Uncertainty and Incentives in Crisis Bargaining: Game-Free Analysis of International Conflict

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Supporting Information

In this appendix, we provide support for the theoretical claims in the Discussion section of the paper. We describe a simple "bargaining while fighting" model and show that all of our main results continue to hold in this setting. The model we define is a simplification of the model of Powell (2004). Suppose there are two states which are bargaining over a pie of size 1. Bargaining occurs over several periods in the game. The outcome of each period is either a peaceful settlement which divides the pie and ends the game or fighting. If the outcome of a period is fighting, then a decisive military result occurs with probability k and country i wins the whole pie with probability p_i. If the fighting is not decisive, play continues to the next period. In each period of fighting, country i pays a cost $c_i \geq 0$ of fighting.

Consider an arbitrary game G with action space A_i for country i. We assume that this game has voluntary settlements in that each side can choose to fight forever if it wants. Thus, every action profile a represents either a settlement in round m on terms $(v_1^m, 1 - v_1^m)$ (with fighting in periods 1 through $m-1$) or fighting forever. Note that if the two sides plan to settle in round m, the game could end earlier as a result of a decisive military victory in any of the rounds 1 through $m-1$. Thus, the outcome function $g(a)$ represents the *planned* outcome of the game and not necessarily the realized outcome of the game.

We next give the expected payoffs of each of these outcomes. It is straightforward to show that the expected payoff to country i from fighting forever is given by

$$W_i = p_i - \frac{c_i}{k}$$

and the expected payoff to settling in period m is given by

$$T_i^m = (1 - (1-k)^{m-1})(p_i - \frac{c_i}{k}) + (1-k)^{m-1} v_i^m.$$

Note that T_i^m is just a convex combination of W_i and v_i^m. If we write our voluntary agreements condition as requiring that any peaceful settlement be at least as good as fighting forever, we have $T_i^m \geq W_i$. But this is equivalent to

$$\begin{aligned}(1-\alpha)W_i + \alpha v_i^m &\geq W_i \\ \alpha v_i^m &\geq \alpha W_i \\ v_i^m &\geq p_i - \frac{c_i}{k}.\end{aligned}$$

So the voluntary agreements condition looks very similar to that given in the paper. The only difference is that the cost term is "scaled" by a factor of $1/k$.

It is clear that $T_1^m + T_2^m < 1$ if $m > 1$ and so the assumption in the paper that settlements are efficient does not hold in this case. But this assumption is not critical for our results, which continue to hold in this setting. Specifically, for any strategy profile s in the game G, we can find a direct mechanism defined by a probability of fighting forever, $\pi(t)$, the probability $s(m)$ of settling in period m, conditional on not fighting forever, and an expected settlement v_i^m for every period m. Of course, the payoff to fighting forever is W_i. We can use the function $s(m)$ and the settlements v_i^m to generate an expected payoff of settling eventually, given by

$$T_i = \sum_{\tau=1}^{\infty} s(\tau) v_i^{\tau}.$$

We thus have a direct mechanism with outcomes of war and peace that occur with probability π and $1-\pi$ and have payoffs W_i and T_i, respectively.

With this structure established, we can now review our results for uncertainty about costs and uncertainty about relative power. The main point in evaluating these results is that the only change from our assumptions in the paper is that the settlements may not be efficient. For the case of uncertainty about costs, the monotonicity results of Proposition 1 continue to hold because the proof does not use efficiency of settlements. The result in Proposition 3 that in any peaceful equilibrium the expected settlement does not depend on the private information of the two sides also continues to hold for the same reason. For uncertainty about relative power, the monotonicity results of Proposition 4 again do not depend on efficiency of settlements. In addition, the proof of Proposition 5 is easily modified to apply to this setting. The new version of the theorem is that if $(c_1 + c_2)/k \leq \bar{c}$, then no always peaceful equilibrium exists. The proof of this result follows the proof of Proposition 5 closely. We need only replace c_i with c_i/k and replace the argument that $\bar{U}_1 + \bar{U}_2 = 1$ by the fact that $\bar{U}_1 + \bar{U}_2 \leq 1$ when $T_1 + T_2 \leq 1$. Lastly, the result in Proposition 7 applies as well because its proof does not use efficiency of settlements.