A Reputational Theory of Two-Party Competition

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ABSTRACT

I study a dynamic game of two-party competition in which party preferences are private information, exhibit serial correlation, and change with higher probability following defeat in elections. Assuming partisans care sufficiently about office, extreme policies are pursued with positive probability by the government when (a) both parties have a reputation for being extreme that exceeds a fixed level, and (b) elections are close in that both parties have similar reputations. Two qualitatively different equilibrium dynamics are possible depending on the speed with which the latent preferences of parties in government shift between moderation and extremism relative to the opposition. One dynamic produces regular government turnover and extreme policies along the path of play, whereas the other involves a strong incumbency advantage and moderate policies.

In the canonical model of two candidate electoral competition, the two contenders for office make platform announcements that the electorate takes at face value. In the classic Hotelling (1929)/Downs (1957) version of this model, equilibrium platforms converge to the median when the policy space is represented in one dimension and candidates are motivated by the pursuit of office. In part to address the criticism that actual elections do not result in identical policy platforms by the candidates, platform divergence is obtained in equilibrium under the alternative assumption that candidates have policy motivations and face electoral uncertainty (see, e.g., Wittman, 1983; but also Calvert, 1985). These models and their variants generate ideals with trivial dynamics, since they imply either persistent policy convergence or (partial) divergence over time. At least with regard to

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partisan competition, empirical observation suggests otherwise. For instance, Downsian convergence seems to be a fair approximation of British politics in the 1950s and 1960s. But this era of the “politics of consensus” (Kavanagh and Morris, 1994) came to an end in the 1980s with the governments of Margaret Thatcher.

Besides their variable success in explaining policy extremism and their inability to account for different degrees of policy moderation over time, there is another count on which static models of electoral competition are at odds with empirical observation. In these models both candidates are in principle able to be competitive in elections because a candidate can perform at least as well as the opponent by adopting the opponent’s platform. Yet, we often observe two-party systems in which one of the two parties contesting for power is widely perceived to have little or no chance of winning office, often over multiple successive elections. For example, the Tories in Britain seemed unable to compete effectively against the Labour party during the second part of the 1990s, because Labour was perceived to occupy a policy position in the center of the policy space while the Conservative party could not convince the electorate that their policies were moderate, given the track record of the preceding governments of Margaret Thatcher and John Major. Similarly, the Labour party in the late 1980s and early 1990s was outperformed by the conservatives in consecutive elections because it was unable to effectively distance itself from the radical left policies and platforms that the electorate had associated with the party since the late 1970s and early 1980s. In these examples, a significant amount of time was required for the parties involved to overcome a reputation for being extreme, despite public assertions or claims by party leaders that they would pursue moderate policies if elected. My objective in this paper is to develop a model of two-party competition which focuses on the dynamics of party reputations and allows for equilibria that are consistent with these empirical observations.

The model I develop shares features with many static models of electoral competition. First, I combine the premises of the models of Downs and Wittman and assume candidates have a mix of office and policy motivations. Second, as in the citizen–candidate models of Osborne and Slivinski (1996) and Besley and Coate (1997), I focus on credible policy choices by the governing parties and dispense with cheap talk policy platform declarations. Third, while the citizen–candidate models focus on individual candidacies, I model electoral competition between political parties which I assume are populated by individuals with different policy preferences as in the party equilibrium model of Roemer (2000). In particular, I assume that individuals with different policy preferences within a party battle for control of the party in each period and the outcome of this battle is probabilistic.

I also depart from the above models in that I assume that the true preferences that prevail within each party are private information. In the absence of policy platform commitment, party preferences that are private information cannot be credibly communicated

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1 This is the case in most standard models with probabilistic voting, but is also true in models with deterministic voting as long as the voter is not exogenously assumed to vote in favor of one of the two parties when indifferent.
to other actors except possibly via costly policy choices while the party is in office. Thus, in lieu of platform declarations, I assume each political party enters the electoral arena with an endogenously formed reputation. These party reputations are the beliefs of the electorate regarding the preferences that prevail within each party, and they reflect the accumulated history of electoral outcomes and policy choices that have transpired prior to the elections. I also assume that political parties, like all organizations, exhibit inertia. I formalize this idea by assuming that latent party preferences are positively serially correlated: extremists (moderates) have a higher probability of prevailing within the party if the party was controlled by extremists (moderates) in the previous period. These assumptions define a stochastic game of incomplete information in which players’ strategies (those of parties and the voters) are conditioned on the two parties’ joint reputations in any given period, and these reputations in turn are rationally updated given past actions.

I first study the case in which parties are impatient or place significant emphasis on policy relative to office, and I find that the only equilibrium involves party types implementing their ideal policy independent of the electorate’s beliefs about the two parties. The more interesting case, though, on which I focus the bulk of the analysis, is when partisans assign significant weight to office utility relative to policy. Under this assumption, I obtain three chief results. First, I find that the policy choice of the parties in government depends on the joint reputation levels of both competing parties, so that the governing party pursues an extreme policy (on or off the equilibrium path) only when it has a relative reputational disadvantage compared to the opposition party. In particular, a party in government that has a worse reputation than the opposition party cannot hope to win re-election by pursuing a moderate policy with probability one because that policy choice reveals no information about the government’s true preferences and maintains the government’s reputational disadvantage. Thus, when the governing party is perceived to be extreme relative to the opposition and it is also controlled by extremists, it implements an extreme policy with positive probability (but not with probability one).

Second, I find that extreme policies are observed along the equilibrium path if the following two conditions are met: (a) both parties are perceived to be relatively extreme with a probability that exceeds a fixed reputation level, and (b) the government is elected in a close race, i.e., in an election in which the two parties have similar reputation levels. When both parties are perceived to be extreme with a relatively high probability, their reputations tend to improve due to a regression to the mean effect. Since opposition parties are more likely than government parties to change policy preferences, they tend to benefit more than the government from the fact that their reputation regresses towards average levels. Thus, when the electorate has to choose between two parties with very similar reputations (as in condition (b)), and both parties are perceived to be extreme with high probability (as in condition (a)), it must elect a party in government that is likely to suffer from a worse reputation than the opposition party in the next election. Faced with such unfavorable electoral prospects, the party elected in government has an incentive to pursue extreme policies with positive probability, as discussed in the previous paragraph.
Finally, I study the dynamics of party reputations, policy choices, and electoral outcomes, and I find that the equilibrium is consistent with two radically different patterns of competition over time. When the odds ratio of party preferences switching from moderate to extreme versus from extreme to moderate is higher for parties in the opposition than for parties in government, equilibrium dynamics are characterized by moderate policies and a strong incumbency advantage for the government, because the government is eventually able to maintain a persistent reputational advantage over the opposition party. But when the comparison of these odds ratios is reversed, I obtain dynamics in which extreme policies and alternation of parties in government are a regular equilibrium phenomenon that occurs infinitely often along the path of play. In an intermediate special case, the equilibrium dynamics depend on the initial reputation levels of the two parties.

A number of other papers explicitly study the dynamics of two-party competition but assume complete information. Kramer (1977) and Wittman (1977) assume that in each period the incumbent is committed to the policy pursued in the previous period, while the challenger myopically chooses a platform that maximizes one-period vote share or policy utility, respectively. Harrington (1992) and Aragones et al. (2007) study equilibrium models of repeated elections with an explicit focus on party reputations. The term reputation has a different meaning in these studies than the one I adopt in the present analysis. In particular, a reputation in the context of these studies is the belief about the policy to be chosen by the candidates in equilibrium. In contrast, a reputation in the present study is the belief of the electorate about the preferences that prevail within the party. Harrington (1992) and Aragones et al. (2007) establish the range of policy choices or reputations other than the candidate’s ideal policy that can emerge in equilibrium. These equilibrium reputations are built on history dependent strategies such that, upon observing a choice different than the one dictated by a candidate’s reputation, the voters expect the candidate to switch to pursuing his/her ideal policy forever after. Alesina (1988) and Duggan and Fey (2006) study different types of history-dependent strategies in repeated election models in which candidates have policy and office motivations, respectively. They characterize the set of subgame perfect equilibria which are consistent with a wide range of policy platform choices by the parties in Alesina (1988), and include all possible policy outcome paths in Duggan and Fey (2006). Dixit et al. (2000) characterize efficient subgame perfect equilibria in a model in which parties’ re-election probabilities follow an exogenous Markov process conditional on the incumbent’s policy choice. Besides the fact that I study a model with incomplete information, another difference with these studies is that I focus on Markovian equilibria, so that the different players (partisans, voters, etc.) do not coordinate on complex history-dependent strategies.

Among models with incomplete information, Alesina and Cukierman (1990) study a two-period model under the assumption that the candidates’ preferences in the second period are serially correlated with their first period preferences, as is assumed in the present infinite period model. Their analysis focuses on the first period policy choice, emphasizing the strategic incentive of the candidates to choose a moderate policy in order to win re-election. Thus, their study does not involve the type of dynamic analysis
I pursue. Repeated elections under incomplete information about candidate preferences are studied by Duggan (2000), Bernhardt et al. (2004), and Banks and Duggan (2008). These models are suitable for the study of elections in which the two contenders for office are individuals so that it is plausible to assume, as these authors do, that challengers to the incumbent are drawn from an identical pool of possible candidates over time. On the other hand, the assumption that challengers are drawn from a stationary distribution seems inappropriate in the case of partisan competition, because inertia within party organizations implies that the past preferences of the opposition party influence the realization of current preferences and the reputation of that party.

MODEL

I study a model of two political parties and an electorate that interact over an infinite number of periods $t = 0, 1, \ldots$. I model the electorate as a pivotal or median voter, $M$, and denote a generic party by $P$, which is either a left-wing party ($P = L$) or a right-wing party ($P = R$). I use $-P$ to denote the party in opposition of party $P$. Implicitly, I assume that each of the two parties contains two groups of individuals with distinct ideological convictions: moderates and extremists. These two groups fight each other for controlling their party in each period, and the group that prevails in this internal battle becomes the type of the party in that period. I denote the type of the party by $\tau$ which can be either extreme ($\tau = e$) or moderate ($\tau = m$). I assume that party types are not observable or verifiable by extra-party players, and are determined according to the following stochastic process: if the governing party in period $t$ is of type $\tau$, then that party is of the same type in period $t + 1$ with probability $\pi^g_{\tau} \in [0, 1]$, while if the opposition party in period $t$ is of type $\tau$, then that party is of the same type in period $t + 1$ with probability $\pi^o_{\tau} \in [0, 1]$. I assume that the probabilities $\pi^g_e, \pi^g_m, \pi^o_e,$ and $\pi^o_m$ that determine latent party preferences satisfy

$$\pi^g_{\tau} > \pi^o_{\tau}, \tau \in \{e, m\}, \text{ and}$$

$$\pi^o_{e} > 1 - \pi^o_{m}. \text{ (2)}$$

According to Inequality (2), parties are more likely to be of type $\tau$ in some period if they were of the same type in the immediately preceding period.\(^3\) Note that because of Inequality (1), this serial correlation in party preferences is assumed whether the party

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\(^{2}\) Other models of incomplete information, such as Ferejohn (1986), focus on the fact that the incumbent’s action while in office is unobserved (hidden action) and can be traced to Barro (1973). Rogoff (1990), Banks and Sundaram (1993, 1998), Ashworth (2005), and Martinez (2009) combine aspects of both the models. Besley’s (2006) monograph contains models of both the types.

\(^{3}\) Literally, Inequality (2) states that a party is more likely to be extreme in period $t + 1$ if it was in the opposition and it was extreme in period $t$ than if it was in the opposition and it was moderate in period $t$ (with probabilities $\pi^o_e$ and $1 - \pi^o_m$, respectively). The analogous statement is true for the probability that the party is moderate in period $t + 1$, since Inequality (2) is equivalent to $\pi^o_m > 1 - \pi^o_e$. 

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is in government or in the opposition.\footnote{In effect, I assume that the ideological group that controls the party in one period is in a better position to fight the battle for controlling the party in the next period (for example, by delaying the retirement of members of its own group, favoring the advancement of like-minded members to different ranks of the party hierarchy, buying off ambivalent opposition members of the party with perks, etc.), thus leading to inertia or serial correlation in party preferences. In addition, Inequality (1) states that the type of the party in government is more likely to persist in the next period, compared to the same type of the opposition party. I view this as a natural assumption, since parties in government have additional resources at their disposal that enhance the ability of the prevailing ideological group within the party to maintain control of the party in the next period. On the other hand, parties in the opposition have experienced defeat in the immediately preceding elections and are more prone to internal shifts of power.}

I should emphasize that throughout the analysis, I assume that parties are strategic players and that party types optimally choose what policies to implement while in control of their party and the government. Yet, I depart from the previous literature on party competition by simultaneously making two assumptions: First, that political parties cannot choose their own preferences, and, second, that these preferences (i.e., the ideological group that prevails within the party) shift with certain probability over time. The first assumption, that parties cannot choose their preferences, is a direct consequence of the assumption that parties have no mechanism to commit to a policy platform. Such an assumption is not novel to this paper. The absence of platform commitment creates a standard adverse selection problem: All parties (and partisans of either type) have an incentive to convince the electorate that they are moderate in order to reap gains in the electoral arena. As a result, partisan preferences cannot be fully or credibly conveyed to extra-party players using, for example, party manifesto announcements, the judicious selection of party leaders, etc.\footnote{It should be clear from the discussion so far, that I view a political party as a more complex entity than one in which, for example, all internal power struggles cease the minute the party elects a new leader. Despite their prominence in the public arena, I view party leaders as agents of the party who are constrained by the prevailing opinion within their party. Party leaders in parliamentary systems can be challenged and replaced even while their party is in government. This rarely happens for the very reason that leaders recognize (and choose not to exceed) the boundaries of their influence.} In effect, I assume that any such (non-modeled) actions that political parties can take in order to improve their reputation are already captured by the probabilities $\pi_o^g$ and $\pi_g^o$. Thus, these probabilities reflect residual uncertainty about party preferences after parties have optimally taken all available publicly observable actions, besides the choice of policy while in government.

