The interplay between discourse patterns and curricular resources: The impact of dialogic tendencies on the activation of resources in a mathematics reform curriculum

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Abstract

This paper explores the relationship between the resources found in a National Science Foundation (NSF) funded mathematics curriculum and discursive practices in classrooms. Discursive practices are considered as indicative of the beliefs and practices students and teachers have related to the use of curricula and as evidence of the ways in which the curricular resources are being realized in classroom activity. The study explores the interaction between *hybrid* discourse practices and curricular resources, particularly resources found in reform curricula, in relation to opportunities students have to learn for understanding. I briefly analyze a conventional curriculum and a reform curriculum to illustrate the resources found in a reform curriculum and then present data from three classrooms using CMP along a continuum of discourse patterns. The results from the three classrooms suggest the resources in the CMP curriculum are partially realized even in cases when the discourse patterns resemble the recitation pattern. However, the results also suggest that when certain dialogic characteristics are present in the discourse patterns, students are able to take greater advantage of the resources. In particular, the aspect of metarepresentational competence that requires students to problematize the features and use of representations, and the outcome of ambiguous tasks in which students collectively generate a common understanding of the underlying mathematical concept, were not realized when dialogic features of discourse were absent. The results from the three cases suggest the possibility of a curriculum effect and a discourse effect.

Introduction

Pea (1993), in his articulation of distributed intelligence, and Wertsch (1998), in his discussion of the dialectic relationship between agent and tool, demonstrate the importance of artifacts in mediating activity. Artifacts, by conveying meaning and by serving as resources, influence the activity of people. Similarly, people, through the lens of their own beliefs, knowledge, and experiences, develop particular relationships with cultural artifacts that influence the potential outcomes involved in their use. The relationship between tool and agent applies to teachers and students in relation to curricula. Curricula convey cultural interpretations and representations of knowledge that serve to afford and constrain the development of particular forms of disciplinary knowledge. Similarly, people develop a set of beliefs and practices that influence the ways they use curricula, which is particularly evident when curricula are introduced that have different kinds of resources than have been historically present (Remillard & Bryans 2004; Sherin & Drake, 2005).

In this paper, I explore the relationship between the resources found in a National Science Foundation (NSF) funded mathematics curriculum and discursive practices in classrooms. I focus on discursive practices as indicative of the beliefs and practices students and teachers have related to the use of curricula and as evidence of the ways in which the curricular resources are being realized in classroom activity. The distinct culture in U.S. mathematics classrooms, which consists of students independently working on repetitive tasks based on established procedures (Stigler & Hiebert, 1999), has been challenged by the recommendations in the National Council of Teachers of Mathematics (NCTM) Standards documents (NCTM, 1989, 2000). These recommendations include suggestions for more active student roles in the classroom discourse, especially in the context of engagement in tasks that are set in a real-world context, promote problem solving, and help students to make connections. A substantial minority of classrooms in the U.S. are presently using curricula (henceforth termed *reform* curricula) designed with the NCTM Standards as a guiding document (The K-12 Mathematics Curriculum Center, 2005), and the teachers in many of those classrooms are establishing more active roles for students than those described by Stigler and Hiebert.

Evaluations of these curricula have generally shown an increase in students' conceptual understanding, but these evaluations have not sufficiently elaborated the conditions in which these curricula have been implemented, including the classroom practices (National Research Council, 2004). This raises some central questions, such as, what kinds of practices are developing in relation to the use of the reform curricula and do these practices sufficiently activate[1] the resources in the new curriculum. To answer these questions, I briefly analyze materials from two curricula, one that is a conventional and popular commercially developed curriculum, and one that is an NSF-funded (reform) middle school curriculum. I present this analysis to illustrate the kinds of resources found in reform curricula and in conventional curricula. Borrowing from the work of Voigt (1995), I note the occurrence of established patterns of discourse in mathematics classrooms. I pay attention in particular to the recitational pattern predominant in U.S. classrooms (Cazden, 2001) and to discourse patterns that have dialogic qualities that promote sense-making and interaction (Nystrand, 1997).

Discourse patterns

[1] I interchange the verbs activate and realize throughout the paper. Both of these verbs indicate the extent to which classroom practices are able to take advantage of the resources identified in the curriculum materials.

Drawing from an interactionist perspective developed for mathematics classrooms, Voigt (1985) theorized that stable discourse patterns emerge in mathematics classrooms, which he termed *patterns of interaction*. These regularities in the discourse serve to minimize the risk "of a collapse and disorganization of the interactive process" (Voigt, 1995, p. 178) and influence how students construct mathematical knowledge as well as their beliefs about what it means to learn mathematics (Wood, 1994).

Voigt (1985) characterizes patterns of interaction in the following way:

- The structure serves to reconstruct a specific regularity of interaction focusing on a topic;
- The structure refers to the concerted actions, interpretations, and mutual perceptions of at least two participants, and cannot be represented as a sum of individual activities;
- The structure cannot be deductively explained by the adherence of given rules in the sense of an implicit generative grammar;
- The subjects participating do not generate the regularity consciously and with a strategic purpose and do not think about it, but constitute it routinely.

Voigt (1985, 1995) observed patterns of interaction that indicate a perspective of mathematics as rule-bound, which constrains the manner in which student are able to reflect on their own thinking, and which Bauersfeld (1988) notes can easily lead to a misinterpretation of student learning. In particular, Voigt noted the elicitation pattern that "represents the structure of an interaction in mathematics instruction which mainly serves to constitute a result considered to be shared, but which does not ensure a shared understanding" (Voigt, 1985, p. 111). Bauersfeld discusses the impact of the elicitation pattern and the funneling pattern as creating an illusion that understanding has been accomplished, when in fact a teacher has not provided the opportunity for students to unpack their thinking. These patterns of discourse constitute particular “language games” unique to mathematics classrooms and which convey a cultural norm of what it means to know and do mathematics (Bauersfeld). Cobb et al. (1992), for example, distinguish between two discourse patterns, which they term ‘classroom mathematics traditions’, and their impact on the construal of mathematics by the students in the two classrooms. In one classroom, students posed questions and engaged in discussions about mathematics and problem solving strategies, while in the other classroom, students mainly mimicked the procedures described by the teacher.

