

Democracy and the Logic of Political Survival[§]

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Abstract

Although democracy is a key concept in political science, debate continues over definitions and mechanisms. Bueno de Mesquita, Smith, Siverson, & Morrow (2003) make the important claim that most of democracy's effects are in fact due to something conceptually simpler and empirically easier to measure than democracy: the size of the minimum winning coalition that selects the leader. The argument is intuitively appealing and supported by extensive data analysis. Unfortunately, the statistical technique they use induces omitted variable bias into their results. They argue that they need to control for democracy, but their estimation is equivalent to omitting democracy from their analysis. When we reestimate their regressions controlling for democracy, most of their important findings do not survive. As a final check, we replicate their analysis using more appropriate measures derived by Powell (2000). The results again fail to support the predictions of the selectorate model.

Democracy is a central concept in political science. Scholars have linked it to patterns of war and peace, alliance formation, conflict escalation, success in war, political stability, economic development, and improvements in a wide array of social indicators. These claims are subject to challenge, but the existence of a substantial literature on each of them underlines the importance of the questions. What, however, is the mechanism by which democracy generates these benefits? Is it a matter of norms, behavior, or institutions? Although democracy is a key concept, there is no consensus regarding its theoretical underpinnings.

It is for these reasons that the appearance of *The Logic of Political Survival* (Bueno de Mesquita et al. 2003) was hailed as a breakthrough. The book provides a simple explanation for the effects of democratic institutions: democratic leaders face different incentives than autocrats because democracies have different reselection institutions. The selectorate theory holds that political power depends upon satisfying a winning coalition (W) of the relevant selectorate (S). As the size of the winning coalition increases, leaders shift public policy from providing private benefits to supporters to providing public goods to the masses. Coalitions are most loyal when W is small relative to S . As the ratio W/S increases, defection becomes less risky, and leaders must spend more to keep their coalitions satisfied.

If correct, the selectorate theory has important implications for the long-standing debate over whether democracy should be understood primarily in terms of attitudes and behavior (e.g. Almond & Verba 1963, Dahl 1971, Putnam 1994, Przeworski, Alvarez, Cheibub, & Limongi 2000) or in terms of institutions (e.g. Shugart & Carey 1992, Cox 1997, Lijphart 1999, Powell 2000). According to the theory, the differences between democratic and authoritarian politics and the differences in politics among democracies are reducible to institutional incentives; thus, institutions and not behaviors are the crux of the matter.

Bueno de Mesquita et al. test the predictions of their model by regressing a wide range of dependent variables against winning coalition size (W) and its ratio to the size of the selectorate (W/S). W and S are derived from commonly used measures of democracy, notably the Polity IV score, and are highly correlated with democracy ($r = 0.8$).

Consequently, the authors are sensitive to the criticism that these variables are no more than proxies for democracy. They attempt to show that the effects of these variables are robust to models that include democracy as a control. In fact, one of their key findings is that these variables outperform democracy when included in equations containing democracy.

We demonstrate that these conclusions are incorrect. When controlling for democracy, the size of a polity's selectorate and winning coalition are often not associated with the decisions made by leaders or are associated in ways that contradict the model's predictions. We show that Bueno de Mesquita et al. are misled due to their use of an estimation technique that induces omitted variable bias. This technique, called residualization, exaggerates the importance of their key variables by increasing their coefficients and decreasing their standard errors. When we reestimate the main tables from Chapters 4 and 5 using the correct estimator, their most important findings do not survive. We show that the apparent strength of winning coalition size, for example, is an artifact of the biased estimator and is caused by the correlation between W and democracy.

Bueno de Mesquita et al. are quick to emphasize the “crude and primitive” nature of their measures, although they argue that their approximations “should, however, be adequate to evaluate whether the central tendencies of politics are aligned with the expectations that follow from the selectorate model” [133]. We reestimate their results using a more appropriate measure of W/S . Again, we find that the results do not support the expectations of their model.

The Problem

To give a flavor for Bueno de Mesquita et al.'s results and what is incorrect about their analysis, we initially focus on a single example, public expenditures. The selectorate model implies that institutions that call for large winning coalitions should be associated with more effort to produce public goods, and the authors expect this to be reflected in the level of public expenditures. They write, “when the coalition is small, government expenditures are expected to be relatively small, and when the coalition is larger, so are government expenditures. Small-coalition leaders can survive spending a smaller quantity of what they collect than can large-coalition leaders” (Bueno de Mesquita et al. 2003, 161-162).