The second assumption concerning the randomness in the determination of party types can be attributed to the regular turnover that takes place among influential party members over the course of their career cycles. At the same time that part of the party’s ‘old guard’ retires or becomes less prominent within the party, a new set of powerful members gains influence. If we assume that these new influential members have different
preferences with positive probability, then a switch in the prevailing ideological position within the party can also take place with positive probability. A second source of randomness in party type determination may be attributed to the intransitivity of social preferences and the consequent instability in the coalition formation process within parties. Although in the social-choice-theoretic tradition such instability is synonymous with the absence of equilibrium, an appropriately modeled dynamic game of coalition formation would be consistent with the existence of a mixed strategy Nash equilibrium. Thus, the lotteries I assume in the game can be interpreted as arising from an equilibrium in mixed strategies in some internal coalition formation game within each party. In particular, if winning coalitions within parties are built along both ideological issues and additional dimensions such as, for example, the distribution of offices or other perks within each party, then shifts in internal power alliances can take place behind the scenes, and these shifts may permit previously losing ideological groups to exercise control over party decisions based on a new power arrangement within their party.

Independent of the exact source of these shifts in party preferences, parties have an incentive to present an image of being moderate to extra-partisan players. Thus, while parties know the realization of their own type in each period, other players cannot perfectly discern the balance of power between extremists and moderates within the party. Instead, players rationally update beliefs about the probability that (other) parties are moderate or extreme. I will refer to these beliefs as the reputation of each party, which at the beginning of each period I represent by a pair of probabilities \( b_t = (b_L, b_R) \in [0, 1]^2 \). For example, probability \( b_L \) represents the belief of voter \( M \) (and party \( R \)) in period \( t \) that party \( L \) is extreme.

Interaction within each period represents a complete political cycle. In particular, period \( t \) starts with a pair of party reputations \( b' \) and an incumbent party in government. Elections take place at the beginning of the period, and voter \( M \) chooses whether to re-elect the party in government or not. The party/type elected in government by the voter in these elections implements a policy \( x^e \in X \). Extreme types of party \( P \) can choose between their favorite policy \( x^e \) and a moderate policy \( x^m \), while moderate types of either party always implement the moderate policy of their party \( x^m \). Thus, in general, there are four possible policies given by \( X = \{x^L, x^L, x^R, x^R\} \). Following the policy choice of the governing party, which is publicly observed, nature chooses a new type for each party, players update their beliefs, and the game moves to the next period with these beliefs, \( b_{t+1} \). In that period, the voter elects a new government, the governing party implements a policy, new partisan types are realized, etc.

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6. This can be construed as an argument about overlapping generations of party members, although the changes in the strength of different groups of influential party members that I have in mind need not be solely attributed to the influx of younger members. It is possible, for example, that the group of party extremists becomes stronger in a period if a sufficient number of influential moderates simply switch preferences, or lose their appeal within the party due to personal scandals, the need to attend to health or family issues, etc. Alesina and Spear (1988) study an explicit model of overlapping generations of party members, but their model does not involve incomplete information.

7. I simplify the analysis by exogenously restricting moderates to choose a moderate policy, but this behavior would also arise endogenously in an equilibrium of the type I characterize.
If policy \( x_t^P \in X \) is implemented in period \( t \), then voter \( M \)'s payoff in that period is given by \( r_t^P \in \mathbb{R} \), while extremists of party \( R \) receive \( r_t^R \in \mathbb{R} \) and extremists of party \( L \) receive \( l_t^R \in \mathbb{R} \). I preserve the symmetry of the game by setting \( l_t^P = r_t^P \) and \( l_t^L = v_t^L \) for \( \tau \in \{e, m\} \) and \( P \in \{L, R\} \). I assume that \( v_m^P > v_e^P \) and \( r_m^R > r_e^R \geq r_m^L > r_e^L \), that is, the voter prefers moderate policies and extremists of each party prefer the respective partisan policy most, moderate policies next, and they least prefer the partisan policy of the other party. Note that by assuming \( r_m^R \geq r_m^L \), I do not preclude the possibility that the moderate policies of each party coincide, i.e., \( x_m^L = x_m^R \). This would permit a convergence to the median equilibrium to occur, but I also allow \( x_m^L \neq x_m^R \) so that there may exist residual partisanship even if the moderates are the prevailing group within each party. Parties receive additional office payoff when they control the government so that extreme partisans receive utility \( G > 0 \) when their party is in government. I assume that the voter is strategic but cares only about the policy outcome in the current period. Partisan types are (potentially) more far-sighted and care about the electoral and policy outcome in two periods, the current period \( t \) as well as period \( t + 1 \). The weight parties place on the outcome of the next period is given by a discount factor \( \delta \in [0, 1] \).

I will focus the analysis on equilibria in strategies that are appropriately Markovian. Given the structure of the model, the relevant strategic environment for the party in government is summarized by the reputations of the two parties. Thus, a strategy for an extreme type \( e \) of governing party \( P \) is given by a function \( \sigma_P : [0, 1]^2 \rightarrow [0, 1] \), and \( \sigma_P(b) \) denotes the probability that the extreme type \( e \) of party \( P \) implements policy \( x_t^P \) when party reputations are given by \( b \in [0, 1]^2 \). I express the choice of the voter between the two parties as a re-election strategy \( \sigma_M : [0, 1]^2 \times \{L, R\} \rightarrow [0, 1] \), so that this strategy is conditioned on the pair of beliefs about the two parties as well as on the identity of the incumbent government party. Accordingly, the voter re-elects the incumbent party in government, party \( P \), by setting \( \sigma_M(b, P) = 1 \), and elects the opposition party by setting \( \sigma_M(b, P) = 0 \). I denote a strategy profile for all three players by \( \sigma = (\sigma_M, \sigma_L, \sigma_R) \).

Players update their beliefs regarding the extremism of the two parties at two stages within a period. First, players update beliefs about the type of the party in government after observing its policy choice. Let \( \beta(h, x; \sigma) \), where \( \beta : [0, 1]^2 \times X \rightarrow [0, 1] \), denote the updated belief of the electorate about the type of the governing party \( P \) after observed policy \( x \in X \), given party reputations \( b \in [0, 1]^2 \) and strategies given by \( \sigma \). The second change in party reputations between periods \( t \) and \( t + 1 \) occurs due to the possibility of internal ideological shifts within the two parties. In particular, if the opposition party \( P \)

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8 Since moderate partisan types always pursue the same action, I only specify payoffs for the voter and the extreme partisan types of each party.

9 While the office payoff \( G \) is independent of the type that prevails within the party, I can easily accommodate different office payoffs for extreme partisans depending on the type that controls the party when the party is in office.

10 This assumption regarding the time horizon of political parties can be viewed as a consequence of an overlapping generations structure of politicians within parties as suggested in Footnote 6.

11 For a dynamic game in which players’ Markov strategies are conditioned on beliefs in a similar fashion, see Mailath and Samuelson (2001).
has reputation $b'_o$ in period $t$, then the probability that the party is perceived to be extreme in period $t+1$ is given by

$$b'_o \pi'_o + (1 - b'_o)(1 - \pi'_m).$$

The first term in the above sum represents the probability that the party remains extreme in period $t+1$ given that it was extreme in period $t$, while the second term reflects the probability that the party becomes extreme in period $t+1$, given that it was moderate in period $t$. In order to compactly express these changes in party reputations, I define a transition function $T_o: [0, 1] \rightarrow [1 - \pi'_m, \pi'_o]$ for the reputation of the opposition party as $T_o(b) = \pi'_o b + (1 - \pi'_m)(1 - b)$. A similar change in reputations occurs for the governing party, so I similarly define a function $T_g: [0, 1] \rightarrow [1 - \pi'_m, \pi'_o]$ for the governing party given by $T_g(b) = \pi'_o b + (1 - \pi'_m)(1 - b)$. According to the above, if beliefs at the beginning of period $t$ are given by $b' \in [0, 1]^2$, if the party in government in period $t$ is $P'$, and if this party implements a policy $x' \in X$, then the beliefs of the electorate in period $t+1$ are given by $b'^{t+1} = b'(b', x'; \sigma)$, where the coordinate of function $b': [0, 1]^2 \times X \rightarrow [0, 1]^2$ that corresponds to party $P$ takes the form

$$b'_P(b', x'; \sigma) = \begin{cases} T_g(b(b', x'; \sigma)), & \text{if } P = P', \\ T_o(b'_P), & \text{if } P \neq P'. \end{cases}$$

(3)

Let $V(b, P; \sigma)$ denote the expected payoff of voter $M$ from electing party $P$ when party reputations are given by $b \in [0, 1]^2$ and strategies are denoted by $\sigma$. This payoff is given by

$$V(b, P; \sigma) = (1 - b_P \sigma_P(b))v'_m + b_P \sigma_P(b)v'_e.$$  (4)

Similarly, the expected payoff of party $P$ from implementing a policy $x'_e$ while in government in a period with party reputations $b$ is given by

$$U_P(b, x'_e; \sigma) = r'_e + G + \delta[\sigma_M(b', P)(1 - \pi'_e \sigma_P(b'))\pi'_m + \pi'_e \sigma_P(b')G' + G] + (1 - \sigma_M(b', P))(1 - \sigma'_{\sigma_p} - b'_P \sigma_P(b')r'_p]$$  (5)

where $b' = b'(b, x'_e; \sigma)$. Observe that the expected utility calculation in Eq. 5 reflects the uncertainty of extreme types of party $P$ regarding the type prevailing in the next period both in the opposition party, $-P$, as well as within their own party, $P$. I can now state the definition of the equilibrium concept which is a version of Markov Perfect Bayesian Nash equilibrium:

**Definition 1** An equilibrium is a triple of strategies $\sigma^* = (\sigma^*_M, \sigma^*_L, \sigma^*_R)$ such that

$$\sigma^*_M(b, P) = \begin{cases} 1, & \text{if } V(b, P; \sigma^*) > V(b, -P; \sigma^*), \\ 0, & \text{if } V(b, P; \sigma^*) < V(b, -P; \sigma^*). \end{cases}$$

(6)

and

$$\sigma^*_P(b) = \begin{cases} 1, & \text{if } U_P(b, x_P; \sigma^*) > U_P(b, x'_P; \sigma^*), \\ 0, & \text{if } U_P(b, x'_P; \sigma^*) < U_P(b, x'_P; \sigma^*). \end{cases}$$

(7)
for all reputations $b \in [0, 1]^2$ and all parties $P$, and a reputations updating rule $b'$ that satisfies Eq. 3 and is such that the updating function $\beta$ satisfies

$$\begin{align*}
\beta(b, x; \sigma) &= \begin{cases} 
(1 - \sigma_p^P(b))b_P, & \text{if } x = x_m^P, b_P\sigma_p^P(b) < 1, \\
1 - \sigma_p^P(b)b_P & \text{if } x = x_e^P.
\end{cases}
\end{align*}$$

(8)

In Eq. (8), I require players to use Bayes’ rule to update their beliefs about the governing party after observing its policy choice, but I also effectively restrict certain out-of-equilibrium beliefs by assuming that other players believe the government party is extreme with probability one after observing an extreme policy even when its strategy dictates a moderate policy with probability one ($\sigma_p^P(b) = 0$). I do not restrict out-of-equilibrium beliefs in cases in which a moderate policy is observed and $b_P\sigma_p^P(b) = 1$, but the flexibility allowed by the absence of any such restriction is of no consequence for the equilibrium analysis that follows. Indeed, such a restriction is redundant if $\pi_p^e < 1$ since then we cannot have party $P$ with reputation $b_P = 1$ in periods other than (possibly) the first along any path of play.