A version of the elicitation pattern noted by Voigt and Bauersfeld has been well documented in U.S. classrooms (Cazden, 2001; Mehan, 1979). The typical pattern of interaction in U.S. classrooms can be characterized by IRE (initiate-respond-evaluate) or IRF (initiate-respond-feedback) sequences (Mehan, 1979; Wells, 1992). In an IRF sequence, the teacher initiates a query, a student responds, and teacher provides feedback to the response, often in the form of an evaluation. Typically this sequence involves short or known-answer questions associated with a transmissionist learning perspective (Nystrand, 1997; Wertsch & Toma, 1995). The monologic quality of the elicitation or IRF discourse patterns[2] provides little opportunity for collective sense-making and a concomitant lack of opportunity for students to participate in ‘epistemologically important’ roles (Nystrand).

Bakhtin (1986) uses the notion of dialogism to describe how meaning is generated from interactions in specific contexts. He describes dialogism as a struggle between

[2] I will term these patterns of discourse *recitation* or *recitation-style* patterns in the remainder of the paper.

conversants and competing voices, a tension which is evident in any utterance. Meaning is constantly evolving in any interaction, which suggests an active and subjective view of knowledge at odds with recitation-style discourse patterns typically found in US classrooms (Cazden, 2001). Discourse patterns that are more dialogic can be characterized by the presence of interactivity and uptake (Nystrand, 1997). In dialogic discourse, students participate actively in the construction of knowledge by offering and discussing explanations as a means to generate ideas in the classroom community: student explanations serve as ‘thinking devices’ for the class (Wertsch & Toma, 1995).

The discursive features of classrooms influence the ways in which students engage with mathematical content. Traditional discourse patterns leave little room for students to explain their reasoning and to engage in mathematical argumentation, limiting opportunities for students to make sense of and make connections between mathematical representations, procedures, and concepts. Patterns of discourse that are dialogic provide opportunities to generate ideas which then become the focal point for collective reflection: such opportunities help students to learn mathematics with understanding.

Learning mathematics with understanding

The NSF-funded curricula are designed to promote conceptual understanding through a comprehensive set of experiences. These curricula are designed to provide opportunities for students to make connections between concepts and procedures, between various representations, and between students’ informal thinking and conventional mathematical ideas: as such, they represent a substantive departure from traditional curricula (Trafton, Reys & Wasman, 2001).

In order to document the impact of curriculum and pedagogy, I provide a conception of learning that reflects research in cognitive science and mathematics education. I turn to Carpenter et al.'s (2004) conception of learning mathematics with understanding, which they define in terms of mental activity rather than static attributes (see also Carpenter & Lehrer, 1999; Hiebert et al, 1997). They propose four interrelated forms of activity from which understanding emerges: (a) constructing relationships, (b) extending and applying mathematical and scientific knowledge, (c) justifying and explaining generalizations and procedures, and (d) developing a mathematical disposition. Carpenter et al. state that people construct meaning for a new idea by relating to ideas they already understand. In order for knowledge to be generative (that is, for people to be able to generate new knowledge from what they have learned), it must be extended. In order for that to happen, people need to understand how that knowledge can be used, both in terms of applying it and in terms of seeing what new ideas are implicated. Justifying and explaining solutions provides opportunities for sense making and helps student to develop identities as mathematical thinkers. Finally, Carpenter et al. describe developing a mathematical disposition as having a sense of identity related to taking responsibility for making sense of mathematical and scientific knowledge.

Curricula that promote learning with understanding

Doyle (1983, 1984) noted that tasks focus students on particular concepts and representations, in effect structuring the learning activity of students. He distinguished between traditional mathematics tasks and more open-ended tasks in terms of how they offered different opportunities – and challenges – for teachers and students due to differences in cognitive demand and ambiguity (Doyle; Stein, Grover, & Henningsen, 1996). Doyle's discussion emphasizes that task resources provide opportunities for students to engage in particular forms of activity. This is an especially important point when comparing tasks found in conventional mathematics curricula and those found in reform curricula. The NSF-funded curricula structure student activity in ways substantively different from traditional curricula (Trafton, Reys, & Wasman, 2001). For example, many NSF-funded curricula provide opportunities for students to informally explore concepts before development of procedural competency, communicate as means to articulate their understanding and to contribute to intellectual development of classroom community, and to use multiple approaches and interpretations. These curricular characteristics require higher levels of cognitive demand due to the connections students must make and to the ambiguity involved in completing tasks (Stein et al.).

Brown (forthcoming) suggests that curriculum materials offer basic types of resources, including representations of tasks and representations of concepts. Representations of tasks include "descriptions of instructions, procedures, or scripts that are intended for enactment by teachers and students" (p. xx). Representations of concepts refer to "the depiction and organization of domain concepts and their relationships through means such as diagrams, models, explanations, descriptions, and analogies" (p. xx). The NSF-funded curricula are intended to provide different types of resources, both in terms of the base curriculum and the teacher resource materials. The NSF-funded curricula are meant to 'speak to' rather than 'speak through' teachers, in recognition of some of the perceived failures of the New Math era curricula (Ball & Cohen, 1996; Remillard, 2005). The teacher resources, for example, include the 'design rationale'

(Davis & Krajcik, 2005) of the curriculum, in which the authors articulate connections between concepts, student strategies that have been observed during the field tests of the materials, and an elaboration of the mathematical concepts a unit or task is intended to develop. As mentioned above, the resources in the base curriculum are intended to provide opportunities for students to make connections and to provide a sequence of experiences that help students build from informal to formal understandings of mathematical concepts.