The authors test this proposition by regressing expenditures as a proportion of GDP on W , the size of the minimum winning coalition, and separately on W/S , minimum winning coalition size divided by the size of the selectorate, and the results support their expectations. The authors argue that this specification is incomplete; democracies, for example, might spend more than autocracies for reasons other than the logic of the selectorate model. At the same time, governments in wealthier countries are able to spend a greater percentage of GDP. Thus, Bueno de Mesquita et al. run additional regressions with these variables as controls, and it is here that their analysis goes off track. Instead of controlling for democracy and GDP, they attempt to control for these effects by using the residuals from two other regressions. The first auxiliary equation regresses democracy on W and S , and the second regresses the natural logarithm of GDP on W and S .¹

This technique, substituting residuals for control variables, serves to exaggerate the size of the estimated coefficients on W and W/S because both democracy and the log of GDP are positively correlated with W . The first column of Table 1 replicates Model 2 in Table 4.6 of Bueno de Mesquita et al. (2003, 163). The coefficient for W is in the expected

Table 1: Expenditures, $n = 2373$

Variable	Residuals		Controls	
	Coef.	s.e.	Coef.	s.e.
W	2.32	(0.99)	-4.47	(1.74)
S	3.10	(0.85)	4.35	(0.87)
ln(Population)	-1.90	(0.13)	-1.90	(0.13)
Democracy residuals	2.83	(1.15)		
ln(GDP) residuals	1.07	(0.20)		
Democracy			2.83	(1.15)
ln(GDP)			1.07	(0.20)
Constant	53.89	(2.19)	47.49	(2.69)

direction—larger coalitions exhibit higher levels of expenditures—and it is statistically significant ($p = 0.02$). “Societies with high per capita incomes spend more,” they write. “However, even after controlling for these effects (or the independent, positive, and significant impact of democracy), large coalition size and a weak loyalty norm are still powerful indicators of higher government spending” [163]. In the second column, we replace the residuals of democracy and GDP with the original variables. The results no longer support the selectorate model. The coefficient on W changes sign—larger coalitions are associated with lower levels of expenditures—and it is statistically significant ($p = 0.01$). We show that these results are consistent throughout Chapters 4 and 5; the broad pattern of evidence that Bueno de Mesquita et al. find is an artifact of their biased estimation procedure. The strong effects that they attribute to W or W/S are in fact due to their correlation with democracy and the log of GDP. When these two variables are included in the specifications, the effects of W and W/S disappear, change signs, or are substantially attenuated.

The technique used by Bueno de Mesquita et al. induces omitted variable bias. The procedure, known as “residualization,” comes from a suggestion by Goldberger (1964, 196) and is well-known in certain areas of American politics, although recent uses, such as

Hicken, Satyanath, & Sergenti (2005), cite Bueno de Mesquita et al. (2003) directly.

Intuitively, the problem with residualization is that the shared variation between, for example, W , *democracy*, and the dependent variable is added to the shared variation between W and the dependent variable. The effect is to bias the estimated coefficient on W and lower the standard error. The danger of using this technique becomes clear when we realize that the effect on W 's estimated coefficient *is precisely the same as if we had omitted democracy from the regression altogether.*

For the sake of notational simplicity, let us assume a model with only two independent variables: $\mathbf{y} = \beta_0 + \mathbf{w}\beta_w + \mathbf{d}\beta_d + \boldsymbol{\epsilon}$, where \mathbf{w} is *winning coalition size*, and \mathbf{d} is *democracy*. As Bueno de Mesquita et al. are concerned with the correlation between *winning coalition size* and *democracy*, they replace *democracy* in this equation with the residuals from a regression of *democracy* on *winning coalition size*: $\mathbf{y} = \delta_0 + \mathbf{w}\delta_w + \hat{\boldsymbol{\nu}}\beta_d + \boldsymbol{\epsilon}$, where $\hat{\boldsymbol{\nu}}$ are the residuals from the auxiliary regression, $\mathbf{d} = \gamma_0 + \mathbf{w}\gamma_w + \boldsymbol{\nu}$. (We have left the coefficient on $\hat{\boldsymbol{\nu}}$ as β_d to emphasize that the residualization technique does not change this coefficient.) The expected value of this estimator, under the usual assumption that the expectation of the error term is zero, $E[\boldsymbol{\epsilon}] = \mathbf{0}$, is $E[\hat{\delta}_w] = \delta_w + (\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'\mathbf{d}\beta_d$. The bias on the estimated coefficient on W is therefore

$$B(\hat{\delta}_w; \delta_w) = E[\hat{\delta}_w] - \delta_w = (\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'\mathbf{d}\beta_d. \quad (1)$$