Note that the above definition of equilibrium (specifically Condition (6)) does not restrict the voter’s strategy in cases in which the voter is indifferent between the two parties. Such indifference may arise, for example, when both parties are expected to pursue a moderate policy with probability one following the election. In such a case it is reasonable to expect that the electorate’s indifference is resolved in favor of the party that is considered less likely to be extreme. Such would be the resolution of the voter’s indifference if, for example, parties tremble with some fixed probability so that the voter expects such a tremble to an extreme policy with higher probability from the party that is perceived to be more extreme. Instead of refining the equilibrium concept using such trembles, I focus on a simpler class of equilibria. In particular, I define a responsive equilibrium as follows:

**Definition 2** An equilibrium with strategies $\sigma^*$ is responsive if the voting strategy $\sigma^*_M$ satisfies

$$\sigma^*_M(b, P) = \begin{cases} 
1, & \text{if } b_P < b_{-P}, \\
0, & \text{if } b_P > b_{-P}.
\end{cases}$$

(9)

Although more restrictive than the class of refined equilibria I could obtain by invoking trembling-hand arguments, a responsive equilibrium ensures that the electorate votes for the party with the best reputation when indifferent. With the solution concept clarified, I proceed to the analysis of the game. First, I consider analogues of pooling and separating equilibria for this game in the next section. In such equilibria, extreme partisan types pursue the same policy (moderate or extreme, respectively) independent of party reputations, hence I call these equilibria simple. The chief equilibrium results are contained in the ensuing section, and concern responsive equilibria that are not simple and involve parties that place high weight on office (high $G$) and on the future (high $\delta$). Then, in the penultimate section I discuss equilibrium dynamics and other equilibrium properties.

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12 I pursue this approach in the working paper version of this paper.
SIMPLE EQUILIBRIA

In this section, I consider two simple types of equilibria in which the two parties’ strategies do not depend on party reputations. First, in Proposition 1 I give a precise range of parameters for which extreme partisan types implement extreme policies whenever in power:

Proposition 1

(i) An equilibrium in which extreme party types pursue an extreme policy independent of party reputations, that is, an equilibrium with party strategies satisfying \( \sigma_P(b) = 1 \) for all \( b \) and \( P \), exists if and only if

\[
\delta \leq \frac{r_L^R - r_L^m}{G + r_R^R - r_L^m + \pi_c^L (r_L^L - r_L^c) + \pi_c^R (r_R^R - r_R^m)}. \tag{10}
\]

(ii) Every equilibrium with party strategies that satisfy \( \sigma_P(b) = 1 \) for all \( b \) and \( P \) is responsive.

(iii) If Inequality (10) is strict then party strategies satisfy \( \sigma_P(b) = 1 \) for all \( b \) and \( P \) in every equilibrium.

Part (iii) of the proposition establishes that when Condition (10) holds with the inequality strict, all equilibria of the game involve extreme partisan types pursuing their ideal policy, independent of the electorate’s beliefs. This is despite the fact that in these equilibria parties implementing extreme policies lose the election with probability one, as is implied by the fact that these equilibria are responsive by part (ii) of the proposition. Thus, according to Condition (10) such punishment is not sufficient to induce moderation when either (a) parties are impatient (low \( \delta \)), (b) parties place low value on office (low \( G \)), (c) the loss in utility due to the policies pursued by the opposition party controlling the government is small (low \( r_R^R - r_L^m \)), or (d) the ability of extremists to maintain control of their party following electoral defeat is small (low \( \pi_o^c, \pi_g^c \)).

One may conjecture that when these conditions are reversed we may instead obtain a simple pooling equilibrium in which extreme partisan types always imitate the moderate partisan types by pursuing a moderate policy. It is possible to construct such equilibria that exploit voters’ indifference (for high enough \( G \) and \( \delta \)), but these equilibria do not survive the refinement in Definition 2. Indeed I can show that there does not exist a responsive pooling equilibrium:

Proposition 2 There does not exist a responsive equilibrium such that \( \sigma_P(b) = 0 \) for all \( b \) and \( P \).

The reasoning behind Proposition 2 is straightforward. If all party types choose moderate policies independent of the electorate’s beliefs, then the electorate does not update its belief about the governing party upon observing its policy choice because both party

\[\text{[13] In fact, such equilibria do not survive a milder refinement motivated by trembling-hand ideas, as shown in Kalandrakis (2008).}\]
types pursue the same policy with probability one. Since the voter is indifferent between the two parties, the voter elects the party that has a (strictly) better reputation in a responsive equilibrium. Thus, a party that is in government, controlled by extremists, and perceived to be more extreme than the opposition, even after implementing a moderate policy, has no incentive to pursue such a moderate policy. This party faces electoral defeat independent of its policy choice, so types in control of the party might as well pursue their ideal policy.

In combination, Propositions 1 and 2 imply that when Condition (10) fails, a responsive equilibrium must involve some configuration of reputations for which there is a positive probability of extreme policies pursued by extreme types as well as reputations for which such types choose a moderate policy with positive probability. I take up the analysis of these more interesting equilibria in the next section.

EQUILIBRIUM WITH OFFICE MOTIVATIONS

My goal in this section is to establish an equilibrium assuming that Condition (10) fails and parties are sufficiently patient and motivated predominantly by office considerations (high $G$). When parties are strongly motivated by office, they are willing to pursue a moderate policy if such a policy secures them re-election. Note that for any strategies $\sigma$, a governing party’s reputation cannot deteriorate following a choice of a moderate policy (i.e., we have $\beta(b_P, x_m^P; \sigma) \leq b_P$). Hence, the updated pair of party reputations determined by Eq. 3 ensures that the incumbent party enjoys a better reputation than the opposition in the election if $T_g(b_P) < T_o(b_{-p})$, since

$$b'_p(b, x_m^P; \sigma) \leq T_g(b_P) < T_o(b_{-p}) = b'_{-p}(b, x_m^P; \sigma).$$

Thus, in a responsive equilibrium the governing party $P$ will pursue a moderate policy with probability one for all party reputations $b \in [0, 1]^2$ such that $T_g(b_P) < T_o(b_{-p})$. Such a (relative) reputational advantage and a moderate policy lead to re-election with probability one, and partisans who care a lot about office will follow a moderate policy in all such cases.

The situation is rather different when the governing party $P$ is in power with party reputations $b$ in the set $B_P$ defined by

$$B_P = \{ b \in [0, 1]^2 : T_g(b_P) > T_o(b_{-p}) \}.$$ 

If the government party pursues a moderate policy with probability one with such reputations, the policy of the government conveys no information to the electorate

\[14\] Of course, Proposition 2 does not preclude the existence of equilibria with moderate policies implemented independently of party reputations, if we expand the class of equilibria under study and allow players to use more complex history-dependent strategies. As pointed out by a referee, such equilibria and many more can be obtained as long as we expand the time horizon of the players so that they place sufficient weight on the future.
regarding the government’s type (i.e., $\beta(b_P, x^P_m; \sigma) = b_P$). Hence, according to Eq. (3), party reputations in the upcoming election satisfy $b'_P(b, x^P_m; \sigma) = T_g(b_P) > T_o(b_{-P}) = b'_{-P}(b, x^P_m; \sigma)$ and party $P$ loses the election despite its attempt to appear moderate. Thus, pursuing a moderate policy with probability one with these reputation levels is not part of a responsive equilibrium. Similarly, it is not an equilibrium choice for extremists of party $P$ to implement an extreme policy with probability one. If such an extreme policy were expected from extremists with probability one, then extremists would have an incentive to deviate and implement a moderate policy instead, which would convince the electorate that the party is moderate and would secure them re-election. Thus, the only possibility for equilibrium policy making by party $P$ with reputation levels $b \in B_P$ is a mixed strategy. As I show in Proposition 3, by allowing the voter to also use a mixed strategy when the two parties’ reputations are identical, such a responsive equilibrium exists. The equilibrium is such that the governing party’s mixture probability between moderate and extreme policies makes it barely competitive against its opponent in the elections when the realization of the party’s randomization is a moderate policy.

Proposition 3 Assume

$$\delta > \frac{r^R_c - r^R_m}{G + r^R_m - r^L_m}. \quad (11)$$

(i) There exists a responsive equilibrium such that party strategies satisfy

$$\sigma_P(b) = \begin{cases} 
T_g(b_P) - T_o(b_{-P}) \over b_P(\pi^e_m - T_o(b_{-P})), & \text{if } T_g(b_P) > T_o(b_{-P}), \\
0, & \text{otherwise.}
\end{cases} \quad (12)$$

for each $P$.

(ii) The probability of an extreme policy by a government of party $P$ in this equilibrium, $b_P\sigma_P(b)$, (weakly) increases with the governing party’s reputation $b_P$ and (weakly) decreases with the reputation of the opposition, $b_{-P}$.

(iii) The equilibrium probability of an extreme policy in a period with party reputations $b$ is positive if and only if $b \in B_e = B_L \cap B_R \subset (b^*, 1]$, where

$$b^* = \frac{\pi^m_m - \pi^m_o}{\pi^m_m - \pi^m_e + \pi^e_m - \pi^e_e}. \quad (13)$$

(iv) For a fixed reputation level for party $P$, $b_P$, the equilibrium probability of an extreme policy is maximized when party $-P$’s reputation equals that of party $P$ ($b_P = b_{-P}$) and (weakly) decreases with the absolute difference between $b_{-P}$ and $b_P$.

The equilibrium in Proposition 3 holds for arbitrarily large values of the office payoff $G$, as long as parties place some weight on the future ($\delta > 0$). Thus, no matter how much weight parties place on office, there exists a configuration of party reputations that makes
it worthwhile for extreme partisan types to pursue extreme policies when in government. Figure 1(a) displays a contour plot of the equilibrium probability of an extreme policy choice by extreme partisans of party $L$ in the space of party reputations $[0, 1]^2$. As I have already discussed, a party must be perceived to be relatively more extreme than its opposition in order for it to pursue extreme policies with positive probability.

Note that from the perspective of the electorate the expected probability that, say, party $L$ will pursue an extreme policy if elected when reputations are given by $b \in [0, 1]^2$ is equal to $b_L \sigma_L(b)$. In Figure 1(b) I plot this probability. This graph illustrates the comparative static in part (ii) of Proposition 3, that is, that the probability of an extreme policy by the governing party increases with that party’s reputation level and decreases with that of the opposition party. The more disadvantaged the government party is, the more surprising a moderate policy by such a government must be in order for this policy to convince the electorate that the governing party is as likely to be moderate as the opposition, rendering the subsequent election competitive. Thus, the worse the governing party’s reputation is relative to the opposition, the more likely it is that that party will pursue an extreme policy in equilibrium.

The fact that the party strategies in the equilibrium of Proposition 3 are such that governments may pursue extreme policies for some configurations of party reputations is not sufficient to produce extreme policies in equilibrium. Indeed, the voter can ensure that a moderate policy prevails with probability one for reputation levels such that one of the two competing parties pursues a moderate policy with probability one, simply by electing that party. Hence, a positive probability of extreme policies arises in equilibrium only when the voter is forced to choose the better between two evils, that is, for reputation levels such that both parties would pursue extreme policies with positive probability if elected. As stated in part (iii) of Proposition 3, the set of such reputation levels, $B_e$, lies in the upper quadrant of the space of party reputations. Part (iv) of Proposition 3 also states that the probability that the elected party pursues such an extreme policy is higher, the closer the two parties’ reputations are.
In combination, parts (iii) and (iv) of Proposition 3 imply that two conditions produce extreme policies in equilibrium: (a) the two parties are perceived to be relatively extreme (both have reputations above level \( b^* \) defined in Eq. (13) of Proposition 3), and (b) the two parties have similar reputation levels. If we interpret the proximity of the two parties’ reputation levels as a proxy for the closeness of the election, then the second of the above two conditions states that extreme policies emerge with higher probability after close elections. In order to understand the interaction between these two conditions, we must take into account the stochastic nature of latent party preferences determination assumed in Inequality (1). In particular, any time either party’s reputation is extremely high or low, that reputation tends to adjust to more moderate levels due to a straightforward regression to the mean effect.\(^{15}\) Inequality (1) states that the speed of this adjustment is higher for parties in the opposition than for parties in government. As a result, without updating from the government’s policy choice, the opposition party enjoys a larger improvement in reputation when both parties have high reputation levels (i.e., when they are both perceived to be extreme with high probability). If, in addition, both parties’ initial reputation levels are close to each other, then we have a situation in which the government won the previous election with a (small) reputational advantage but faces a new election with a (small) reputational disadvantage. As I have already discussed prior to the statement of Proposition 3, it is exactly the anticipation of this future electoral disadvantage that produces extreme policies in equilibrium. If, on the other hand, both parties have low reputation levels (i.e., they are both perceived to be extreme with low probability), then both parties tend to adjust to higher reputation levels, and the opposition party experiences a larger deterioration in its reputation due to Inequality (1). Thus, in equilibrium, the government party implements a moderate policy with probability one, even if the two parties have identical reputation levels, as long as these reputation levels are lower than the level \( b^* \) given in Eq. (13). Figure 1(c) plots the probability of an extreme policy by the party elected in government as a function of the two parties’ reputation levels.

While Proposition 3 establishes the existence of a responsive equilibrium, it leaves open the possibility that additional equilibria may exist. In the next Proposition, I show that under an additional condition, the equilibrium of Proposition 3 is essentially unique in the class of responsive equilibria.

Proposition 4 Assume Eq. (11) and

\[
\left( r^R_c - r^R_m \right) > \frac{\delta(T_g(\pi^g_e) - T_o(\pi^o_e))}{\pi^g_e - T_o(\pi^o_e)} \left( r^L_m - r^L_c \right),
\]  

Then party strategies satisfy Eq. (12) in every responsive equilibrium.