I describe in greater detail below specific resources that I will highlight in my analysis of the NSF-funded curricula, particularly middle school curricula, as a means to contrast differences with conventional curricula and to elaborate the nature of these resources. These resources should be evident in the task and concept representations found in both the base curriculum and the teacher resource materials. I discuss resources that are associated with the activities that Carpenter et al. (2004) suggest are necessary to learn mathematics with understanding.

Making connections between procedures and concepts. The QUASAR researchers (Stein et al., 1996; Henningsen & Stein, 1997) distinguish between tasks that are ‘procedures without connections’ (PNC) and ‘procedures with connections’ (PWC). Tasks that are categorized as PWC “suggest pathways to follow (either explicitly or implicitly) that are broad general procedures that have close connections to underlying conceptual ideas” (Stein & Kim, 2006, p. 12), while tasks categorized as PNC are “narrow algorithms that are opaque with respect to underlying concepts.” An important design characteristic in an NSF-funded curriculum is that tasks provide opportunities for students to make connections and to understand the conceptual basis behind procedures and algorithms. The PSSM state more generally:

> When students make connections, their understanding is deeper and more lasting. They can see mathematical connections in the rich interplay among mathematical topics, in contexts that relate mathematics to other subjects, and in their own interests and experience. (p. 64)

Metarepresentational competence. The development of students' competencies regarding the creation, use, and interpretation of various representations has been a fundamental feature of the mathematics reform (NCTM, 1989, 2000). The term *metarepresentational competence* denotes the development of knowledge of the functions of different systems of mathematical inscription and notation, and the relations among them (Greeno & Hall, 1997). This terms implies that coming to understand representations involves more than simply reproducing representations that have already been modeled and whose features have already been determined, but rather the development of representations "involves learning to participate in the complex practices of communication and reasoning in which the representations are used" (Greeno & Hall, p. 362). Students should be encouraged to develop nonstandard forms to organize, communicate, and reflect on their thinking in relation to problem, in addition to coming to understand standard forms (Greeno & Hall; diSessa & Sherin, 2000).

A strong component of metarepresentational competence is the development of the ability to "select, produce, and productively use" various representational forms as well as the ability to "critique and modify representations and even to design completely new representations" (diSessa and Sherin, 2000, p. 386). Thus, developing metarepresentational competence goes beyond reproducing standard forms of representation, such as tables, graphs, and equations, but also requires that students understand and critique the use of these representations.

Ambiguity. A task that is ambiguous affords multiple pathways or multiple solutions that are not dictated in advance. Ambiguity serves two purposes: it increases the cognitive demand of a task (cf. the QUASAR task framework) and it provides an opportunity for diverse solutions that can be used to collectively negotiate meaning and expand student participation (Stroup, Ares, & Hurford, 2005). The QUASAR task framework has four categories, from rote application of facts to 'doing mathematics.' These categories are marked by an increase in the complexity of the task (the number of steps required to arrive at a solution), as well as the ambiguity of the task (the requirement that students make decisions that are not indicated in advance).

Tasks that have some ambiguity provide an opportunity to orchestrate "classroom activity in ways that occasion productive and creative engagement by participants, characterized by increased personal and collective agency"(Stroup, Ares, & Hurford, 2005, p. 188). Ambiguous tasks have some underlying mathematical similarity and offer opportunities for students to discuss approaches which are different but which serve to elaborate the underlying concept.

Analyses of a conventional curriculum and an NSF-funded curriculum

I explore task and concept representations found in materials from a commercially developed textbook and from an NSF-funded textbook. I explore these representations in light of the three characteristics elaborated above in order to articulate the resources (opportunities to engage in activities related to learning for understanding, view of mathematics) conveyed in each set of materials.

The first analysis will be based on materials from the McDougal Littell Math Course 2 (grade 7) textbook (Larson, Boswell, Kanold, & Stiff, 2007). This is a standard commercial textbook from one of the major textbook publishers. The second analysis will be based on materials from the Connected Mathematics Project (CMP) (Lappan et al., 1998), an NSF-funded curriculum based on the recommendations in the National Council of Teachers of Mathematics (NCTM) Standards documents (NCTM, 1989, 1991, 1995).

McDougal Littell Course 2 materials

I analyze two units from a version of the Course 2 book designed to prepare students for the New York state grade 7 mathematics assessment. The NY state version is organized to fit the sequence of topics on the NY state test, highlights the performance indicators that a particular section addresses, and provides test prep resources. The analysis of the units primarily focuses on the three characteristics elaborated above; however, I also discuss the view of mathematics suggested by the materials by looking at how tasks and concepts are represented, how context is used, and the emphasis on skills and procedures relative to problem solving.

I randomly selected two units from the Course 2 book. One unit was entitled “Investigating factors” and the other was entitled “Ratios and rates.” Each unit was comprised of a brief introduction of a topic, several examples followed by guided practice, and then a set of exercises. In the guided section, answers are listed just below the question and all terms are immediately defined. The exercises begin with about 40 to 50 problems of ‘Skills practice,’ followed by 5 to10 problems in “Problem solving.” The

“Skills” section includes several blocks of similar problems. The problem-solving section includes problems that require writing, applications, explicit connections to other mathematical topics, and a “challenge” problem meant to extend the learning of the topic. Most of these problems fit problems in the ‘Skills’ section closely once they are interpreted. The “Extended response” problem found in each of the “Problem solving” sections is the most ambiguous problem and the one most likely to engender connections.