Equation (1) is precisely the bias that would result if *democracy* were left out of the specification completely (Greene 2003, 148-49). The residualization procedure amounts to generating omitted variable bias on the coefficient of interest while still getting an unbiased estimate on the residualized variable. (The authors could have achieved an unbiased estimate of W by residualizing W instead of *democracy*.) The formal derivation of equivalence between this estimator and omitted variable bias is in Appendix A.

This equivalence cannot be observed directly in the results Bueno de Mesquita et al. report. When they move from models that include only W or W/S to models that include residualized variables, they include additional covariates, such as $\log(\text{population})$. We can observe the equivalence, however, if we compare a regression of *expenditures* on W and *democracy residuals* to a regression of *expenditures* on just W . Table 2 shows that the estimated coefficients on W are precisely the same in both cases.

Table 2: Expenditures, $n = 2500$

Variable	Residuals		Controls	
	Coef.	s.e.	Coef.	s.e.
W	7.76	(0.954)	7.76	(0.959)
Democracy residuals	-9.03	(1.636)		
Constant	23.43	(0.683)	23.43	(0.687)

Democracy is not the only variable for which Bueno de Mesquita et al. substitute residuals. They use, in various specifications, the “residual impact” of GDP, civil liberties, political rights, investment, and savings. How these biases interact with one another is demonstrated easily with an example. Consider the results in Table 3, in which we replicate and reestimate the results of a regression of *savings* on W and controls (the replication is of model 2 in Bueno de Mesquita et al. (2003, 144)’s Table 4.1).

Table 3: Savings, $n = 3942$

Variable	Residuals		Controls	
	Coef.	s.e.	Coef.	s.e.
W	12.64	(0.93)	-2.58	(1.51)
S	-1.43	(0.75)	5.69	(0.76)
ln(Population)	0.70	(0.13)	0.70	(0.13)
Democracy residuals	-7.74	(1.04)		
ln(GDP) residuals	6.24	(0.19)		
Democracy			-7.74	(1.04)
ln(GDP)			6.24	(0.19)
Constant	0.34	(2.10)	-39.94	(2.53)

When we substitute the actual controls for the residuals used by the authors, the estimated

coefficient on W changes sign. Equation (1) tells us why. Consider first the effect of *democracy residuals*. The estimated coefficient in the regression of *democracy* on W , $(\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'\mathbf{d}$, is 1.01. $\hat{\beta}_d$, reading off Table 3, is -7.74. The bias is therefore $(\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'\mathbf{d}\hat{\beta}_2 = 1.01 * -7.74 = -7.82$. Now consider the effect of *GDP residuals*. The estimated coefficient in the regression of *GDP* on W is 3.69. The effect of *GDP*, reading from the table, is 6.24. The bias in this case is $3.69 * 6.24 = 23.04$. The total bias is therefore $-7.82 + 23.04 = 15.22$, and the effect on the estimated W coefficient is $12.64 - 15.22 = -2.58$, which is precisely what we find in column three of Table 3. (The difference in the constants is a function of the different $\hat{\beta}$ s in the two equations and the “demeaning” of *democracy residuals* and $\ln(\text{GDP})$ residuals.)

These results cut through the arguments Bueno de Mesquita et al. advance in favor of residualization. The fundamental question is quite simple: does some measure of democracy belong in their specifications? If it is necessary to control for democracy or some function of democracy, then residualization is incorrect because it is equivalent to leaving democracy out altogether. On the other hand, if it is not necessary to control for democracy or some function of democracy, there is no reason to use residualization. The authors make it clear, however, that democracy does belong in their regressions; they repeatedly make a point of emphasizing the independent effects of W and W/S after controlling for democracy.