Condition (14) is a mild sufficient condition for uniqueness that is guaranteed to hold if, for example, \( T_g(\pi^g_e) \leq T_o(\pi^o_e) \) or \( (r^R_c - r^R_m) \geq \delta(r^L_m - r^L_c) \). Proposition 4 enhances the significance of the comparative statics established in Proposition 3. In the next section, I turn to a study of the dynamics induced by this equilibrium.

\(^{15}\) This is established formally in parts (i) and (ii) of Lemma 1.
REPUTATION AND POLICY DYNAMICS

The equilibrium I have established in Proposition 3 produces extreme policies following elections in which both parties are perceived to be relatively extreme and have similar reputation levels. Whether the combination of these two conditions on party reputations arises frequently in equilibrium depends on the dynamics induced by this equilibrium. In order to decipher these dynamics, we must determine the direction and rate of change of the reputation of the government and the opposition along the path of play. To that end, observe first that irrespective of equilibrium strategies, the reputation of the opposition party adjusts monotonically toward a (non-equilibrium) steady-state level

\[ b^o = \frac{1 - \pi^o_m}{2 - \pi^o_m - \pi^o_e}, \]

where \( b^o \) uniquely solves the equation \( T_o(b^o) = b^o \). Second, if we assume that there is a positive probability that party preferences change while the party is in government (i.e., if \( \pi^g_m + \pi^g_e < 2 \)) and that the government’s policy choice is not informative about its type, then the governing party’s reputation monotonically adjusts towards a level given by

\[ b^g = \frac{1 - \pi^g_m}{2 - \pi^g_m - \pi^g_e}, \]

where in this case \( b^g \) uniquely solves \( T_g(b^g) = b^g \). In order to combine these two remarks to determine the reputation dynamics induced in equilibrium, I first establish a lemma concerning the two reputation levels \( b^o \) and \( b^g \).

**Lemma 1** For all \( \pi^o_e, \pi^o_m \) and all \( \pi^g_e, \pi^g_m \):

(i) If \( b > b^o \) then \( b > T_o(b) > b^o \) and if \( b < b^o \) then \( b < T_o(b) < b^o \).

(ii) If \( \pi^e_m + \pi^e_m < 2 \), then \( b > T_g(b) > b^g \) if \( b > b^g \) and \( b < T_g(b) < b^g \) if \( b < b^g \).

(iii) If \( \pi^e_m + \pi^e_m < 2 \) and for \( b^* \) defined in Eq. (13), \( b^g > b^o > b^* \) if and only if

\[ \frac{1 - \pi^g_m}{1 - \pi^o_m} < \frac{1 - \pi^e_m}{1 - \pi^o_e}. \] (15)

Parts (i) and (ii) of Lemma 1 establish the monotonic convergence of reputations towards the non-equilibrium steady-state levels \( b^o \) and \( b^g \), respectively. Condition (15) in part (iii) provides a criterion that ranks the non-equilibrium steady-state reputation levels \( b^o \) and \( b^g \). The ratio \( (1 - \pi^e_m)/(1 - \pi^o_m) \) is the relative probability with which parties switch from being extreme to being moderate when they are in government versus when they are in opposition, while the ratio \( (1 - \pi^e_m)/(1 - \pi^o_m) \) represents the relative probability with which parties switch from being moderate to being extreme when they are in government versus when in opposition. Thus, part (iii) of Lemma 1 states that the non-equilibrium steady-state reputation for parties in opposition, \( b^o \), is lower than that for parties in government, \( b^g \), when parties in government tend to switch from
being moderate to extreme with higher relative frequency than the relative frequency of switching from being extreme to being moderate.

The next step in this analysis is to show that, despite the fact that $b^o$ and $b^g$ constitute non-equilibrium steady-state levels of reputation, these quantities constrain the possible long-term dynamics on party reputations in the equilibrium of Proposition 3 as follows:

**Lemma 2** Assume party strategies given by Eq. (12). For every (possibly non-equilibrium) voting strategy $\sigma_M$, every pair of initial reputations $b^0 \in [0,1]^2$, and every sequence of party reputations $b^0, b^1, \ldots, b^t, \ldots$ induced by Eq. (3) and Eq. (8):

(i) If $\pi^g_{e} + \pi^g_{m} < 2$, then

$$\lim_{t \to +\infty} \min \{b^L_t, b^R_t\} \geq \min \{b^o, b^g\}$$

and

$$\lim_{t \to +\infty} \max \{b^L_t, b^R_t\} = \max \{b^o, b^g\}.$$

(ii) If $\pi^g_{e} = \pi^g_{m} = 1$, then

$$\lim_{t \to +\infty} \min \{b^L_t, b^R_t\} \geq \min \{b^0_L, b^0_R, b^o\}$$

and

$$\lim_{t \to +\infty} \max \{b^L_t, b^R_t\} = \max \{\min \{b^0_L, b^0_R\}, b^o\}.$$

Note that Lemma 2 is general in that it allows the voter to employ a strategy that differs from the equilibrium strategy in Proposition 3. The implications of part (i) of Lemma 2 are depicted graphically in Figure 2. In particular, the lemma ensures that in the long-run, party reputations are confined to the dark gray areas of that figure. Figure 2 also depicts the area of joint party reputations in which extreme policies occur with positive probability in equilibrium. Note that on the basis of Figure 2 there are at least two (apparently) substantively different dynamics that can emerge in the long-run, as the area of long-run reputations and the area of reputations resulting in extreme policies do not intersect in the case of Figure 2(a), while the two areas intersect in the case of Figure 2(b). In either case, Lemma 2 places significant restrictions on the possible location of long-run reputations and, in combination with part (iii) of Lemma 1, it permits a detailed characterization of equilibrium dynamics. In particular, Inequality (15) of Lemma 1 provides a succinct criterion to categorize the dynamics that are possible in the equilibrium of Proposition 3 into three main cases which I now discuss. These cases are illustrated in Figure 3.

**Case I** ($\pi^g_{e} + \pi^g_{m} < 2$ and $b^g > b^o > b^*$): Policy Extremism

When Inequality (15) holds then the non-equilibrium steady-state reputation levels of both the opposition and governing parties ($b^o$ and $b^g$, respectively) exceed the reputation...
Figure 2. Implications of Lemma 2(i). In the long-run, party reputations are confined to the dark gray area. (a) If Inequality (15) holds, then the area of possible long-run reputations overlaps with the area of reputations (in light gray) for which extreme policies are implemented with positive probability in equilibrium. (b) If Inequality (15) is reversed, then party reputations in the long-run do not reach the area of reputations for which extreme policies are implemented with positive probability in equilibrium.

level $b^o$ defined in equation Eq. 13 of Proposition 3. By part (i) of Lemma 2, we conclude that in the long-run the minimum party reputation, $\min\{b_L^t, b_R^t\}$, varies in the interval $[b^o, b^g]$ so that the two parties’ reputations are absorbed in the area depicted in Figure 2(a). Since the reputation of the opposition party adjusts toward the level $b^o$ that is lower than the corresponding level for the governing party, $b^g$, the party in government must eventually find itself in a reputational disadvantage vis-a-vis the opposition party. Thus, a pattern of policy making emerges whereby governments come into power with a reputational advantage, implement moderate policies during their initial terms in office, but eventually pursue extreme policies with positive probability as their initial advantage over the opposition party dissipates. These dynamics then lead to regularly occurring extreme policies and incumbent governments that are replaced by the opposition party. Figure 3(I) displays a sample path of play exhibiting these dynamics. In that figure, each point represents the pair of reputations for the two parties in each period along with the party in government in that period (solid circles correspond to a government by party L, open circles to a government by party R), and arrows represent reputation transitions in successive periods. Periods with extreme policies
Figure 3. Equilibrium dynamics. Each point corresponds to the pair of reputations for the two parties in each period and indicates the party in government in that period. Arrows represent reputation transitions in successive periods. (I) When $b^g > b^o > b^*$, equilibrium features extreme policies and alternation of parties in government in the long-run. (II) When $b^g < b^o < b^*$, moderate policies prevail in the long-run and the government enjoys a strong incumbency advantage. (IIIa) When $b^o = b^*$ and $\pi_{m}^{g} + \pi_{m}^{o} < 2$, policy and electoral dynamics (but not long-term reputations) depend on initial reputations. (IIIb) When $b^o = b^*$ and $\pi_{m}^{g} = \pi_{m}^{o} = 1$, policy and electoral dynamics and long-term reputations depend on initial reputations.
are followed by a transition to the highest possible reputation level for the incumbent party.

Case II ($\pi_g^e + \pi_m^g < 2$ and $b^g < b^o < b^r$): Policy Moderation & Incumbency Advantage

Unlike the equilibrium dynamics I characterized in the previous case, part (i) of Lemma 2 now implies that when $b^g < b^o < b^r$, party reputations eventually lie outside the subset $[b^o, 1]^2$ with probability one, since the minimum party reputation, $\min \{b^o_L, b^o_R\}$, eventually lies in the interval $[b^g, b^o]$. As a consequence, except possibly for some initial periods, governments always implement a moderate policy with probability one, by part (iii) of Proposition 3. Furthermore, once party reputations satisfy $b^g \notin [b^o, 1]^2$, the government party wins re-election with probability one by pursuing a moderate policy. Note that in this case the reputation of the opposition party adjust toward the level $b^o$, that is higher than that of the governing party, $b^g$. As a result, we obtain dynamics such that either party reputations converge to $(b^g, b^o) \notin B_L$ and a government by party $L$ implements a moderate policy and wins re-election perpetually, or party reputations converge to $(b^o, b^g) \notin B_R$ and party $R$ implements a moderate policy and wins re-election perpetually, that is, the equilibrium exhibits policy moderation and an extreme form of incumbency advantage. Figure 3(II) displays two paths of play with these dynamics. Note that either party can benefit from these dynamics in the long-term, even though the two parties are treated symmetrically in the model except possibly for their initial reputation levels.

Case III ($b^o = b^r$)

In the special case when the ratios in Inequality (15) are exactly equal, the equilibrium induces two qualitatively different dynamics that depend on initial reputations. These dynamics share features with the dynamics I described in case I when the initial reputations satisfy $b^0 \in [b^o, 1]^2$. In particular, if $b^0 \in [b^o, 1]^2$, then party reputations may reach the set $B_o$ along the path of play, so that extreme policies are possible in equilibrium. Nevertheless, the minimum party reputation converges towards the long-run level $b^g$ by Lemma 2 so that a moderate policy prevails with probability one in the long-run and party reputations converge in probability to the pair $(b^g, b^o)$. These dynamics are depicted in the upper quadrant of the space of party reputations in Figures 3(IIIa) and (IIIb). If, on the other hand, initial reputations satisfy $b^0 \notin [b^o, 1]^2$ then equilibrium dynamics share features with the dynamics I described in case II. Specifically, if $b^0 \notin [b^o, 1]^2$, then governments implement moderate policies with probability one, and one of the two parties wins office perpetually. If $\pi_g^e + \pi_m^g < 2$, then party reputations converge to the level $(b^g, b^o)$, as is the case in the path of play depicted in the lower quadrant of Figure 3(IIIa). If $\pi_g^e + \pi_m^g = 2$, then party reputations may converge either to $(b^0_L, b^o)$ or to $(b^o, b^0_R)$, as is the case in the path of play depicted in the lower quadrant of Figure 3(IIIb).

I summarize the dynamics of the equilibrium in Propositions 3 and 5.
Proposition 5 Consider the sequence of reputations, parties in government, and policies induced by the equilibrium in Proposition 3. In the long-run:

(i) If \( b^g > b^o > b^* \), then party reputations lie in the set \([b^o, 1]^2\), and alternation of parties in government and extreme policies occur regularly along the path of play.

(ii) If \( b^g < b^o < b^* \), then party reputations converge to one of two reputation pairs \((b^o, b^g)\) or \((b^*, b^o)\), moderate policies prevail with probability one, and one of the two parties is perpetually in office.

(iii) If \( b^o = b^* \) and if initial reputations \( b^0 \in [b^o, 1]^2 \), then the probability of extreme policies converges to zero and party reputations converge in probability to \((b^o, b^o)\). If initial reputations satisfy \( b^0 \in [0, 1]^2 \setminus [b^o, 1]^2 \) then extreme policies are pursued with probability zero in all periods, and:

(a) If \( \pi^g_{1r} + \pi^o_{1r} < 2 \), then party reputations converge to \((b^o, b^o)\) with one of the two parties perpetually in power.

(b) If \( \pi^g_{1r} = \pi^o_{1r} = 1 \), then either party reputations converge to \((b^0_L, b^o)\) with a government by party \(L\), or party reputations converge to \((b^o, b^0_R)\) with a government by party \(R\).