Making connections. The examples and problems in the text make few explicit connections to other mathematical content. The connections to context tend are mainly superfluous in that understanding the context does not support understanding the mathematics. In some cases, the context can be completely ignored in order to solve the problem. In other cases, the procedure in an application problem closely matches one found in the skills section; consequently, the student may predict with some degree of certainty the nature of the procedure he or she will be using to solve the problem.

The most important sense of connection identified by the QUASAR task framework was whether tasks connected procedures to concepts. According to the QUASAR framework, most of the tasks in the skills section would be rated as ‘procedures without connections.’ The tasks closely follow a clearly defined procedure that is demonstrated on the page or the page before; there is no attempt to develop experiences in these tasks to connect factoring or ratios to underlying concepts (e.g. unique arrays or multiplicative reasoning, respectively). The concepts, if they are mentioned, are directly stated (versus being explored) or are explored in one of the tasks in the “Problem solving’ section.

Metarepresentational competence. There is a strong emphasis on numeric or symbolic representations, with little attempt to connect to geometric representations, even when the term *scale* is introduced. Even when a tabular representation was introduced in the "Ratios" unit, it was used as a means to correctly set up a symbolic representation. There is little opportunity for students to problematize how factors or ratios should be represented. Nearly all of the problems utilize only one representation and there is little opportunity to develop fluency between representations.

Ambiguity. As stated above, the tasks lend themselves to predictable procedures. The tasks in the 'Skills' section contain some complexity through the use of large numbers, negative numbers, or decimals, but the procedures are well-defined and follow the examples presented earlier in the section. A number of the tasks in the 'Problem solving' section also contain little ambiguity in that they can be solved by using one of the procedures emphasized in the 'Skills' section. Each 'Problem solving' section does, however, contain one or more problems that are ambiguous in that there is not an obvious procedure to solve them or they can be solved using multiple approaches. These problems typically occur at the very end of the unit.

The view of mathematics presented in the Course 2 materials is that of mathematics as a set of facts, rules, and procedures with mastery of these as the goal of student activity. The location of problem solving at the end of the unit and the emphasis in the introduction and early examples on the development of skills and procedures reinforces this view. The concept and task representations in the base curriculum emphasize efficient procedures and precise terminology. Students could use these representations as models for their own strategies when factoring or solving a ratio problem.

Other units in the Course 2 materials are strikingly similar in form to the ones described above. There is not a great deal of deviation in terms of the emphasis on procedure, the use of guided examples as explicit models to follow, the lack of connection between topics and from procedures to concepts, the use of context to generate interest rather than understanding, and the placement of ambiguous tasks toward the end of the unit.

CMP materials – Bits and Pieces II

CMP is a 3-year curriculum for grades 6-8 developed with funding from NSF. Like other NSF-funded curricula, it is designed to provide students with multiple opportunities to explore and formalize their understanding of key mathematical ideas within five major “strands” (numbers and operations, geometry, measurement, data analysis and probability, and algebra). The curriculum is organized into units, each comprising of 3-5 “investigations” where students explore a key mathematical concept or process. Each investigation begins with the presentation of a meaningful real-life problem/situation that embodies the mathematical concept/process under study (“launch”), followed by a combination of whole class and small group guided explorations, and concluding with a discussion in which the mathematical concept/process at the core of the investigation is explicitly identified and its understanding reinforced.

I selected the Bits and Pieces II (BP II) unit from the 6th grade series to analyze. This unit is designed to develop students’ understanding of operations with fractions. The investigations develop the notion of equivalent fractions and then proceed toward addition and subtraction of fractions.

Making connections. All of the investigations are set in context. Context is used to help students reason about the mathematical concepts, not simply to recruit students' interest in the problem. In the Tupelo investigation, for example, the context is two square miles of land (literally two squares whose length is 1 mile each) that are divided into irregular plots of land. The students are then asked to determine the fractional part each plot of land represents. The lack of realism and reference to agriculture in this problem suggest that the use of context is not meant to relate to most students' everyday experiences. Instead, the labeling of fractions with names and the idea of comparing farmers' plots provides resources for students to discuss and think about the fractions.

In terms of whether tasks connected procedures to concepts, the adding of fractions, for example, was not established initially as an algorithm or procedure presented to students. Instead, students, through the Tupelo activity, establish a number of ways of representing fractions equivalently. As students consider what happens when two farm plots are combined, they generate strategies for determining how to accomplish that addition. The adding of plots is tied to both the geometric representation as well as to multiple forms of equivalent fractions. Students have the opportunity to establish the need for equivalent fractions and to connect that need to a diagram and an activity. Ultimately, the curriculum is designed for students to establish the procedures for adding and subtracting fractions themselves.

Metarepresentational competence. The BP II unit requires students to move between geometric, numeric, verbal, and ultimately symbolic representations. There is an explicit attempt in the Tupelo investigation to relate the numeric generation of equivalent fractions and the adding of fractions to the diagram. In the following investigation, the connection is between numeric and verbal representations. The students have an opportunity to problematize how to write the fractions in the Tupelo problem (e.g. to select denominators for each plot of land) and to consider when fractions are equivalent and how one can make them equivalent.

Ambiguity. The tasks as written contain significant ambiguity. There is little indication of a specific way to approach a task and most of the tasks are accessible to a variety of approaches. Most of the tasks are ‘open-middled’ rather than ‘open-ended,’ meaning that there is one right answer but a number of ways of getting to that answer.

The view of mathematics in BPII is that of a sense making process that connects to experiences but that ultimately has rules and properties that govern its structure. The development of concepts through problem solving signifies the importance of sense-making as the means to arrive at mathematical understanding. The ambiguity of tasks creates an opportunity to negotiate a collective sense of how various solutions relate to each other and to some underlying idea. Thus, the social and collaborative aspects of mathematical activity are emphasized.

