4}{\partial y^4}\mathrm{DZ}(x,y)\right)$$

The out of plane stresses

```
STUDENT > tau1:=-Y/2/(1-nu^2)*diff(LW,x)*((h/2)^2-z^2);
          tau2:=-Y/2/(1-nu^2)*diff(LW,y)*((h/2)^2-z^2);
          sigma3:=Y/6/(1-nu^2)*LLW*(2*(h/2)^3+3*(h/2)^2*z-z^3);
```

$$\tau 1 := -\frac{1}{2}\,\frac{Y\left(\left(\frac{\partial^3}{\partial x^3}\mathrm{DZ}(x,y)\right) + \left(\frac{\partial^3}{\partial y^2\,\partial x}\mathrm{DZ}(x,y)\right)\right)\left(\frac{1}{4}h^2 - z^2\right)}{1-\nu^2}$$

$$\tau 2 := -\frac{1}{2}\,\frac{Y\left(\left(\frac{\partial^3}{\partial y\,\partial x^2}\mathrm{DZ}(x,y)\right) + \left(\frac{\partial^3}{\partial y^3}\mathrm{DZ}(x,y)\right)\right)\left(\frac{1}{4}h^2 - z^2\right)}{1-\nu^2}$$

$$\sigma 3 := \frac{1}{6}\,\frac{Y\left(\left(\frac{\partial^4}{\partial x^4}\mathrm{DZ}(x,y)\right) + 2\left(\frac{\partial^4}{\partial y^2\,\partial x^2}\mathrm{DZ}(x,y)\right) + \left(\frac{\partial^4}{\partial y^4}\mathrm{DZ}(x,y)\right)\right)\left(\frac{1}{4}h^3 + \frac{3}{4}h^2 z - z^3\right)}{1-\nu^2}$$

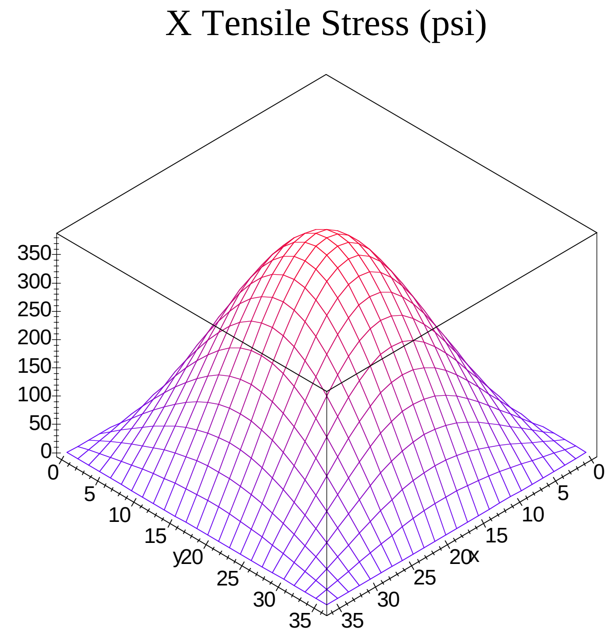
To apply all of this we need to put in the actual expression for w

```
STUDENT > DZ(x,y):=w:
```

The in plane stresses take on their maximum values on the two surfaces. Let's look at the lower surface, z = -h/2, where the normal stresses will be tensile, and plot the normal stress (it doesn't matter which because they are symmetric)

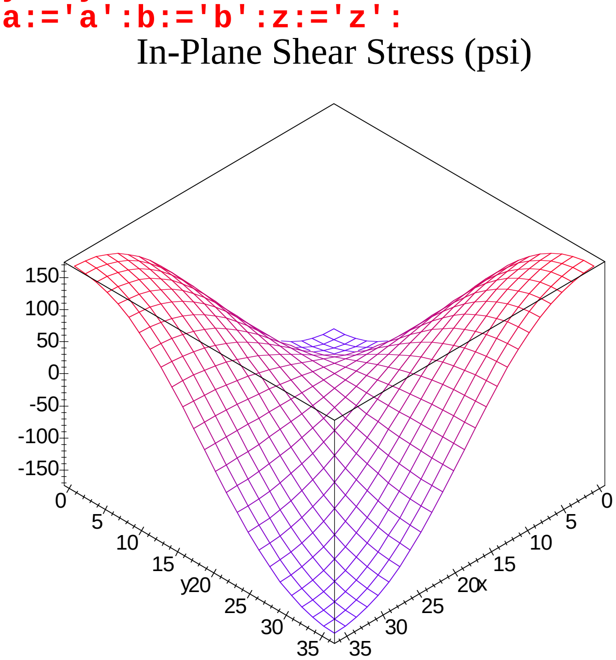
```
STUDENT > Y:=10500000:nu:=334/1000:h:=1/2:
```

```
STUDENT > s1:=eval(sigma1):
          P:=-150:a:=36:b:=36:z:=-h/2:
          plot3d(s1,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`X
          Tensile Stress (psi)`);x:=a/2:y:=b/2:sigmaMAX:=evalf(s1);
          x:='x':y:='y':
          P:='P':a:='a':b:='b':z:='z':
```



$sigmaMAX := 380.1925581$

```
STUDENT > t:=eval(tau):
          P:=-150:a:=36:b:=36:z:=-h/2:
          plot3d(t,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`In-Plan
          e Shear Stress (psi)`);x:=a:y:=b:tauMAX:=evalf(t);
          x:='x':y:='y':
          P:='P':a:='a':b:='b':z:='z':
```