Parts (i), (ii), and (iii) of Proposition 5 correspond to the three Cases I, II, and III, respectively, that I identified in the discussion of the dynamics induced by the equilibrium of Proposition 3. According to Proposition 5, the equilibrium is consistent with two radically different patterns of policy-making by the government depending on the values of the persistence of preferences probabilities \(\pi^o\) and \(\pi^g\). One pattern of policy-making involves regularly occurring extreme policies and alternation of parties in government as described in the modal Case I, and the other involves a strong incumbency advantage and moderate policies as in the other modal Case II. The driving force behind these dynamics is the relative probability with which parties switch from being moderate to being extreme when in government versus when in opposition, as quantified in Inequality (15). The dependency of equilibrium dynamics on the direction of Inequality (15) suggests an obvious question for future research, that is, in order to identify the relevant equilibrium dynamic we must further study the process of internal party competition and the forces that determine the transition probabilities \(\pi^o\) and \(\pi^g\).

The dynamics identified in Case II (part (ii) of Proposition 5) involve a stark version of an incumbency advantage for the government, since (eventually) the government remains in power with probability one in all periods. Kalandrakis (2008) studies an extension of the baseline model I have considered that allows for the possibility of probabilistic elections. In this extension, there is some exogenous probability that the outcome of the election favors either party due to surprise events such as scandals, wars, etc., independent of the underlying (systematic) preferences of the voter. Obviously, once we assume this type of probabilistic elections, the strong incumbency advantage obtained in part (ii) of Proposition 5 is no longer possible and any government must lose the election with positive probability in equilibrium. In fact, with this assumption it is even possible that governments that implement extreme policies are re-elected in
Kalandrakis (2008) shows that the dynamics induced in the model that allows for probabilistic election, although more realistic, are otherwise consistent with the dynamics I identify in Proposition 5.

CONCLUSIONS

I have analyzed a dynamic model of two-party competition based on the assumption that political parties enter the electoral arena with endogenously formed reputations regarding latent policy preferences within each party. Instead of relying on campaign promises in order to infer the policies of future governments, the electorate forms expectations about these policies based on the two parties’ reputations and equilibrium strategies. In turn, equilibrium government policies are determined not only by calculations of their effect on the incumbent party’s present and future reputation and the likely behavior of the electorate, but also by taking into account the opposition party’s reputation and strategy.

I have showed that in responsive equilibria in which parties care sufficiently about office, the ruling party pursues extreme policies when it has a relatively worse reputation compared to the opposition. Extreme policies occur in equilibrium when (a) both parties’ reputations are above some benchmark level and (b) elections are close, that is, both parties have similar reputations. The model is consistent with two radically different electoral and policy dynamics. One possible pattern of dynamics involves regular government turnover and the recurrence of party reputations such that the party that wins the election implements extreme policies with positive probability. The second pattern of dynamics involves moderate policies and a strong incumbency advantage for the governing party. Either dynamic may prevail depending on the relative speed with which parties switch preferences while in government or in the opposition, according to Inequality (15).

The radically different equilibrium dynamics that can be generated by the model suggest that in order to fully understand the nature of two-party competition we must further study the manner in which competition between different ideological groups is resolved within political parties. This shift of focus from inter-party competition to intra-party competition would allow us to develop insights on the forces that determine the relative size of quantities that are exogenous in the present analysis, such as the probabilities of persistence of latent party preferences \( \pi^g \) and \( \pi^o \). Kalandrakis and Spirling (2009) take up the empirical task of estimating these quantities using observed data and a likelihood function derived from the equilibrium of the model.

Besides prompting a shift of focus from inter-party competition to intra-party competition, the present model leaves a number of other open avenues for improvement. One such improvement involves an increase of the time horizon which affects the strategic calculations of political actors. Perhaps the most important extension of the current model, though, would be to enrich the policy/type space by allowing more than two policy choices and party types per party. More party types and policy choices open the possibility for richer dynamics such that, for example, the governing party pursues a
relatively extreme policy even when it has a large reputational advantage over the opposition party. If the opposition party has a really bad reputation, then the governing party may be able to afford to pursue such intermediate policies and maintain a smaller advantage over the opposition. In particular, the worsening of the government’s reputation due to the fact that it does not pursue the most moderate policy need not come at the cost of losing the elections.

APPENDIX

In this section I prove Propositions 1–4, and Lemmas 1 and 2. I start with Proposition 1:

Proof of Proposition 1. I show parts (ii) and (iii) first, and I conclude with the proof of part (i) which relies on arguments used to prove parts (ii) and (iii). I start with part (ii).

Consider an equilibrium with strategies \( \sigma = (\sigma_M, \sigma_L, \sigma_R) \), such that party strategies satisfy \( \sigma_P(b) = 1 \) for all \( b \in [0, 1]^2 \) and \( P \in \{L, R\} \). Using Eq. 4, the voter’s expected utility in equilibrium \( \sigma \) is given by \( V(b, P; \sigma) = b_P (v_e^P - v_e^m) + v_P \), and it follows that

\[
V(b, P; \sigma) > V(b, -P; \sigma) \iff b_P < b_{-P}.
\]

Since \( \sigma \) is an equilibrium, it must also be the case that

\[
b_P < b_{-P} \iff V(b, P; \sigma^0) > V(b, -P; \sigma^0) \iff \sigma_M(b, P) = 1 - \sigma_M(b, -P) = 1.
\]

Hence, the voter’s strategy \( \sigma_M \) satisfies Eq. 9 and equilibrium \( \sigma \) is responsive.

Next, I show part (iii). Suppose that Inequality 10 is satisfied strictly and there exists an equilibrium \( \sigma^* \) with \( \sigma_P^*(b) < 1 \) for some \( b \) to get a contradiction. Then it must be that \( U_P(b, x_m^P; \sigma^*) \geq U_P(b, x_L^P; \sigma^*) \) for these reputations \( b \). Note that there exists a logical lower bound on expected utilities which must satisfy \( U_P(b, x_m^P; \sigma) \geq r_e^R + G + \delta(T_o(b-p)) r_e^R + (1 - T_o(b-p)) r_m^L \) for all \( \sigma \). Since \( T_o(y) \leq \pi_v \) for all \( y \in [0, 1] \) and \( r_m^L > r_e^L \), the lower bound inequality above implies

\[
U_P(b, x_m^P; \sigma) \geq r_e^R + G + \delta(\pi_v x_m^L + (1 - \pi_v) r_m^L)
\]

for all \( b \in [0, 1]^2 \) and all \( \sigma \). Similar reasoning yields an upper bound for \( U_P(b, x_m^P; \sigma) \) so that

\[
U_P(b, x_m^P; \sigma) \leq r_m^R + G + \delta(\pi_v r_e^R + (1 - \pi_v) r_m^R + G)
\]

for all \( b \in [0, 1]^2 \) and all \( \sigma \). Now, the strict version of Inequality (10) yields

\[
\delta < \frac{r_e^R - r_m^R}{\pi_v (r_m^L - r_e^L) + \pi_v (r_m^R - r_e^R) + G + r_m^R - r_m^L}
\]

\[
< \frac{r_m^R + G + \delta(\pi_v r_e^R + (1 - \pi_v) r_m^R + G)}{r_e^R + G + \delta(\pi_v r_e^R + (1 - \pi_v) r_m^R)}
\]
and the latter implies, using Inequalities (16) and (17), that
\[ \delta < \frac{r_e^R - r_m^R}{\pi_e^R(r_m^L - r_e^L) + \pi_e^L(r_e^R - r_m^R) + G + r_m^R - r_m^L} \Rightarrow U_P(b, x_m^p; \sigma) > U_P(b, x_e^p; \sigma) \]
for all \( b \in [0, 1]^2 \) and all \( \sigma \). But this conclusion contradicts the working hypothesis that
\[ U_P(b, x_m^p; \sigma^*) \geq U_P(b, x_e^p; \sigma^*) \]
for some \( b \in [0, 1]^2 \), proving part (iii).

Lastly I prove part (i). First, I verify that there exists an equilibrium with \( \sigma_P(b) = 1 \) for all \( b \) and \( P \) when Inequality (10) holds. Note that Inequalities (16) and (17) yield
\[ \text{Inequality (10)} \Rightarrow U_P(b, x_e^p; \sigma) \geq U_P(b, x_m^p; \sigma) \quad (18) \]
for all \( b \in [0, 1]^2 \) and all \( \sigma \), so that strategy \( \sigma_P(b) = 1 \) for all \( b \in [0, 1]^2 \) is a (weak) best response independent of the voting strategy \( \sigma_M \). As a result, in order to establish existence of equilibrium I need only specify a voting strategy that satisfies Condition (6). By the arguments proving part (ii), any voting strategy that satisfies Eq. 9 also satisfies Condition (6), since the voter is indifferent for all \( b \in [0, 1]^2 \) such that \( b_L = b_R \). Thus, the above strategy profile constitutes a responsive equilibrium when Inequality (10) is true.

It remains to show that there cannot exist an equilibrium with these party strategies when Inequality (10) fails. Suppose instead that Inequality (10) fails and there exists an equilibrium \( \sigma^* \) with \( \sigma^*_P(b) = 1 \) for all \( b \) and \( P \), to get a contradiction. Consider a period \( t \) with party \( P \) of type \( e \) in government. I will show that there exist reputations \( b' \in [0, 1]^2 \) with \( b_P' < 1 \) for which party \( P \) has a profitable deviation from strategy \( \sigma^*_P \). If the governing party follows its strategy and implements a policy \( x_m^p \), then \( b'_{t+1} = b'(b', x_m^p, \sigma^*) \) which by Eqs. (3) and (8) translate to beliefs \( b_p'^{t+1} = \pi_e^R \) and \( b_p'^{t+1} = T_o(b_{t-p}') \) in period \( t + 1 \). Using Inequalities (1) and (2) we then conclude that \( b_p'^{t+1} > b_p'^{t+1} \) for all \( b_{t-p}' \). Furthermore, by part (ii), the voter’s strategy \( \sigma^*_M \) satisfies Condition 9, hence \( \sigma^*_M(b''_{t+1}, P) = 0 \) and the expected payoff of type \( e \) of party \( P \) implementing policy \( x_e^p \) is given by
\[ U_P(b', x_e^p, \sigma^*) = r_e^R + G + \delta(T_o(b_{t-p}')(r_e^L - r_m^L) + r_m^L) \]
On the other hand, a one-period deviation to a moderate policy \( x_m^p \) results in beliefs \( b'_{t+1} = b'(b', x_m^p, \sigma^*) \) in period \( t + 1 \), which by Eqs. (3) and (8) take the values \( b_p'^{t+1} = 1 - \pi_e^R \) and \( b_p'^{t+1} = T_o(b_{t-p}') \). Now \( b_p'^{t+1} = 1 - \pi_e^R < b_p'^{t+1} = T_o(b_{t-p}') \) for all \( b_{t-p}' \) by Inequalities (1) and (2). Thus, implementing \( x_m^p \) leads to \( \sigma^*_M(b'(b', x_m^p, \sigma^*), P) = 1 \), and accrues payoff
\[ U_P(b', x_m^p, \sigma^*) = r_m^R + G + \delta(\pi_e^R(r_e^L - r_m^L) + r_m^R + G) \]
Since \( \sigma^* \) is an equilibrium, it must be that \( U_P(b', x_e^p, \sigma^*) \geq U_P(b', x_m^p, \sigma^*) \) for all \( b' \in [0, 1]^2 \), which (substituting for the expected utilities computed above) satisfies
\[ U_P(b', x_e^p, \sigma^*) \geq U_P(b', x_m^p, \sigma^*) \iff \delta \leq \frac{r_e^R - r_m^R}{T_o(b_{t-p}')(r_m^L - r_e^L) + \pi_e^R(r_e^R - r_m^R) + G + r_m^R - r_m^L} \]
Note that we have assumed that Inequality (10) is violated, i.e., there exists \( \epsilon > 0 \) such that
\[
\frac{r_c^R - r_m^R}{\pi_c(r_m^L - r_c^L) + \pi_c^L(r_m^R - r_c^R) + G + r_m^R - r_m^L} + \epsilon = \delta,
\]
so that we obtain
\[
\frac{r_c^R - r_m^R}{T_o(b_{-p}^t)(r_m^L - r_c^L) + \pi_c^L(r_m^R - r_c^R) + G + r_m^R - r_m^L}
\geq \frac{r_c^R - r_m^R}{\pi_c(r_m^L - r_c^L) + \pi_c^L(r_m^R - r_c^R) + G + r_m^R - r_m^L} + \epsilon,
\]
which is false for any \( b_{-p}^t \) sufficiently close to 1. Thus there exist reputations \( b' \) for which
\[
U_P(b', x_P^b; \sigma^*) < U_P(b', x_m^b; \sigma^*),
\]
contradicting the hypothesis that \( \sigma^* \) is an equilibrium when Inequality (10) is violated.

I continue with the proof of Proposition 2.