Other units in CMP contain qualities similar to BPII. A big difference can be found in units emphasizing algebra in which there is more use of graphical and symbolic representations. In these units, there is considerable effort to develop fluency between representations, though, like in BPII, there is not much of an opportunity to problematize the nature of particular representations. In almost all CMP investigations, tasks are set in context and the context is used to help students reason mathematically. Procedures are generally developed by students. This provides the opportunity for students to link procedures back to a foundational experience and to the underlying mathematical concepts.

Summary of curricular analyses

These brief analyses are intended to illustrate potential differences in terms of the kinds of resources that exist in the two curricula. The conventional curriculum provides resources for students to develop procedural fluency while the reform curriculum provides resources for students to make connections and to collectively negotiate mathematical understanding.

Interaction between curricula and discourse patterns

I consider four possibilities, some of which I discuss using studies reported in the literature and some of which I discuss using data from studies that I have conducted. The four possibilities are intended to represent the interaction between the two types of curricula and the two types of discourse patterns described above. The analysis is intended to consider the ways in which the discourse patterns influence the extent to which the curricular resources described above are realized in the process of enactment. Quadrant I represents the kinds of instruction and curriculum extensively documented in U.S. classrooms (cf. Jacobs et al., 2006; Schmidt, McKnight, & Raizen, 1997; Stigler & Hiebert, 1999). The curriculum presents mathematics as a series of disjointed facts and procedures; the discourse patterns emphasize recall and short answers focused on developing fluency with the facts and procedures. This quadrant has been well-covered in the literature and bears little need for further discussion, except to say that this combination has been frequently associated with the relatively poor performance of U.S.

on national and international assessment, especially in relation to problem solving and conceptual understanding (Braswell et al., 2001; Kenney & Silver, 1997; Schmidt, McKnight, Cogan, Jakwerth, & Houang 1999).

Table 1. Combinations of discourse and curriculum

Quadrant I Conventional curriculum Recitation-style discourse	Quadrant III NSF-funded curriculum Recitation-style discourse
Quadrant II Conventional curriculum Dialogic discourse	Quadrant IV NSF-funded curriculum Dialogic discourse

The other three quadrants present more complex and less documented cases, especially quadrants II and IV. Large scale studies of U.S. mathematics classrooms indicate an absence of dialogic discourse practices (Jacobs et al., 2006; Spillane & Zeuli, 1999; Stigler & Hiebert, 1999). Studies situated in classrooms of teachers who have attempted more dialogic or interactive forms of discourse have shown that, although teachers succeed in having students talk more, it does not indicate that students' explanations have served as 'thinking devices' for the classroom community (Forman, McCormick, & Donato, 1998; Gutierrez, 1994; Nathan & Knuth, 2003; Williams & Baxter, 1996).

In terms of Quadrant II, Hiebert et al. (1997) discuss the notion of problematizing mathematics, regardless of the task design. However, the QUASAR research that studied the cognitive demand of tasks as designed versus the cognitive demand as implemented

(Henningsen & Stein, 1997; Stein et al., 1996) suggests that it is extremely rare that tasks focusing on procedures get enacted at a more complex level consistent with dialogic discourse patterns.

The combination of a reform curriculum being implemented in a classroom whose discourse practices are predominantly recitational (Quadrant III) has been documented in a number of classrooms, to varying degrees (Choppin, 2006; Cohen, 1990; Collopy, 2003). Although extreme cases in this quadrant may easily illustrate how recitation-style discourse does not allow students to make connections, to develop fluency between representations or to problematize representational forms, or to generate and discuss multiple solutions, it is the hybrid practices – those that cannot be fully characterized by the descriptions I provided earlier – that need greater scrutiny. These hybrid practices will likely serve as bridge or transition practices for teachers whose goal is to develop dialogic discourse but who have not yet become competent at doing so.

Studies in Quadrant IV can demonstrate the ways in which dialogic discourse patterns help to realize or activate curricular resources in reform curricula. There are some model cases in the literature (Forman et al., 1998; Lampert, 2001; O’Connor & Michaels, 1996; Yackel & Cobb, 1996) of teachers who were able to engage their students in dialogic forms of discourse using rich tasks: in these cases, students reflected on properties of solutions and engaged in forms of mathematical argumentation. These cases illustrate to some degree the extent to which dialogic patterns allowed students to make connections, develop metarepresentational competence afforded by the tasks, and to collectively consider various approaches afforded by ambiguous tasks. However, the analyses did not explicitly consider these features, so I will present data below to illustrate how hybrid or dialogic discourse patterns serve to activate curricular resources.

I present three cases in which teachers were using units from the CMP curriculum. I present the cases as a continuum of discourse patterns, ranging from patterns that have characteristics of the recitation style to those having more dialogic properties. All three cases characterize hybrid practices, practices that diverge in important ways from the descriptions of classroom practice of traditional mathematics classrooms but which do not fully take on dialogic characteristics. For example, students in all three classrooms were offered opportunities to participate frequently and in ways that contributed to the development of mathematical ideas. However, the extent to which discussions were interactive and the extent to which students were able to fully consider the ideas and strategies of their peers varied, and it is along these two characteristics that I distinguish the three classrooms. I provide an episode from each class in relation to one of the three curricular resources (connections, metarepresentation competence, ambiguity). In relation to the quadrants in Table 1, the three cases below gradually slide from Quadrant 3 to Quadrant 4, though I am reluctant to situate any of them squarely in one of those quadrants.

Case One – Predominantly recitational discourse pattern

The discourse patterns in this classroom were marked by the predominance of IRF sequences in which the teacher asked a series of questions, mostly of the short-answer variety, and provided rapid feedback to the responses, mostly in the form of evaluations. Class discussions flowed through the teacher and there were no instances in nine observed class sessions of extended discussions (more than 1 minute) and very few instances of student to student interaction.