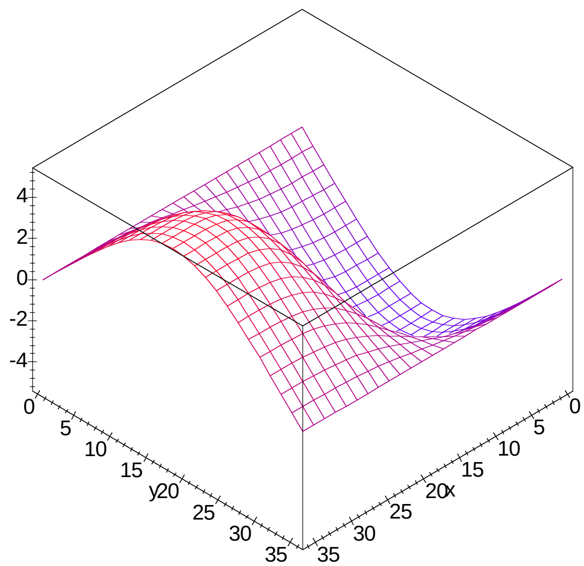


$tauMAX := -167.4073231$

```
STUDENT > t1:=eval(tau1):
          P:=-150:a:=36:b:=36:z:=0:
          plot3d(t1,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`XZ
          Shear Stress (psi)`);x:=a:y:=b/2:tauxzMAX:=evalf(t1);
```

```
x:='x':y:='y':
P:='P':a:='a':b:='b':z:='z':
```

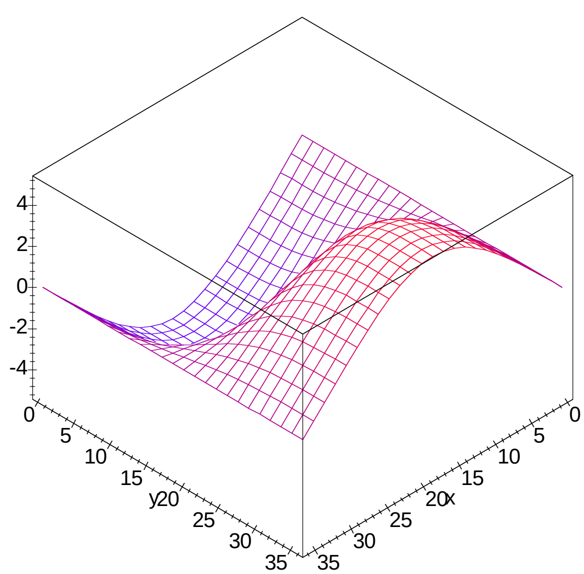
XZ Shear Stress (psi)



tauxzMAX := 5.198469547

```
STUDENT > t2:=eval(tau2):
          P:=-150:a:=36:b:=36:z:=0:
          plot3d(t2,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`YZ
          Shear Stress (psi)`);x:=a/2:y:=b:tauyzMAX:=evalf(t2);
          x:='x':y:='y':
          P:='P':a:='a':b:='b':z:='z':
```

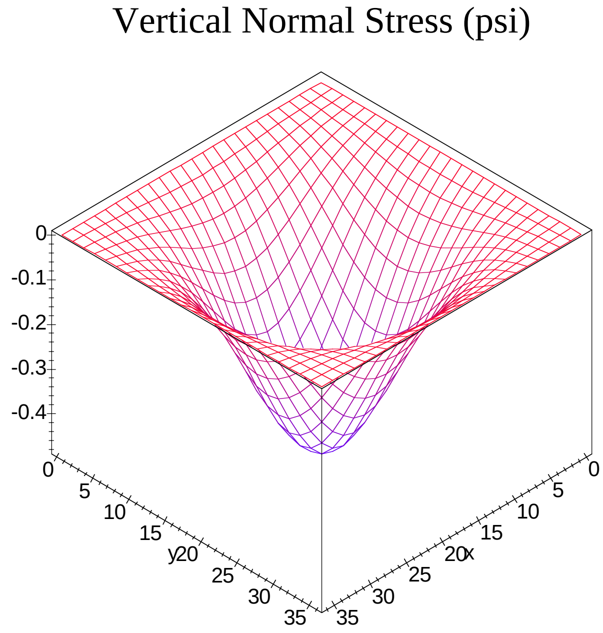
YZ Shear Stress (psi)



tauyzMAX := 5.198469547

```
STUDENT > s3:=eval(sigma3):
          P:=-150:a:=36:b:=36:z:=h/2:
```

```
plot3d(s3,x=0..a,y=0..b,axes=BOXED,shading=Z,title=`Vertic
al Normal Stress
(psi)`);x:=a/2:y:=b/2:sigmazMAX:=evalf(s3);
x:='x':y:='y':
P:='P':a:='a':b:='b':z:='z':
```



$$sigmazMAX := -.4803374630$$

Note that the z normal stress is a few tenths of a psi, the transverse shear stresses are a few psi, and the in plane stress are a few hundred psi. This is not strictly factors of psilon, but certainly of the appropriate order, showing a clear separation between the in-plane stresses, the transverse shear stresses, and the transverse normal stress that matches the applied load.

```
STUDENT > save `example 7.1m`:
STUDENT >
```