**Proof of Proposition 2.** Assume there exists a responsive equilibrium \( \sigma^* \) with strategies \( \sigma_p^*(b) = 0 \), for all \( b \) and \( P \). In order to prove the proposition I will show that there exist party reputations such that \( \sigma_p^* \) is not a best response. Consider a government by party \( P \) in period \( t \) with party reputations \( b' \) such that \( T_g(b_p^t) > T_o(b_{-p}^t) \). With party strategy \( \sigma_p^*(b') = 0 \), reputations in period \( t + 1 \) along the equilibrium path are given by \( b_{-p}^{t+1} = T_o(b_{-p}^t) \) and \( b_p^{t+1} = T_g(b_p^t) \), by application of Eq. (3) and by use of the fact that the posterior belief obtained from Eq. (8) following a moderate policy is \( \beta(b', x_m^b; \sigma^*) = b'_p \). Since \( T_g(b_p^t) > T_o(b_{-p}^t) \) by assumption, we conclude that \( \sigma_p^*(b_{-p}^{t+1}, P) = 0 \), since the equilibrium \( \sigma^* \) is responsive satisfying Eq. (9). If party \( P \) implements an extreme policy \( x_m^b \) in period \( t \) instead, then \( \beta(b, x_m^b; \sigma^*) = 1 \) and it follows that \( b_{-p}^{t+1} = T_o(1) = \pi_c^L > b_{-p}^{t+1} = T_g(b_p^t) \), where the last inequality follows from Inequalities (1) and (2). Thus, when beliefs in period \( t \) are given by \( b' \) such that \( T_g(b_p^t) > T_o(b_{-p}^t) \), it must be that \( \sigma_p^*(b_{-p}^{t+1}, P) = 0 \) whether party \( P \) pursues a moderate or an extreme policy. Substituting in the expected utility expression of Eq. (3) we obtain
\[
U_P(b', x_m^b, \sigma^*) = r_m^P + G + \delta r_m^P < r_c^P + G + \delta r_m^P = U_P(b', x_m^b, \sigma^*).
\]
Thus, there exist \( b' \in [0, 1]^2 \) such that \( \sigma_p^*(b') = 0 \) is not a best response, and there does not exist a responsive equilibrium with such party strategies.

Before proving Proposition 3, I prove two lemmas.

**Lemma 3** Consider party strategy \( \sigma_P \) given by Eq. (12) and reputations \( b \in [0, 1]^2 \) such that \( T_g(b_P) > T_o(b_{-p}) \). Then,
\[
(i) \quad \frac{\partial \sigma_P(b)}{\partial b_R} > 0 \text{ and } \frac{\partial b_P \sigma_P(b)}{\partial b_R} > 0, \quad \text{and}
\]
\[
(ii) \quad \frac{\partial b_P \sigma_P(b)}{\partial b_{-p}} \leq 0 \text{ and } \frac{\partial b_P \sigma_P(b)}{\partial b_{-p}} \leq 0.
\]
Proof: To show part (i), note that
\[
\frac{\partial \sigma_p(b)}{\partial b_p} = \frac{T_o(b_{-p}) - (1-\pi_m^e)}{b_p (\pi_v^e - T_o(b_{-p}))} > 0,
\]
since \(T_o(b_{-p}) \geq 1 - \pi_m^a > 1 - \pi_m^e\) by Inequality (1) and, since the numerator is positive by Inequalities (1) and (2). Part (ii) is similarly obtained since
\[
\frac{\partial b_p \sigma_p(b)}{\partial b_p} = \frac{\pi_v^e + \pi_m^e - 1}{\pi_v^e - T_o(b_{-p})} > 0,
\]
since the numerator is positive by Inequalities (1) and (2). Part (ii) is similarly obtained since
\[
\frac{\partial \sigma_p(b)}{\partial b_p} = \frac{(T_o(b_{-p}) - \pi_v^e)(\pi_m^a + \pi_m^e - 1)}{b_p (\pi_v^e - T_o(b_{-p}))^2} \leq 0, \quad \text{and}
\]
\[
\frac{\partial b_p \sigma_p(b)}{\partial b_p} = \frac{(T_o(b_{-p}) - \pi_v^e)(\pi_m^a + \pi_m^e - 1)}{(\pi_v^e - T_o(b_{-p}))^2} \leq 0,
\]
because \(T_o(b_{-p}) \leq \pi_v^e\).

The next Lemma is:

Lemma 4 Consider any equilibrium strategy profile \(\sigma\) and reputations \(b \in [0,1]^2\).

(i) If \(b_p \sigma_p(b) < 1\) then \(b_p'(b, x_m^p; \sigma) < b_{-p}'(b, x_m^p; \sigma) \iff b_p \sigma_p(b) > \frac{T_o(b_{-p}) - T_o(b_{-p})}{(\pi_v^e - T_o(b_{-p}))}\).

(ii) \(b_p'(b, x_m^p; \sigma) > b_{-p}'(b, x_m^p; \sigma)\).

(iii) If party strategies are given by Eq. (12) and \(b_p < 1\), then \(b_p'(b, x_m^p; \sigma) = b_{-p}'(b, x_m^p; \sigma)\).

Proof: To show part (i) note that since \(\sigma_p(b) b_p < 1\) and from Eqs. (3) and (8) it follows that
\[
b_p'(b, x_m^p; \sigma) < b_{-p}'(b, x_m^p; \sigma) \iff T_o(b_{-p}) < T_o(b_{-p}) \iff T_o(b_{-p}) \iff T_o(b_{-p}) \iff b_p \sigma_p(b) = \frac{T_o(b_{-p}) - T_o(b_{-p})}{(\pi_v^e - T_o(b_{-p}))}.
\]

For part (ii), note that from Eqs. (3) and (8) and Inequalities (1) and (2) it must be that
\[
b_p'(b, x_m^p; \sigma) = T_o(b_{-p}) = \pi_v^e \quad \text{and} \quad b_{-p}'(b, x_m^p; \sigma) = b_{-p}(b, x_m^p; \sigma).
\]

Finally, part (iii) follows immediately from part (i) and Eq. (12).
I now prove Proposition 3.

Proof of Proposition 3 To show part (i), I establish the existence of a responsive equilibrium with party strategies \( \sigma^*_p \) given by Eq. (12), and any voting strategy \( \sigma^*_M \) that satisfies Eq. (9) and

\[
\sigma^*_M(b, P) = \frac{r^R_c - r^R_m + \delta b_R (\sigma^*_R(b) - \sigma^*_R(\pi^*_L, b_R)) (r^L_m - r^L_e)}{\delta[G + r^R_m - r^L_m + \sigma^*_M(b) (\pi^*_R(r^R_e - r^R_m) + b_R(r^L_m - r^L_e))]} \tag{19}
\]

for all reputations \( b = (h_L, h_R) \) such that \( h_L = h_R \in [1 - \pi^*_m, \pi^*_e] \). Let \( \sigma^* \) denote a profile of such strategies and specify out-of-equilibrium beliefs \( \beta(b, x^p_m, \sigma^*) \) for reputations \( b \in [0, 1]^2 \) with \( b_p = 1 \) to \( \beta(b, x^p_m, \sigma^*) = T_s(b_{-p}) \). The proof now proceeds in four steps. First, I show that the probability specified in Eq. (19) is well-defined.

1. The value of \( \sigma^*_M(b, P) \in [0, 1] \) for all \( b \in [0, 1]^2 \) such that \( b_p = b_{-p} \in [1 - \pi^*_m, \pi^*_e] \).

Assume such reputations \( b \). Note that the denominator in Eq. (19) is greater than zero since \( \delta > 0 \) from Inequality (11) and \( r^R_m > r^L_m > r^L_e \). Also, observe that

\[ r^R_c - r^R_m + \delta b_R (\sigma^*_R(b) - \sigma^*_R(\pi^*_L, b_R)) (r^L_m - r^L_e) \geq 0 \Rightarrow \sigma^*_M(b) \geq 0. \]

The first inequality is true since \( \sigma^*_R(b) \geq \sigma^*_R(\pi^*_L, b_R) \), because \( \sigma^*_R(b) \) is weakly increasing in \( b_R \) and weakly decreasing in \( b_L \) by Lemma 3, so we conclude that \( \sigma^*_M(b) \geq 0 \) as desired. Furthermore, we have

\[ \sigma^*_M(b) \leq 1 \Leftrightarrow \delta \geq \frac{(r^R_c - r^R_m) - \delta (\pi^*_L \sigma^*_R(b) (r^R_e - r^R_m) + b_R \sigma^*_R(\pi^*_L, b_R) (r^L_m - r^L_e))}{G + r^R_m - r^L_m}, \]

and the latter (using the fact that \( h_L = h_R \in [1 - \pi^*_m, \pi^*_e] \)) is implied by Eq. (11), completing the proof of this step.

2. The voter’s strategy \( \sigma^*_M \), is a best response. Note that by symmetry we have

\[ V(b, P; \sigma^*) = V(b, -P; \sigma^*) \]

for all \( b \in [0, 1]^2 \) such that \( b_{-p} = b_{-p} \), so that any value of \( \sigma^*_M(b) \in [0, 1] \) forms part of a best reputation for such reputations. Furthermore, the party strategy \( \sigma^*_p(b) \) is weakly increasing in \( b_p \) and weakly decreasing in \( b_{-p} \) by Lemma 3. Since \( \sigma^*_p(b) = \sigma^*_p(b) \) if \( b_p = b_{-p} \), we have

\[ b_p < b_{-p} \Rightarrow \sigma^*_p(b) \leq \sigma^*_p(b) \Leftrightarrow V(b, P; \sigma^*) \geq V(b, -P; \sigma^*), \]

and \( \sigma^*_M \) that satisfies Eq. (9) is a best response.

We show that party strategies constitute best responses in Steps (3) and (4).

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16 A range of values for \( \beta(b, x^p_m, \sigma^*) \) is consistent with equilibrium. Once more, observe that such beliefs with \( b_p = 1 \) cannot occur along the path of play when \( \pi^*_L < 1 \).
3. If $b \in [0,1]^2$ is such that $T_o(b_p) < T_o(b_{-p})$, then $U_P(b, x_{p_m}^P; \sigma^*) \geq U_P(b, x_{c}^P; \sigma^*)$. By part (ii) of Lemma 4, if party $P$ implements policy $x_{p}^P$, then $b_p^p(b, x_{p}^P; \sigma^*) = T_p(1) = \pi_x^x > T_o(b_{-p}) = b_{-p}^d(b, x_{c}^P; \sigma^*)$. Thus, $\sigma^*_M(b(x_{p}^P, \sigma^*), P) = 0$ by Eq. (9). Using Eq. (5) we conclude that

$$U_P(b, x_{c}^P; \sigma^*) = r_c^R + G + \delta[T_o(b_{-p})\sigma^*_p(b(b, x_{p}^P; \sigma^*))](r_c^L - r_m^L) + r_m^L].$$

Since $\sigma^*_p(b) = 0$, we also have from Eq. (8) that $b(b, x_{p}^P, \sigma^*) = b_p$. As a consequence, if party $P$ implements a moderate policy, $b_p^p(b, x_{p}^P, \sigma^*) = T_p(b_p) < T_o(b_{-p}) = b_{-p}^d(b, x_{c}^P, \sigma^*)$, hence $\sigma^*_M(b(b, x_{p}^P, \sigma^*), P) = 1$ from Eq. (9). So, using Eq. (5), the expected payoff from pursuing a moderate policy is

$$U_P(b, x_{m}^P; \sigma^*) = r_m^R + G + \delta[\pi_x^p\sigma^*_p(b(b, x_{p}^P; \sigma^*))](r_c^R - r_m^R) + r_m^R + G].$$

Comparing the expected payoffs from the two policy choices it follows, after a bit of algebra, that $U_P(b, x_{p}^P, \sigma^*) \geq U_P(b, x_{c}^P, \sigma^*)$ is equivalent to

$$\delta \geq \frac{(r_c^R - r_m^R) - \delta[\pi_x^p\sigma^*_p(b(b, x_{p}^P; \sigma^*))](r_c^R - r_m^R) + T_o(b_{-p})\sigma^*_p(b(b, x_{p}^P; \sigma^*))]}{G + r_m^R - r_m^L}.$$

But the last inequality is implied by Inequality (11), and we conclude that $U_P(b, x_{p}^P, \sigma^*) \geq U_P(b, x_{c}^P, \sigma^*)$ as desired.