I provide an example below in which the students are asked to compare two graphs from the textbook, one of Andrea doing jumping jacks and one of Ken doing jumping jacks. The graph of Andrea's data is scaled at intervals of 20 on the dependent axis while Ken's data is scaled at intervals of 10. On the page, Ken's graph is higher though, due to the differences in scales, he has done fewer jumping jacks than Andrea. I include discourse around this task because it offers an opportunity for students to problematize a feature of graphical representations, which is whether the visual impact of one graph being higher relays information of whether that constitutes a larger quantity.

1. Teacher: How many agree that Ken DID do the most jumping jacks? [No students raise their hands.]
2. Teacher: Okay. How many of you disagree? [Most students raise their hands] Why do you disagree with the statement? The statement in here by the way says Ken says that he did more jumping jacks than Andrea in a hundred twenty seconds, do you agree, why or why not? Kendall?
3. Kendall: I said um .. Ken's go up by tens, while Andrea's go up 20.
4. Teacher: Ken's go up by tens and Andreas go up by 20's. Okay. So, in other words, it went from twenty to forty and so on and in the same distance, Ken went ten, twenty and so on. How many people agree with that? The intervals are different.

5. Teacher: And by going up only ten at a time, at the same point...this is about 80 right here I believe...that's not 80, that's going to be about 110 right here...and at that same point 120, you get on this graph, 70?

By providing most of the substantive comments, the teacher removes the opportunity for students to collectively consider the competing qualities in the representation (height of graph, interval of scale on the dependent axis) and how these qualities influence the information conveyed by graphs in general. The opportunity to develop one dimension of metarepresentational competence – to problematize the functional and visual qualities of representations – is not afforded by the recitation-style pattern exhibited in this example.

By contrast, the students had multiple opportunities to develop fluency between representations in the Variables and Patterns unit. Despite the lack of dialogic properties in the whole-class features of the discourse, the number and sequence of tasks devoted to considering connections between various representations provided resources for students to develop representational fluency. That is, by simply following the tasks as written in the curriculum, the teacher was able to promote fluency regardless of the kinds of discussions he was able to engender around those tasks.

The results from this classroom raise the possibility that some resources in the curriculum are less affected by patterns of discourse. The development of metarepresentational competence is at least partially accomplished without the necessity of dialogue conducted collectively by the whole class or other group (though Bahktin (1986) would argue that the students are in dialogue with the designers of the curriculum, and by extension with a much larger community of mathematicians, when they are working independently). However, there is a component of metarepresentational competence that “involves learning to participate in the complex practices of communication and reasoning in which the representations are used” (Greeno & Hall, 1997, p. xx). It is this component that is difficult to realize in the absence of dialogic forms of discourse. The episode illustrates how an opportunity to reason and communicate about representations, while supported by the task design, was not realized in the discursive enactment of the task.

Similarly, the Variables and Patterns unit of the CMP curriculum provides opportunities in many tasks for multiple approaches to be used to solve a problem (in an open-middle rather than open-ended design) (Choppin, 2006), and to connect procedures with concepts. There was little evidence of multiple approaches in the classroom described above, as discussions around specific tasks usually ended when a single appropriate response was established. As with the case of developing fluency between representations, students had the opportunity to connect procedures with connections without engaging in interactive discussions, due to the nature of the task design and sequencing.

It bears repeating that this class, though the discourse predominantly consisted of IRF sequences and was largely directed by and through the teacher, diverged from descriptions of classrooms described in Stigler and Hiebert (1999), for example, because there was a greater emphasis on conceptual development and a reduced emphasis on repetitive exercises aimed at skill development.

Case Two – Emphasis on interactivity

The case two classroom exhibited a greater degree of interactivity than the previous case. Earlier, I characterized dialogic discourse as using students' explanations as 'thinking devices:' this entails a collective reflection of one or more explanations and evidence that this reflection helps to establish a common understanding of the mathematics underlying the explanations. Despite the emphasis on interactivity in this classroom, the discussions around student explanations were less about developing a collective understanding and more about convincing one or more students of the accuracy of a particular explanation.

The teacher emphasized the importance of spontaneous discussions. To this end, she consistently sought comments about explanations and would allow discussions to proceed, with minimal intervention, for as long as five minutes. Nystrand's (1997) large-scale study of English classes suggests that discussions of more than one minute are exceedingly rare in U.S. classrooms. These discussions were supported by the ambiguity present in the CMP tasks, as will be illustrated in a brief episode below, but did not achieve the goal of using this ambiguity to stress some underlying mathematical idea. This case suggests that more interactive forms of discourse do not necessarily serve to activate all of the resources in a reform curriculum.

In the episode presented below[3], from the Variables and Patterns unit, the students were asked to estimate the number of knee bends a person would do in 25 seconds if the person had completed 18 knee bends in 20 seconds and 25 knee bends in 30 seconds.

6. Teacher: Alright. Estimate the number of knee bends Derrick could do in 25 seconds.

[3] In all of the episodes presented for this paper, I have edited the transcripts to shorten their length, for purposes of readability and overall manuscript length, while preserving the qualities I wish to emphasize.

7. Raymond: I got 23. [Raymond writes 23 on the overhead]

8. Teacher: Okay. What do you guys think about that?

9. Student: I said 21.

10. Teacher: You said 21? [To Raymond] Why did you say 23?

11. Raymond: Cause right here it says in 20 seconds he had 18.

12. Teacher: So it would have to be more than 18. What do you think about that? Melinda? Yeah.

13. Melinda: (Raymond) says … he does 2 knee bends in 5 seconds so the other 5 seconds he … had 5.

14. Teacher: Okay, let (Raymond) explain why.

15. Raymond: I put 23 because if they added 5 seconds on to it, I think they would add 5 more to 18.

16. Teacher: Okay. So you're thinking they're doing 1 knee bend per second? They started out that way, didn't they? Doing one knee bend per second.