To be specific about their reasoning, Bueno de Mesquita et al., in a private communication, argue that democracy is a function of W , and residualization was used to deal with the resulting endogeneity. Residualization was used by Soper (1976) for the same reason. He was criticized by Swan (1978) and Kennedy (1982) and the same criticisms apply. Briefly, if democracy is a function of W , and the various dependent variables are functions of W , democracy, and other variables, then the system of equations is recursive (all variables in

the democracy equation plus some additional variables appear in the main equation). Recursive systems are problematic for linear regression only if the error terms are correlated. If the error terms are correlated, however, it makes no sense to include an estimate of the first error term (the democracy residuals) in the main equation. The error terms would remain correlated.

Bueno de Mesquita et al.'s concerns about the interdependence of democracy and W can be thought of in terms of multicollinearity, which was also a concern of Soper's [42]. In fact, the residualization technique has traditionally been thought of as a "fix" for multicollinearity (see Goldberger 1964). We ran multicollinearity diagnostics for all of our replications, and the results demonstrate that multicollinearity is not a problem in these data. (The full diagnostics are available from the authors upon request.) Thus, whether we think of the interrelatedness of winning coalition size and democracy as endogeneity or multicollinearity, multiple regression is the correct way to proceed.

Replication and Reestimation Results

Bueno de Mesquita et al.'s (2003) model is attractive because it generates a wide range of observable implications, which the authors use to conduct a series of ambitious statistical tests. Space does not allow us to replicate all of the results, so we focus on chapters 4 and 5, which are the two core chapters of the book. In Chapter 4, Bueno de Mesquita et al. (2003) test the implications of their model regarding taxation, labor, leisure, and total spending and "assess the implications of the theory regarding personal income, economic growth, and kleptocracy..." [129]. In chapter 5, Bueno de Mesquita et al. (2003) test the implications of their model for the provision of "core public goods" (civil liberties, political rights, transparency, and peace), "general public goods" (public health, education, social

security, and international trade), and private goods (black markets, construction, and corruption). We replicate and reestimate the results in these two chapters, and we present them briefly. Few of their important results survive reestimation when the residuals are replaced with the original variables.

Table 4 summarizes our results, which are categorized by dependent variable. For instance, when we reestimate the effect of W on savings, the resulting coefficient is no longer statistically significant and has, in fact, changed signs (last row, column 2). The results show that of the 70 coefficients reported by Bueno de Mesquita et al., 33 (47%) are either no longer significant (28 cases) or have changed signs and are significant (5 cases). In 64 of the 70 regressions (91%), use of the residualization technique exaggerated the substantive effect of coalition size. For example, the authors incorrectly conclude that W accounted for most of the cross-national variation in economic growth, civil liberties, political rights, literacy, and public health. The broad pattern of evidence that Bueno de Mesquita et al. marshal in support of their theory does not exist.

There are a few cases (investment, trade, the number of doctors, and immunity to measles) where the biased procedure makes the authors' results appear to be weaker than they should be. However, the results that the authors themselves indicate are the most important are overturned. These key findings include the results on government expenditures and economic growth in chapter 4 and the results on core public goods in chapter 5. These important results are overturned because these tests involve dependent variables that are correlated with democracy and thus are most strongly affected by omitted variable bias. Bueno de Mesquita et al. obtain their misleading results because W is correlated with democracy, and the biased procedure they use exploits this fact to inflate the apparent effect of W and decrease its standard error. Even in cases where W remains statistically significant, its effects turn out to be quite modest. The combination of W 's

Table 4: Reestimation Results for Chapters 4 and 5

Change in Coefficient	W Not Significant [†]	W Significant [‡]	W/S Not Significant [†]	W/S Significant [‡]
No Change or Larger		Doctors, Trade Measles Immunity % Invest		Trade % Invest
Smaller	Black Market Construction	Income ⁵ , Low Birth Wt. Female Secondary Ed. DPT Immunity LogIncome ^{4.1}	Black Market Construction	Female Secondary Ed. Low Birth Wt., Doctors Measles Immunity DPT Immunity LogIncome ^{4.1}
Smaller: by at least 50%	Beds Kleptocracy ^{4.8}	Political Rights ⁵ , Tax ^{2,5} Income ² , Death Rate Education Spending	Kleptocracy ^{4.8}	Tax, Income, Illiteracy Education Spending Death Rate
Smaller: by at least 75%	Health Spending Corruption	Political Rights ² Illiteracy Infant Mortality Life Expectancy	Health Spending Yrs. of Education Beds, Corruption Expenditures	Political Rights Life Expectancy Infant Mortality % Savings Consumption
Smaller: by at least 95%	Civil Liberties ^{2,5} Yrs. of Education Water			
Change of Sign	War, % Savings Consumption LogIncome ^{4.4} Econ. Growth Kleptocracy ^{4.7}	Expenditures War Social Security	Civil Liberties War, Water LogIncome ^{4.4}	Social Security Kleptocracy ^{4.7}

[†]Superscripts refer to the particular model when there is a question.