4. If $b \in [0,1]^2$ is such that $T_o(b_p) \geq T_o(b_{-p})$, then $U_P(b, x_{m}^P, \sigma^*) = U_P(b, x_{c}^P, \sigma^*)$. As in the previous step, if party $P$ implements policy $x_{p}^P$, then $b_p^p(b, x_{p}^P, \sigma^*) = b_{-p}^d(b, x_{c}^P, \sigma^*)$ by part (ii) of Lemma 4. Thus, $\sigma^*_M(b(b, x_{p}^P, \sigma^*), P) = 0$ by Eq. (9). Note that part (iii) of Lemma 4 and Eq. (12) ensure that if $b_p < 1$, then $b_p^p(b, x_{m}^P, \sigma^*) = b_{-p}^d(b, x_{m}^P, \sigma^*) = T_o(b_{-p})$ and the same is true by the specified out-of-equilibrium beliefs when $b_p = 1$. Thus, $\sigma^*_M(b(b, x_{p}^P, \sigma^*))$ is given by Eq. (19). Now, making use of Eq. (5) and of the symmetry $\sigma^*_M(b_{L}, b_{R}) = \sigma^*_M(b_{R}, b_{L})$, it follows that

$$U_P(b, x_{p}^P, \sigma^*) = U_P(b, x_{c}^P, \sigma^*) 
\iff \begin{cases} r_c^R + G + \delta[T_o(b_{-p})\sigma^*_p(b(b, x_{p}^P; \sigma^*))](r_c^L - r_m^L) + r_m^L] 
= r_c^R + G + \delta[\sigma^*_M(b', P)(\pi_x^p\sigma^*_p(b'))(r_c^R - r_m^R) + r_m^R + G] 
+ (1 - \sigma^*_M(b', P)) \delta(r_c' - r_m^L) + r_m^L] 
\iff \sigma^*_M(b', P) \delta(G + r_m^R - r_m^L + \sigma^*_p(b')(\pi_x^p \sigma^*_p(b')(r_c^R - r_m^R) + b'(r_m - r_c^L)] 
= r_c^R - r_m^R + \delta b'(r_m - r_c^R)] 
\iff \text{Eq. 19,} 
\end{cases}$$

where $b' = (b'_L, b'_R) = b'(b, x_{p}^P, \sigma^*) = (T_o(b_{-p}), T_o(b_{-p}))$. This completes the proof of this step.
With Steps 3 and 4, I have established that the party strategy $\sigma_p^*(b)$ is a best response. Thus, there exists a responsive equilibrium with the specified party strategies, and it remains to show parts (ii) and (iii). Part (ii) follows from Lemma 3. For part (iii), note that $(b^*, b^*)$ constitutes the unique solution of the system of linear equations
\[
\begin{align*}
    T_b(b_L) - T_o(b_R) &= 0 \\
    T_b(b_R) - T_o(b_L) &= 0
\end{align*}
\]
for the unknowns $(b_L, b_R)$. Furthermore, the equation $T_b(b_p) = T_o(b_{-p})$ is linear in parties’ reputations so that $B_p$ is formed as the intersection of the open half-space defined by $T_b(b_p) > T_o(b_{-p})$ and the unit square $[0, 1]^2$. In addition, reputations $(b^*, b^*)$ lie at the boundary of $B_o$, reputations $(1, 1) \in B$, since $T_b(1) = \pi_e^1 > \pi_o^1 = T_o(1)$ by Inequalities 1 and 2, and reputations $(b^*, 1), (1, b^*) \notin B_p$ since $T_b(b^*) = T_o(b^*) < \pi_e^1 = T_o(1)$. As a result, the set $B_e = B_L \cap B_R \subset [b^*, 1]^2$. Part (iv) follows from part (ii) and the fact that a voter pursuing a responsive equilibrium induces an equilibrium probability of an extreme policy equal to $\min\{b_1, \sigma_L(b), b_R, \sigma_R(b)\}$.

Next I prove Proposition 4.

**Proof of Proposition 4.** I prove the Proposition in two steps. In the first step I show that each party’s equilibrium probability of pursuing an extreme policy cannot be larger than that implied by the strategies in Eq. 12.

1. For any responsive equilibrium $\sigma^*$, party strategies $\sigma'_p$ satisfy $\sigma'_p(b) \leq \sigma_p^*(b)$ for all $b \in [0, 1]^2$, where $\sigma_p^*(b)$ is given by Eq. (12). This is obvious for $b \in [0, 1]^2$ such that $b_p > 1$, since $\sigma_p^*(b) = 1$ in these cases. For reputations $b \in [0, 1]^2$ with $b_p < 1$, assume that $\sigma'_p(b) > \sigma_p^*(b)$ in order to get a contradiction. Since $b_p < 1$, from part (i) of Lemma 4 we have that $\sigma'_p(b) > \sigma_p^*(b) = (T_b(b_p) - T_o(b_{-p}))/b_p(\pi_e^b - T_o(b_{-p})) \implies b'_p(b, x_{m'}^p, \sigma') < b_{-p}(b, x_{e}^p, \sigma')$. Similarly, from part (ii) of Lemma 4 we conclude that $b_{-p}(b, x_{e}^p, \sigma') < b_{-p}(b, x_{e}^p, \sigma')$. Since the voting strategy satisfies Eq. (9), then $\sigma_M(b, x_{e}^p, \sigma')$, $P = 0$ and $\sigma'_M(b, x_{e}^p, \sigma', P) = 1$. Using Eq. (5), it follows that $U_p(b, x_{m'}^p, \sigma') > U_p(b, x_{e}^p, \sigma')$ is equivalent to

\[
\begin{align*}
    r_m^R + G + \delta(\pi_e^b \sigma_p^*(b, x_{m'}^p, \sigma'))(r_e^R - r_m^R) + r_m^R + G \\
    > r_e^R + G + \delta(T_o(b_{-p})\sigma_{-p}(b, x_{e}^p, \sigma'))(r_e^L - r_m^L) + r_m^L,
\end{align*}
\]

which in turn is equivalent to

\[
\begin{align*}
    r_m^R - r_e^R + \delta(G + r_m^R - r_m^L) > -\delta[\pi_e^b \sigma_p^*(-b, x_{m'}^p, \sigma']) (r_e^R - r_m^R) \\
    + T_o(b_{-p})\sigma_{-p}(b, x_{e}^p, \sigma'))(r_m^L - r_e^L)].
\end{align*}
\]

The right-hand side of the last inequality is less than or equal to zero, while the left-hand side satisfies

\[
\begin{align*}
    r_m^R - r_e^R + \delta(G + r_m^R - r_m^L) > 0 \Leftrightarrow \text{Eq. (11)}.
\end{align*}
\]
Thus, if \( \sigma'_p(b) > \sigma_p^*(b) \geq 0 \), we conclude that \( U_p(b, x^P_m; \sigma') > U_p(b, x^P_e; \sigma') \), which contradicts the assumption that \( \sigma' \) constitute equilibrium strategies. Thus we must have \( \sigma'_p(b) \leq \sigma_p^*(b) \).

We conclude the proof by showing:

2. For any responsive equilibrium \( \sigma' \), party strategies \( \sigma'_p \) satisfy \( \sigma'_p(b) \geq \sigma_p^*(b) \) for all \( b \in [0, 1]^2 \), where \( \sigma_p^*(b) \) is given by Eq. (12). The claim is obviously true for \( b \in [0, 1]^2 \) such that \( T_g(bP) \leq T_a(bP) \) since \( \sigma_p^*(b) = 0 \) in those cases. Thus, it remains to consider reputations \( b \in [0, 1]^2 \) such that \( T_g(bP) > T_a(bP) \), and the proof for these reputations proceeds by contradiction as in Step 1. By part (i) of Lemma 4 it follows that if \( \sigma_p(b) < \sigma_p^*(b) = (T_g(bP) - T_a(bP))/(bP(\pi^g_e - T_a(bP))) \), then \( \sigma_p(b, x^P_m; \sigma') > \sigma_p(b, x^P_m; \sigma') \).

Also, \( \sigma_p(b, x^P_e; \sigma') > \sigma_p(b, x^P_e; \sigma') \) from part (ii) of Lemma 4. Thus, if \( \sigma'_p(b) < \sigma_p^*(b) \), then \( \sigma'_M(b, x^P_m; \sigma'), P) = \sigma'_M(b, x^P_e; \sigma'), P) = 0 \), since \( \sigma'_M \) constitutes an equilibrium strategy that satisfies Eq. 9. Thus the expected payoff from pursuing either policy, \( x^P_e \), \( \tau \in \{e, m\} \), satisfies

\[
U_p(b, x^P_e; \sigma') = r^R_e + G + \delta(T_a(bP)\sigma_p(b, x^P_e; \sigma'))(r^L_e - r^L_m) + r^L_m. \tag{20}
\]

From Step 1 and the fact that \( \pi^g_e \geq T_a(bP) \) it follows that

\[
\sigma'_p(b, x^P_e; \sigma') \leq \frac{T_g(T_a(bP)) - T_a(\pi^g_e)}{T_a(bP) - T_a(\pi^g_e)} \leq \frac{T_g(\pi^g_e - T_a(\pi^g_e))}{T_a(bP) - T_a(\pi^g_e)}. \tag{21}
\]

I now start with Eq. (14) and use Eq. (20) and Inequality (21) to deduce:

\[
\begin{align*}
& r^R_e - r^R_m > \frac{\delta(T_g(\pi^g_e) - T_a(\pi^g_e))}{(\pi^g_e - T_a(\pi^g_e))}(r^L_m - r^L_e) \overset{(21)}{\Rightarrow} \\
& r^R_e - r^R_m > \delta T_a(bP)\sigma'_p(b, x^P_e; \sigma')(r^L_m - r^L_e) \Rightarrow \\
& r^R_e - r^R_m > \delta T_a(bP)\sigma'_p(b, x^P_e; \sigma') \\
& \quad - \sigma_p(b, x^P_m; \sigma')(r^L_m - r^L_e) \Leftrightarrow \\
& r^R_m + \delta T_a(bP)\sigma_p(b, x^P_m; \sigma')(r^L_m - r^L_e) < r^R_e + \delta T_a(bP)\sigma'_p(b, x^P_e; \sigma') \\
& \quad \times (r^L_m - r^L_e) \overset{(20)}{\Leftrightarrow} \\
& U_p(b, x^P_m; \sigma') < U_p(b, x^P_e; \sigma').
\end{align*}
\]

Thus, since \( U_p(b, x^P_m; \sigma') < U_p(b, x^P_e; \sigma') \) it cannot be that \( \sigma'_p(b) < \sigma_p^*(b) = (T_g(bP) - T_a(bP))/(bP(\pi^g_e - T_a(bP))) \leq 1 \).

I now prove Lemmas 1 and 2.
Proof of Lemma 1

(i) First \( b > T_o(b) \iff b > \pi_o b + (1 - \pi_m)(1 - b) \iff b > (1 - \pi_m)/(2 - \pi_m - \pi_o) \iff b > b^o \). Also

\[
T_o(b) > b^o \iff \pi_o b + (1 - \pi_m)(1 - b) > \frac{(1 - \pi_m)}{(2 - \pi_m - \pi_o)}
\]

\[
\iff b > \frac{(1 - \pi_m - (1 - \pi_m)(2 - \pi_m - \pi_o))}{(2 - \pi_m - \pi_o)(\pi_o + \pi_m - 1)}
\]

\[
\iff b > \frac{(1 - \pi_m)}{(2 - \pi_m - \pi_o)}.
\]

where use is made of the fact that \( \pi_o + \pi_m - 1 > 0 \) from Inequality 2.

(ii) Same as in part (i) \( \text{mutatis mutandis} \).

(iii) Starting with the first inequality,

\[
b^o > b^o \iff \frac{1 - \pi_m}{(1 - \pi_o) + (1 - \pi_m)} > \frac{1 - \pi_m}{(1 - \pi_o) + (1 - \pi_m)}
\]

\[
\iff (1 - \pi_m)(1 - \pi_o) + (1 - \pi_m) > (1 - \pi_m)(1 - \pi_o) + (1 - \pi_m)
\]

\[
\iff 1 - \pi_o < 1 - \pi_m.
\]

The second inequality is obtained similarly:

\[
b^o > b^o \iff \frac{1 - \pi_m}{(1 - \pi_o) + (1 - \pi_m)}
\]

\[
\iff (1 - \pi_m)(1 - \pi_o) + (1 - \pi_m) - (1 - \pi_m)(1 - \pi_o) + (1 - \pi_m)
\]

\[
\iff (1 - \pi_m)(1 - \pi_o) - (1 - \pi_o) + (1 - \pi_m) - (1 - \pi_m)
\]

\[
\iff -(1 - \pi_m)(1 - \pi_o) > -(1 - \pi_m)(1 - \pi_o)
\]

\[
\iff 1 - \pi_o < 1 - \pi_m.
\]

Proof of Lemma 2. Consider party reputations \( b^o \in [0, 1]^2 \) at the beginning of period \( t \). We distinguish three cases:

Case 1, a government by party \( P \) and \( T_o(b^o_p) \leq T_o(b^o_{-p}) \): Then by Eq. (12) party \( P \) implements a moderate policy \( x^o_m \) and updated reputations \( b^{t+1}_P \) satisfy \( b^{t+1}_P = T_o(b^o_p) \) and \( b^{t+1}_{-p} = T_o(b^o_{-p}) \). Hence \( \min\{b^{t+1}_L, b^{t+1}_R\} = T_o(b^o_p) = \min\{T_o(b^o_p), T_o(b^o_{-p})\} \).
Case 2, a government by party $P$, $T_g(b^o_P) > T_o(b^-_P)$, and party $P$ implements a moderate policy, $x^o_m$. Then $b^o_P + 1 = b^-_P = T_o(b^-_P)$ by part (iii) of Lemma 4. Hence, \( \min\{b^o_L, b^o_R\} = T_o(b^-_P) = \min\{T_g(b^o_P), T_o(b^-_P)\} \).

Case 3, a government by party $P$, $T_g(b^o_P) > T_o(b^-_P)$, and party $P$ implements an extreme policy, $x^o_P$. In this case $b^o_P + 1 = \pi^* > b^-_P = T_o(b^-_P)$ by Eq. (8) and Inequalities (1) and (2), so that \( \min\{b^o_L, b^o_R\} = T_o(b^-_P) = \min\{T_g(b^o_P), T_o(b^-_P)\} \).

