1 Chapter 13 review exercises

13.2

Option (i) is better, as the chance that you will win $1 are diminished by the requirement in (ii) that the second card be a queen: $\frac{4}{52} > \frac{4}{52} \times \frac{4}{51}$.

13.4

The chance that the first four cards are aces and the fifth is a king is very small in this setup (note that there is no replacement here):

$$\frac{4}{52} \times \frac{3}{51} \times \frac{2}{50} \times \frac{1}{49} \times \frac{4}{48} = \frac{3.07}{10,000,000}.$$

13.5

Color and number are independent. Recall the definition for independence, which states that the following must hold:

$$\Pr(1|\text{Black}) = \Pr(1).$$

We can now check whether this equality holds or not. First, find Pr(1):

$$\Pr(1) = \frac{\text{number of tickets with a 1 on them}}{\text{total number of tickets}} = \frac{4}{6} = \frac{2}{3}.$$

Next, find Pr(1|Black):

$$\Pr(1|\text{Black}) = \frac{\Pr(1 \text{ and Black})}{\Pr(\text{Black})} = \frac{\frac{2}{6}}{\frac{3}{6}} = \frac{2}{3}.$$

Thus, we find that

$$\Pr(1|Black) = \frac{2}{3} = \Pr(1),$$

and we can similarly show that

$$\Pr(1|\text{White}) = \Pr(1),$$
$$\Pr(8|\text{Black}) = \Pr(8),$$
$$\Pr(8|\text{White}) = \Pr(8).$$

Therefore color and number are independent.

13.8

(a) The chance that all four rolls show 3 or more spots is $(\frac{4}{6})^4 \approx 0.20$.

(b) The chance that none of the rolls show 3 or more spots is $(\frac{2}{6})^4 \approx 0.0123$.

(a) The chance that not all the rolls show 3 or more spots is $1-(\frac{4}{6})^4 \approx 0.80$.

13.10

For option (i) if you get a head, you gain a dollar and if you get a tail, you lose a dollar. The expected value from (i) is then $100 * (\frac{1}{2}(1) + \frac{1}{2}(-1)) = 0$. For option (ii) you get a dollar if you draw a 1 and you neither gain nor lose a dollar for drawing a 0, so the expected value is $100 * (\frac{1}{2}(1) + \frac{1}{2}(0)) = 50$. So option (ii) is the better choice.

2 Chapter 14 review exercises

14.2

There are two ways to get 11 with two dice: 5,6 or 6,5. The sum of their probabilities is:

$$\frac{1}{36} + \frac{1}{36} = \frac{2}{36}.$$

14.3

(a) False. The three events described are not disjoint, so that we cannot simply add the probability of getting an ace as done in the question. The modified addition rule has to be used to find the correct probability. Alternatively, note that Pr(an ace is drawn at least once) = 1 − Pr(an ace is never drawn). Hence,

$$\begin{aligned}
\text{Pr(an ace is drawn at least once)} &= 1 - \text{Pr(an ace is never drawn)} \\
&= 1 - \text{Pr(each time 2, 3, 4, 5, or 6 is drawn)} \\
&= 1 - (5/6)^3 \\
&= \frac{91}{216} < \frac{1}{2}.
\end{aligned}$$

(b) False. The chance of getting at least one head is lower than 100%, namely 75% (one of the four possible outcomes, (T,T), contains no heads).