[‡]We treat 0.05 as the threshold for significance.

apparent invincibility—it seems to never fail a hypothesis test—and its inflated substantive effects lead the authors to conclude that coalition size is the core of the matter where democracy is concerned. What appeared to be a conceptually cleaner measure that outperformed democracy is really an artifact of a biased estimation technique.

A Better Measure

One possible interpretation of our findings is that the data collected by Bueno de Mesquita et al. are inadequate to test the theory. As W and S do not directly measure coalition size or selectorate size, it is possible both for the theory to be true and for W and W/S to have no significant effects. To test this conjecture, we reestimate Bueno de Mesquita et al.'s regressions using a theoretically more appropriate measure of one of their key variables, the ratio of coalition size to selectorate size, W/S .

W and S do not discriminate among the variety of political institutions in the way Bueno de Mesquita et al. desire. Their measure of the size of the selectorate, S , takes three values depending upon whether a legislature exists, is appointed, or is elected. Their measure of W takes five values and is constructed from three components of the polity score plus an indicator for military regimes. A maximum value means that the government is not a military regime, the executive is not selected in unopposed elections and is not hereditary, and there are stable political groups that compete for influence (Bueno de Mesquita et al. 2003, 134-135). These measures do not capture variations among democracies, and it is not clear that the variations among autocracies correspond to coalition sizes. Furthermore, these variables do not allow the authors to test the claims they make about discriminating the effect of W from that of W/S because the correlation between these variables is 0.99. For statistical purposes, these measures are identical.

Fortunately, an alternative measure of W/S exists. In a study of representation in democracies, Powell's (2000) theoretical interest is to determine how alternative electoral institutions affect the degree to which elections represent the preferences of the electorate in the political process. He constructs a variable that measures the proportion of the electorate that voted for parties that subsequently joined the governing coalition. This is a direct measure of coalition size divided by electorate size, or W/S . The data exist for twenty countries and elections ranging from 1967 to 1994. All twenty countries are advanced democracies, which represents a limitation of this test.

Table 5: Results when W/S is replaced by *Votegov*.

Table	Supports	Opposes	Insignificant	Insufficient
Table 4.1	LogIncome		% Invest % Savings	Consumption
Table 4.4			LogIncome	
Table 4.6		Expenditures		
Table 4.7		Kleptocracy		
Table 4.8				Kleptocracy
Table 5.1			Civil Liberties Political Rights	
Table 5.2			Tax	Income
Table 5.3		War		
Table 5.5		Education Spending Female Secondary Ed.	Yrs. of Education Illiteracy	
Table 5.6		Infant Mortality Measles Immunity DPT Immunity Social Security	Life Expectancy Death Rate Health Spending Doctors Beds Low Birth Wt. Water	
Table 5.7	Trade			
Table 5.8		Black Market	Corruption Construction	

The results in Table 5 were calculated using the Powell measure of representation in place of W/S . Of the 32 regressions we reestimate using *votegov*, only two provide evidence that

supports Bueno de Mesquita et al.'s hypotheses. On the other hand, ten provide statistically significant results that reject their hypotheses, and seventeen provide insignificant results. There was insufficient data to conduct a meaningful test in three cases.

Proponents of the selectorate model might object that Powell's measure captures voting and coalition-building behavior, rather than directly measuring institutional constraints. Thus, *votegov* may measure these constraints with error if leaders and voters do not form minimum winning coalitions. The model assumes, however, that leaders form minimum winning coalitions, and *votegov* is a good measure of W/S to the extent that this assumption is correct. If the effective electoral constraint is a function of the distribution of preferences—as the effectiveness of gerrymandering demonstrates—then the effective minimum winning coalition may vary over time within a political system.