Hence, in all three cases \( \min\{b^o_L, b^o_R\} = \min\{T_g(b^o_P), T_o(b^-_P)\} \), where $P$ is the governing party. Now it follows that

\[
\min\{b^o_L, b^o_R\} = \min\{T_g(b^o_P), T_o(b^-_P)\} \\
\geq \min\{\min\{T_g(b^o_P), T_o(b^-_P)\}, \min\{T_g(b^o_P), T_o(b^-_P)\}\} \\
= \min\{T_g(\min\{b^o_L, b^o_R\}), T_o(\min\{b^o_L, b^o_R\})\}.
\]

Similarly

\[
\min\{b^o_L, b^o_R\} = \min\{T_g(b^o_P), T_o(b^-_P)\} \\
\leq \max\{\min\{T_g(b^o_P), T_o(b^-_P)\}, \min\{T_g(b^o_P), T_o(b^-_P)\}\} \\
\leq \max\{T_g(\min\{b^o_L, b^o_R\}), T_o(\min\{b^o_L, b^o_R\})\}.
\]

The last inequality follows since, assuming $b^o_P \leq b^-_P$, we have

\[
\max\{T_g(\min\{b^o_L, b^o_R\}), T_o(\min\{b^o_L, b^o_R\})\} = \max\{T_g(b^o_P), T_o(b^-_P)\},
\]

and \( \max\{T_g(b^o_P), T_o(b^-_P)\} \geq \max\{\min\{T_g(b^o_P), T_o(b^-_P)\}, \min\{T_g(b^o_P), T_o(b^-_P)\}\} \) because, if

\[
\max\{\min\{T_g(b^o_P), T_o(b^-_P)\}, \min\{T_g(b^o_P), T_o(b^-_P)\}\} = T_o(b^-_P)
\]

for some $\ell \in \{a, g\}$, then $T_o(b^-_P) = \min\{T_g(b^o_P), T_o(b^-_P)\}$, where $-\ell \in \{a, g\}$, $-\ell \neq \ell$. Part (i) now follows from Inequalities (22) and (23) and parts (i) and (ii) of Lemma 1. Part (ii) is similarly obtained, noting that if $\pi^*_k = \pi^*_m = 1$, then $T_k(b) = b$. ■

I conclude this appendix by formally restating and then proving Proposition 5:

**Proposition 5** Let $b^t, P^t, x^t, t = 0, 1, \ldots$ be the sequence of reputations, parties in government, and policies, respectively, induced by the equilibrium in Proposition 3.

(i) If $b^t > b^o > b^s$, then there exists $\bar{t}$ such that party reputations satisfy $b^t \in [b^o, 1]^2$ for all periods $t > \bar{t}$. In addition, for all $\varepsilon > 0$ there exists $\bar{k}$ such that for all periods $t > \bar{t}$, the sequence of policies $\{x^{t+1}, \ldots, x^{t+k}\}$ satisfies

\[
\text{Prob}\{x^{t+1}, \ldots, x^{t+k}\} \cap \{x^L_t, x^R_t\} = \emptyset < \varepsilon.
\]

(24)
(ii) If \( b^t < b^o < b^s \), then there exists \( \bar{t} \) such that \( \sigma_P(b^t) = 0 \) for all periods \( t > \bar{t} \), and either \( \lim_{t \to +\infty} b^t = (b^s, b^t) \) and \( \lim_{t \to +\infty} P^t = L \) or \( \lim_{t \to +\infty} b^t = (b^o, b^s) \) and \( \lim_{t \to +\infty} P^t = R \).

(iii) If \( b^o = b^s \) and if initial reputations \( b^0 \in [b^o, 1]^2 \), then \( \lim_{t \to +\infty} \sigma_P(b^t) = 0 \) and \( \lim_{t \to +\infty} b^t = (b^o, b^s) \). If initial reputations satisfy \( b^0 \in [0, 1]^2 \cup [b^o, 1]^2 \), then \( \sigma_P(b^t) = 0 \) for all periods \( t \to +\infty \) in all periods \( t \), and:

(a) If \( \pi^x_p + \pi^x_m < 2 \), then \( \lim_{t \to +\infty} b^t = (b^o, b^o) \) and \( \lim_{t \to +\infty} P^t = L \) or \( \lim_{t \to +\infty} P^t = R \).

(b) If \( \pi^x_p = \pi^x_m = 1 \), then either \( \lim_{t \to +\infty} b^t = (b^o, b^o) \) and \( \lim_{t \to +\infty} P^t = L \) or \( \lim_{t \to +\infty} b^t = (b^o, b^R) \) and \( \lim_{t \to +\infty} P^t = R \).

Proof of Proposition 5. I prove each of the three parts (i)–(iii) separately.

Part (i): When \( b^t > b^o > b^s \), Inequality (15) holds and reputations are absorbed in \((b^o, 1]^2\) by part (i) of Lemma 2. Let \( t \geq 1 \), \( P^t = P \), and let party reputations \( b^t \in (b^o, 1]^2 \). If \( b^t \in B_P \), then \( \sigma_P(b^t) > 0 \), and since \( t \geq 1 \), it must be that \( b^t \in B_P \) by part (iii) of Proposition 3. If, on the other hand, \( b^t \notin B_P \) then I will show that \( b^{i+k} \in B_P \) for some finite number of \( k \) periods. First, since party \( P \) is in government and \( b^t \notin B_P \), it must be that \( b^{i+k} \leq b_{-P}^t \) and \( T_{s_P}(b^{i+k}) \leq T_{s_P}(b_{-P}^t) \) by the definition of the set \( B_P \). If set \( B_P \) is not reached in \( k \) periods, then \( b^{i+k} = T_{s_P}^k(b^{i+k}) \equiv b^{i+k} \) where \( T_{s_P}^k \) and \( T_{s_P}^k \) are the \( k \)-times compositions of the mappings \( T_{s_P} \) and \( T_{s_P} \), respectively, and party \( P \) is re-elected with probability one and implements a moderate policy with probability one in all periods \( t+1, \ldots, t+k \). But by Lemma 1

\[
\lim_{k \to +\infty} T_{s_P}^k(b^{i+k}) = b^s > b^0 = \lim_{k \to +\infty} T_{s_P}^k(b_{-P}^t),
\]

so that there must exist finite \( k \) such that \( b^{i+k} \in B_P \).

As a result, with probability one some government by party \( P \) must implement an extreme policy at some period \( t \). Following such an extreme policy, party \( -P \) is elected to government in period \( t+1 \) and \( b_{-P}^{i+1} = \pi^x_p \geq b^s \). I will show that \( b^{i+1} \in (b^o, 1]^2 \subset (b^o, 1]^2 \) for all \( i' > t + k' \) for some finite \( k' \). This is because \( b^{i+1} > b^o \) for all \( i' \geq 1 \) such that party \( P \) remains in the opposition, by part (i) of Lemma 1, while

\[
b_{-P}^{i+1} = \begin{cases} 
  b_P^{i+1}, & \text{if } b^{i+1} \in B_{-P} \text{ and } s^{i+1} = x_{-P}, \\
  \pi^x_p, & \text{if } b^{i+1} \in B_{-P} \text{ and } s^{i+1} = x_{-P}, \\
  T_{s_P}(b_{-P}^{i+1}), & \text{if } b^{i+1} \notin B_{-P}. 
\end{cases}
\]

i.e., the governing party’s reputation either matches the reputation of the opposition party (by part (iii) of Lemma 4 in the Appendix), or it becomes equal to \( \pi^x_p > b^o \) following an extreme policy by the government, or is given by \( T_{s_P}(b_{-P}^{i+1}) \), adjusting monotonically towards \( b^s > b^o \) by part (ii) of Lemma 1. It follows that party reputations are absorbed in \([b^o, 1]^2 \subset (b^o, 1]^2 \) once the minimum reputation of the parties satisfies \( \min(b_{-P}^{i+1}, b_P^{i+1}) > b^o \).
Next consider any period \( t \) with reputations \( b^t \in [b^o, 1]^2 \) and assume that moderate policies occur in all periods \( t, t+1, \ldots, t+k \) for some finite \( k \). If \( k \) is large enough, then by our earlier arguments it must be that \( b^{t+k} \in B_r \) for some \( k < \bar{k} \). As a result, for all periods \( t+k+1, \ldots, t+\bar{k} \) in which the government pursues a moderate policy, we have from Lemma 1 and part (iii) of Lemma 4 that

\[
\frac{\pi t^{k+1}}{\pi t^p} = \frac{\pi t^{k+1}}{\pi t^p} > \frac{\pi t^{k+2}}{\pi t^p} = \frac{\pi t^{k+2}}{\pi t^p} > \cdots > \frac{\pi t^{k+\bar{k}}}{\pi t^p} = \frac{\pi t^{k+\bar{k}}}{\pi t^p} > b^o.
\]

Thus, invoking parts (ii)–(iv) of Proposition 3, it follows that the probability of an extreme policy in all periods \( t^* = t + k + 1, \ldots, t + \bar{k} \) satisfies \( \pi t^{k+\bar{k}} = \pi t^p \sigma_p(b^{t^*}) > \sigma_p(b^*, b^o) > 0 \). Thus, the probability that all policies in periods \( t + 1, \ldots, t + k \) are moderate is smaller than \( [1 - \sigma_p(b^*, b^o)]^{k-k} \), a quantity that goes to zero as \( \bar{k} \) increases. Since this is true for any initial period \( t \) with any reputations \( b^t \in [b^o, 1]^2 \), the above argument establishes part (i).

Part (ii): Since \( b^o < b^o < b^* \), \( b^t \not\in [b^o, 1]^2 \) for all large enough \( t \) by part (i) of Lemma 2. As a consequence, by part (iii) of Proposition 3, governments implement a moderate policy with probability one. In particular, when \( b^t \not\in [b^o, 1]^2 \), we must have either \( b^o_L \leq b^o_R \) and a government by party \( L \) with \( b^t \not\in B_L \), or \( b^o_L \geq b^o_R \) and a government by party \( R \) with \( b^t \not\in B_R \). In the former case, \( \pi t^{k+1} = T_o(b^o_L) < T_o(b^o_R) = \pi t^{k+1} \), and we conclude that \( \lim_{t \to \infty} b^t = (b^o, b^o) \) and \( \lim_{t \to \infty} P^o = L \) by parts (i) and (ii) of Lemma 1. In the latter case we similarly conclude that \( \lim_{t \to \infty} b^t = (b^o, b^o) \) and \( \lim_{t \to \infty} P^o = R \).

Part (iii): Consider first the case in which initial reputations satisfy \( b^0 \in [b^o, 1]^2 \). If \( \pi^o_c + \pi^o_m < 2 \), then the minimum party reputation converges to \( b^o \) by Lemma 2, part (i). If, on the other hand, \( \pi^o_c + \pi^o_m = 2 \), then \( T_o(b^o) = b^o \) and using Eq. (3) we obtain that

\[
\min\{b^o_L, b^o_R\} = \begin{cases} b^o_p, & \text{if } P^o = P \text{ and } b^o_{-p} > T_o(b^o_{-p}) = b^o_p, \\
T_o(b^o_{-p}), & \text{if } P^o = P \text{ and } T_o(b^o_{-p}) < T_o(b^o_p) = b^o_p. 
\end{cases}
\]

As a result, the minimum party reputation either remains constant or decreases, converging towards the long-run level \( b^o \) by Lemma 2. Thus, since \( \lim_{t \to \infty} \min\{b^o_L, b^o_R\} = b^o \), for any \( \epsilon > 0 \), there exists a \( t \) such that for all periods \( t > t \) extreme policies can occur in equilibrium only for party reputations \( b^t \not\in B_e \cap [b^o, b^o + \epsilon]^2 \). But the maximum probability of an extreme policy in set \( B_e \cap [b^o, b^o + \epsilon]^2 \) is given by \( \sigma_p(b^o + \epsilon, b^o + \epsilon) \) by part (iv) of Proposition 3; hence (since \( \lim_{t \to 0} \sigma_p(0, 0) = 0 \)) a moderate policy prevails with probability one in the long-run in this case. Furthermore, party reputations converge in probability to the pair \((b^o, b^o)\).

Second, we consider the case in which initial reputations satisfy \( b^0 \not\in [b^o, 1]^2 \). In this case part (i) of Lemma 1 ensures that \( \min\{b^o_L, b^o_R\} \leq b^o \) for all \( t \), and policies remain moderate with probability one along the path of play, by part (iii) of Proposition 3. If \( \pi^o_c + \pi^o_m < 2 \), then part (i) of Lemma 2 ensures that party reputations converge to the pair \((b^o, b^o)\). If, on the other hand, \( \pi^o_c + \pi^o_m = 2 \), then if the first party in government (say party \( P = P^0 \)) enjoys a reputation that is lower than that of the opposition, \( b^0_p < b^0_{-p} \), or, if \( b^0_p \leq b^0_{-p} < b^0 \), then the government is re-elected with probability one and the
reputation of the government party remains constant at its initial level \( b_p = b_p^0 = T_g(b_p^0) \) for all \( t > 0 \), while the opposition party’s reputation converges to \( b_\ast = b_\ast^0 \).

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