14.7

$$
\begin{aligned}
\Pr(2 \text{ is drawn at least once}) &= 1 - \Pr(2 \text{ is never drawn}) \\
&= 1 - \Pr(\text{each time a 1 or 3 is drawn}) \\
&= 1 - (3/5)^4 \\
&= 0.8704.
\end{aligned}
$$

14.9

(a)

$$\Pr(a = 2, b = 1) + \Pr(a = 3, b = 1) + \Pr(a = 3, b = 2)$$

$$\frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} = 3 * \frac{1}{12} = \frac{1}{4}$$

(b)

$$\Pr(a = 1, b = 1) + \Pr(a = 2, b = 2) + \Pr(a = 3, b = 3)$$

$$\frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} = 3 * \frac{1}{12} = \frac{1}{4}$$

(c)

$$\Pr(a = 1, b = 2) + \Pr(a = 1, b = 3) + \Pr(a = 1, b = 4) + \Pr(a = 2, b = 3)$$
$$+ \Pr(a = 2, b = 4) + \Pr(a = 3, b = 4)$$

$$\frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} + \frac{1}{3} * \frac{1}{4} = \frac{6}{12}$$

14.11

(a) The chance that all three cards are diamonds is

$$\frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} = 0.012941$$

(b) The chance that none of the cards are diamonds is

$$\frac{39}{52} \times \frac{38}{51} \times \frac{37}{50} = 0.413529$$

(c) The chance that not all the cards are diamonds is

$$1 - \frac{13}{52} \times \frac{12}{51} \times \frac{11}{50} = 1 - 0.012941 = 0.987059$$

3 Chapter 16 review exercises

1. (iii) is correct. Remember, chance error is expected. So the absolute number of 1's will not be exactly the number in the population (or the box). However, even though there will be chance error, the size of chance error relative to the number of repetitions decreases as one repeats the experiement more and more. Therefore, (iii) is the best answer.

4. (a) 60 rolls give you a better chance of winning the dollar. According to the law of averages, in the long run we expect the percentage of rolls of a fair die that show an ace (one dot) to be ever closer to 1/6. Thus, the more often you roll the die, the less likely it becomes that an ace will show up more than 20% of the time, so we choose the option with fewer rolls.
   (b) Here 600 rolls are better. Over time, i.e., with more rolls, the percentage of rolls that yield an ace will converge to 1/6, and as $1/6 = 16.7\% > 15\%$, more rolls make it increasingly likely that you'll win the dollar.
   (c) Again, you should go for 600 rolls, as based on the law of averages we expect that the longer the run, the closer the fraction of observed aces will be to 1/6.
   (d) Here you are more likely to win the dollar with 60 rolls. Saying that you win a dollar if the percentage of aces is exactly equal to $16\frac{2}{3}\% = \frac{1}{6}$ means that in 60 rolls you'd need exactly 10 aces, while in 600 rolls you'd need exactly 100.

Obtaining the exact number of the one outcome you are interested in declines the more often you repeat the experiment. What converges to 1/6 is the *fraction* of "successful" outcomes, but *not the absolute number* of these outcomes. As an analogy, just think of how likely it is that in 10 tosses of a coin you'll obtain 6 heads (1 above the expected value of 5) as opposed to having exactly 5001 heads in 10000 tosses.

6. As the number of couples in the sample becomes larger, the closer the percentage of families with two children of the same sex should be to 50%, according to the law of averages. In both (i) and (ii), the percentage of couples with same-sex children is at least 67%, greater than the expected value. We would expect larger percentage error in a smaller sample, so (i) is more likely.

7. The score will be like the sum of 25 draws from the box

$$\boxed{4} \quad \boxed{-1} \quad \boxed{-1} \quad \boxed{-1} \quad \boxed{-1}$$

We are dealing with 25 random draws with replacement. For each draw, there are five possibilities: four wrong answers (any of which would subtract 1 from the student's score) and one right answer (which would add 4 to the score).

10. (a) If the sum is 30, the average is $30/200 = 0.15$.

(b) If the sum is -20, the average is $-20/200 = -0.1$.

(c) Recall that the definition of the average is

$$\bar{X} = \frac{1}{N} \sum_{i=1}^{N} X_i.$$

Therefore, if 200 draws are made and we are given their sum, the average is the sum divided by 200.

(d) These bets are the same. If the sum is between -5 and 5, then the average by definition will be between -0.25 ($= -5/200$) and 0.25 ($= 5/200$).