17. Teacher: Melinda's still shaking her head no. Why not, Melinda?

18. Melinda: I don't know. It just seems too close to 25.

19. Jackie: Then you only get 2 knee bends in (5) seconds [referring to the second five-second interval].

20. Raymond: Look, he did 25 in 30 seconds. Now take 5 seconds away and he'd be at what? [Raymond is changing his answer but using the same strategy that indicates the number of knee bends is the same as the

number of seconds for one of the 5-second intervals, in this case the second interval.]

21. Melinda: Like 21.

22. Jackie: So then the first estimate, it'd be like 21 or 22.

23. Raymond: He'd be at 20 if he'd just take away 5 [referring to the number of knee bends]. Okay, right here, he'd be at 25 if you just take away 5 seconds [referring to the number of seconds]. Now, he'd be at 20 [knee bends].

24. Adrian: Yeah but then from 20 to 25 seconds, he's only done 2 knee bends.

25. Melinda: What about at 30, there are only 2 knee bends? From 20 to 25 he did 5? [Melinda is referring to Raymond's first explanation, which has more knee bends in first five seconds than in the second interval of five seconds.] Why? I don't think so.

26. Teacher: She's asking, why did he do 5, right? Cause you're saying he did 5 knee bends between 20 and 25 seconds, right? [Referring to the first explanation.]

27. Jackie: And then he did 2.

28. Teacher: And then he only did 2 between 25 and 30. Why the difference?

29. Raymond: Well. I'm sort of saying that it's running a pattern like you start with this and you just drop down to and then you start up again and start back down then drop down to it. [Raymond is offering an alternative explanation to the ones he presented above, one that might account for variation across intervals.]

The task does not indicate a specific procedure to follow and there are multiple interpretations for determining the number of jumping jacks at 25 seconds, especially because the data are non-linear (the ratio of knee bends to seconds is not constant and one can assume that Derrick began doing knee bends when the stop watch was activated). Although the teacher and several students argue for an averaging strategy, one could also argue that Derrick was slowing down (from 0.9 knee bends per second to 0.84 knee bends per second), that there would be more knee bends in the first 5 second interval. If one follows the teacher's lead, the underlying mathematical idea is that there should be equal numbers of knee bends in each interval (constant rate of knee bends across the 10-second time interval), but the resolution of this discussion does not clarify this position.

This case demonstrates that discourse patterns that contain one characteristic of dialogic discourse – that of interactivity – only partially activate the resources in this CMP task. In this example, the ambiguity of the task potentially offers itself to multiple solutions, whose varied mathematical properties can be used to collectively arrive at an understanding of the underlying mathematical idea. However, despite the interactive engagement of multiple students, the mathematical resolution is vague and the only discernible outcome is that Raymond recognizes the need to search for a new explanation to his answer.

Case Three – Purposeful dialogue

The third classroom provides an example of discourse patterns that, while more structured than the prior example, accomplish purposeful dialogue. In this classroom, the teacher is very deliberate in the way she structures opportunities for students to consider the multiple solutions that arise from engaging in tasks from the CMP curriculum. She utilizes her knowledge of students' thinking in relation to tasks, gleaned from the teacher resource materials as well as from prior implementations of the tasks, to anticipate the variety of solutions and plans the ways in which she will allow these solutions to publicly emerge for collective reflection.

The episode occurred during enactment of the Tupelo problem, described earlier, from the Bits and Pieces II unit of the 6^{th} grade CMP books. The teacher focused the first day of the Tupelo problem on having the students determine the fractional sizes of each plot of land, as well as the number of acres for each plot [there are 640 acres per square mile]. The teacher had taught this investigation, or a close version of it, five different times in the previous three years, and had made some subtle changes to the way in which she organized the summary discussion of the investigation. To prepare for the summary discussion, the teacher created separate maps divided into fourths, sixteenths, thirty-seconds, and sixty-fourths, because she had seen students represent the plots using those denominators during the exploration phase of the task the day before. She also cut out each plot of land (which were of varying irregular shapes, but which could be decomposed into rectangles) and had a large version of Tupelo posted to her blackboard.

In the episode in which the excerpt below takes places, the teacher turns the focus of the discussion to finding the fraction for the plot of land for the farmer Wong. Wong's plot was 3/32nds, which meant that students who had divided their plots into eighths or sixteenths or who were comparing Wong to other plots that were one eighth or one sixteenth would need to make some adjustment for Wong. The discussion on this problem lasted nine minutes, during which the strategies of six students are either mentioned.

30. Teacher: What did you do differently? Annie?

31. Annie: Well, first we figured out Krebs, and I saw that he was half of Bouck or Fuentes. [Wong would be equal to a Krebs plus a Bouck or Fuentes]

32. Teacher: So, how big was a Fuentes?

33. Annie: One sixteenth.

34. Teacher: But again, what's our problem with that? What's our problem with writing half of a sixteenth? Ellen?

35. Ellen: It's in a decimal form.

36. Teacher: It's in a decimal form. And I'm not saying that's wrong, I believe 100 per cent that that is mathematically correct. It is half of a sixteenth. But I know a lot of you did something different. Amanda?

37. Amanda: We got a 32nd. [With some assistance from the teacher, Amanda then explains how she multiplied the numerator and denominator of the fraction 0.5/16 to get 1/32.]

38. Teacher: You got a 32nd.

39. Teacher: OK. So I could write Krebs as 1/32nd. But, a lot of you found it a different way. Anybody have a different way of getting a 32nd? [writes 1/32 in the Krebs row, next to .5/16] Than what Amanda did? Oh, hoh, Ellen.