Conclusion

Bueno de Mesquita and coauthors have made an important contribution by providing a theoretical model that generates numerous testable hypotheses and then subjecting them to a wide range of empirical tests. Our reanalysis indicates, however, that Bueno de Mesquita et al.'s enthusiasm about the predictive power of their key variables, W and W/S , is unwarranted. The authors are misled because their statistical evidence suffers from induced omitted variable bias. The authors do not control for democracy, but rather, they control for the residuals of democracy from auxiliary regressions of democracy on W or W/S . The effect of this residualization on their coefficients of interest is precisely that of omitted variable bias; their estimates of W and W/S are biased and inconsistent. The biased procedure generated false confirmations for half of their hypothesis tests and substantially exaggerated the substantive effects of their key variables 90% of the time.

The incorrect results are consistently the ones that they regard as the best tests of their theory. Thus, the main effects that they attribute to W and W/S are due to their correlation with other variables, and the apparent robustness of their results to the inclusion of a variety of control variables is attributable to the biased estimation procedure.

The authors acknowledge that their key variables are crude and do not directly measure the quantities of interest, but at the same time, they claim that the data are nevertheless adequate to test their claims. As a check, we repeated as many tests as possible using an alternative measure of coalition size as a proportion of the electorate carefully developed by Powell (2000). Again, we found virtually no evidence to support Bueno de Mesquita et al.'s hypotheses and substantial refuting evidence.

The Logic of Political Survival makes the arresting claim that it has isolated the key mechanism by which democracy generates its benefits, thereby resolving the debate between the advocates of institutions, behavior, and political culture. The empirical evidence, however, does not support this claim because the effects that they ascribe to coalition size are attributable to democracy. A lack of evidence does not, of course, mean that the selectorate theory is necessarily wrong. It is possible that confirmatory evidence awaits better measures of W and S or the extension of Powell's measure beyond advanced democracies. In the meantime, scholars will no doubt find Bueno de Mesquita et al.'s model a useful starting point, but they must build on it with the understanding that there is little empirical evidence to support it.

A Proofs

First, consider expectation and consistency. (We abstract from Bueno de Mesquita et al.'s model slightly by ignoring the panel data set-up and the additional control variables. Our conclusions are unaffected by these simplifications.) The true equation is

$\mathbf{y} = \beta_0 + \mathbf{w}\beta_1 + \mathbf{d}\beta_2 + \boldsymbol{\epsilon}$, where \mathbf{y} is an $n \times 1$ vector of observations on any number of dependent variables, \mathbf{w} is an $n \times 1$ vector of observations on *winning coalition size*, and \mathbf{d} is an $n \times 1$ vector of observations on *level of democracy*, and $\boldsymbol{\epsilon}$ is an $n \times 1$ vector of disturbances. Bueno de Mesquita et al. substitute the residuals, $\hat{\boldsymbol{\nu}}$, from a regression of *level of democracy* on *winning coalition size* ($\mathbf{d} = \mathbf{w}\gamma + \boldsymbol{\nu}$) for *level of democracy*,

$$\begin{aligned}\mathbf{y} &= \beta_0 + \mathbf{w}\beta_1 + \hat{\boldsymbol{\nu}}\beta_2 + \boldsymbol{\epsilon} \\ &= \beta_0 + \mathbf{w}\beta_1 + \mathbf{M}\mathbf{d}\beta_2 + \boldsymbol{\epsilon},\end{aligned}$$

where $\mathbf{M} = (\mathbf{I} - \mathbf{w}(\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}')$. The estimated coefficients are

$$\begin{aligned}\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} &= \begin{pmatrix} \mathbf{w}'\mathbf{w} & \mathbf{w}'\mathbf{M}\mathbf{d} \\ \mathbf{d}'\mathbf{M}\mathbf{w} & \mathbf{d}'\mathbf{M}\mathbf{d} \end{pmatrix}^{-1} \begin{pmatrix} \mathbf{w}' \\ \mathbf{d}'\mathbf{M} \end{pmatrix} [\mathbf{w}\beta_1 + \mathbf{d}\beta_2 + \boldsymbol{\epsilon}] \\ &= \begin{bmatrix} (\mathbf{w}'\mathbf{w})^{-1} & 0 \\ 0 & (\mathbf{d}'\mathbf{M}\mathbf{d})^{-1} \end{bmatrix} \begin{pmatrix} \mathbf{w}'\mathbf{w}\beta_1 + \mathbf{w}'\mathbf{d}\beta_2 + \mathbf{w}'\boldsymbol{\epsilon} \\ \mathbf{d}'\mathbf{M}\mathbf{w}\beta_1 + \mathbf{d}'\mathbf{M}\mathbf{d}\beta_2 + \mathbf{d}'\mathbf{M}\boldsymbol{\epsilon} \end{pmatrix}.\end{aligned}$$

Taking expectations, with $E[\boldsymbol{\epsilon}] = \mathbf{0}$,

$$E \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} (\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'\mathbf{d}\beta_2 \\ 0 \end{pmatrix}$$

because $\mathbf{d}'\mathbf{M}\mathbf{w} = 0$. The inconsistency of $\hat{\beta}_1$ follows from the fact that the bias on $\hat{\beta}_1$ is precisely that of omitted variable bias.