40. Ellen: I divided our board into 32nds.

41. Teacher: Oh, into Krebs, right?

42. Ellen: Yeah.

43. Teacher: They divided [showing a map of Tupelo divided into 32 equal rectangles], and Amy and Emily did this from the very beginning, right? They did this from the very beginning, so everything on their paper was already broken into Krebs. So, all of your denominators, except for Lapp, I think, was what? Okay. So if Krebs is one 32nd, how can I fix my Wong so that he's not one and a half 16ths? How can I fix my Wong now so that Wong is not one and a half 16ths? What can I do? J.D.?

44. J.D.: If you put that [the Bouck cutout] on Wong, you have like almost like the whole thing [JK places the cutout of Bouck on the cutout of Wong], and then you could add Krebs on.

45. Teacher: Yeah, you guys should see J.D.'s paper. J.D. has all of these sentences, they would say like 'a Bouck plus a Krebs equals a Wong' and then he had like 'two and a half Bouck’s equals a Stewart' and he would have all these different things and that's how he figured them out.

In the course of this episode, the teacher utilized visual resources to help students make connections to the various explanations and to connect these explanations explicitly to the underlying mathematical idea, which is the notion of equivalent fractions. Like the teacher in O’Connor and Michaels (1996) study, the teacher uses revoicing to position students as participants in a mathematical dialogue. By structuring the discussion and anticipating the strategies and necessary resources, the teacher was able to focus the class on the various strategies, which she yoked together to further develop the idea of equivalent fractions, all the while attributing each strategy to the student who introduced it.

Discussion

This paper set out to investigate the kinds of practices that develop in relation to the use of CMP and whether these practices sufficiently activate the resources in CMP, with the broad perspective of looking at how cultural traditions in relation to the use of curricula as tools are changing in relation to the introduction of new tools, in the form of reform curricula. One conclusion suggested by the data is that new practices are developing, features of which make full use of the resources the tool offers. These new practices emerge from a focus on changing how students interact with each other and with mathematical content (more explanation and exploration), but also from the nature and sequence of tasks in the curricula. However, it is also clear that students and teachers are still learning about the resources the tool offers, a finding that mirrors Wertsch's (1998) articulation of tool as stable at a given point in time but evolving as agents develop more experience with the use of the tool.

The results from the three classrooms suggest the resources in the CMP curriculum are partially realized even in cases when the discourse patterns resemble the recitation pattern. However, the results also suggest that when certain dialogic characteristics are present in the discourse patterns, students are able to take greater advantage of the resources. In particular, the aspect of metarepresentational competence that requires students to problematize the features and use of representations, and the outcome of ambiguous tasks in which students collectively generate a common understanding of the underlying mathematical concept, were not realized when dialogic features of discourse were absent. Although I did not present examples from classrooms using conventional curricula, my analysis suggests that the resources in those curricula are less sensitive to dialogic discourse.

In order to conduct this analysis, I suggested that the discourse practices in the classrooms I observed did not neatly fit into the two patterns I introduced earlier in the paper. The result of this characterization of the practices as *hybrid* – neither predominantly of the IRF or elicitation patterns nor fully dialogic – makes it difficult to provide clear-cut conclusions of the impact of two distinct types of discourse patterns. However, it does afford a more nuanced view of the interaction between curriculum and discourse that lends itself to a broader range of practices and, most importantly, helps to define the nature and impact of formative efforts to change discourse practices.

The goal of my curriculum analysis was to illustrate the substantive distinctions between commercially-developed and NSF-funded curriculum. Although the conventional textbook had some interesting features, particularly in the ‘Problem solving’ section of each unit, the implicit view of mathematics in the introduction and skills section of each unit suggest that mathematics primarily consists of well-defined rules and procedures that need to be mastered. The resources in the conventional curriculum are primarily the explicit definitions and modeling of skills and procedures: students who wish to review their facts and procedures can use these resources to solve problems that are constrained and predictable, with the presumed goal of developing procedural fluency. I contrast these resources with those in CMP, which provide opportunities for students to make connections, particularly between procedures and concepts, to develop representational fluency if not metarepresentational competence, and to use the ambiguity of tasks to collectively develop mathematical understanding.

The results from the three cases suggest the possibility of a curriculum effect and a discourse effect. The design and sequencing of tasks provides abundant opportunities for students to make connections between procedures and concepts, to use context as a means to understand the mathematics, and to move fluently between different representations, regardless of whether the teacher is able to initiate discourse patterns that have dialogic tendencies. Similarly, if one subscribes to the perspective that mathematics is a social practice, then dialogic interactions are required to help students problematize and make sense of complex ideas, including understanding the nature of use of representations (i.e. going beyond reproducing conventional forms), and being able to participate in the collective negotiation of mathematical ideas (i.e. knowing what is mathematical evidence or what is a good mathematical explanation).

The examples provided above show that, in order to establish discourse patterns with dialogic properties, teachers must have a firm grasp of the mathematical goals and of resources that help students to be able to think about the ideas of others. It is not simply enough to let discussions spontaneously evolve (although this can provide an intellectually stimulating and risk-taking classroom culture), the teacher must help students to resolve the competing ideas in a way that develops a collective understanding. In other words, one should not confuse interactivity with dialogism because one can interact while paying little attention to the implications of others' utterances (witness talk radio).

I had originally intended to demonstrate the importance of dialogic discourse patterns in activating all of the noted resources in the CMP curriculum. My analysis, however, illustrates that it is not all about discourse. The design and sequencing of tasks in CMP offer distinct opportunities from those in the conventional curriculum and allows for certain learning trajectories to develop even in cases when the discourse patterns are limited.

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