Second, consider the variance. The estimated coefficients are, from above,

$$\begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} = \begin{pmatrix} \beta_1 \\ \beta_2 \end{pmatrix} + \begin{pmatrix} (\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'\mathbf{d}\beta_2 + (\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'\boldsymbol{\epsilon} \\ (\mathbf{d}'\mathbf{M}\mathbf{d})^{-1}\mathbf{d}'\mathbf{M}\boldsymbol{\epsilon} \end{pmatrix}.$$

Thus, we can write the variance as

$$\begin{aligned} E \begin{bmatrix} \hat{\beta}_1 - E(\hat{\beta}_1) \\ \hat{\beta}_2 - E(\hat{\beta}_2) \end{bmatrix} \begin{bmatrix} \hat{\beta}_1 - E(\hat{\beta}_1) \\ \hat{\beta}_2 - E(\hat{\beta}_2) \end{bmatrix}' &= \begin{pmatrix} (\mathbf{w}'\mathbf{w})^{-1}\mathbf{w}'E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}']\mathbf{w}(\mathbf{w}'\mathbf{w})^{-1} \\ (\mathbf{d}'\mathbf{M}\mathbf{d})^{-1}\mathbf{d}'\mathbf{M}E[\boldsymbol{\epsilon}\boldsymbol{\epsilon}']\mathbf{M}\mathbf{d}(\mathbf{d}'\mathbf{M}\mathbf{d})^{-1} \end{pmatrix} \\ \text{Var} \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix} &= \begin{pmatrix} \sigma^2(\mathbf{w}'\mathbf{w})^{-1} \\ \sigma^2(\mathbf{d}'\mathbf{M}\mathbf{d})^{-1} \end{pmatrix}. \end{aligned}$$

We can compare the variance of the estimated coefficients from the Bueno de Mesquita et al. model to the variance of the estimated coefficients of the correctly specified model by comparing their inverses (Greene 2003, 150). Let the estimated coefficients from the correctly specified model be $\hat{\beta}_{i.c.}$. Then,

$$\begin{aligned} \text{Var} \begin{pmatrix} \hat{\beta}_1 \\ \hat{\beta}_2 \end{pmatrix}^{-1} - \text{Var} \begin{pmatrix} \hat{\beta}_{1.c} \\ \hat{\beta}_{2.c} \end{pmatrix}^{-1} &= \frac{1}{\sigma^2} \begin{pmatrix} \mathbf{w}'\mathbf{w} - [\mathbf{w}'\mathbf{w} - \mathbf{w}'\mathbf{d}(\mathbf{d}'\mathbf{d})^{-1}\mathbf{d}'\mathbf{w}] \\ (\mathbf{d}'\mathbf{M}\mathbf{d}) - (\mathbf{d}'\mathbf{M}\mathbf{d}) \end{pmatrix} \\ &= \frac{1}{\sigma^2} \begin{pmatrix} \mathbf{w}'\mathbf{d}(\mathbf{d}'\mathbf{d})^{-1}\mathbf{d}'\mathbf{w} \\ 0 \end{pmatrix}. \end{aligned}$$

As $(1/\sigma^2)\mathbf{w}'\mathbf{d}(\mathbf{d}'\mathbf{d})^{-1}\mathbf{d}'\mathbf{w}$ is nonnegative definite, the variance of $\hat{\beta}_1$ is never larger than the variance of $\hat{\beta}_{1.c.}$. Thus, the estimator is biased, and the variance is too small. The variance of $\hat{\beta}_2$ remains unaffected.

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Notes

¹In the text, Bueno de Mesquita et al. write, “To assess the independent impact of democracy on our dependent variables, we create a variable called *WS:DemRes* by regressing *W* and *S* on *Democracy* and then saving the residuals...” [137]. Their .do files, however, show that they actually regress *Democracy* on *W* and *S*.