The Swing Voter's Curse with Adversarial Preferences

Jaehoon Kim   Mark Fey *

October 5, 2005

Abstract

We analyze voting behavior in a large electorate in which voters have adversarial state-contingent preferences with incomplete information about the state of the world. We show that one type of voter can suffer from the swing voter's curse *à la* Feddersen and Pesendorfer (1996), and go on to characterize the symmetric Nash equilibria of this model under different parameter values. We prove that unlike settings with non-adversarial preferences, there are equilibria in which in one state of the world, a minority-preferred candidate almost surely wins the election and thus the election may fail to correctly aggregate information. Indeed, we show that the fraction of the electorate dissatisfied with the result can be as large as 66 2/3%.

Journal of Economic Literature Classification Numbers: C72, D72.

1 Introduction

In an influential paper, Feddersen and Pesendorfer (1996) examine the decisions of voters with private information about candidates and show that an election almost surely aggregates the dispersed information in the electorate.

In their model, there are two candidates and two states of the world. Voters are either partisans, who prefer a particular candidate in either state, or independents, who prefer the candidate that matches the state of the world. Before voting, each voter has some probability of learning the true state of the world. Thus, a portion of the electorate is perfectly informed while the remainder is uninformed.

Feddersen and Pesendorfer show that indifferent uninformed independents have a strict incentive to abstain rather than vote for either candidate, even without a cost of voting. Borrowing from auction theory, the authors refer to this phenomenon as the "swing voter's curse." It follows from the fact that a voter should condition her action on being pivotal and, for an uninformed independent, this is most likely to occur because informed voters are voting against her. Continuing the analysis, the authors show that, in equilibrium, uninformed independents mix between abstaining and voting for one of the candidates. In this way, the uninformed independents attempt to compensate for the difference in partisan support for the candidates and thus maximize the chance that informed voters (whose preference they share) will decide the outcome of the election. Finally, Feddersen and Pesendorfer point out that even though there can be substantial levels of strategic abstention, with sufficiently large electorates the winner of the election is almost surely the same as with complete information.

An important assumption of Feddersen and Pesendorfer is that independent voters all share the same preference that the candidate match the state of the world. We refer to this common values assumption as congruent state-contingent preferences. *A priori*, however, there is no reason to restrict independent voters to such preferences. In this paper, we relax this assumption and examine how the conclusions of Feddersen and Pesendorfer are affected. Specifically, we assume there are two kinds of independent voters with opposing state-contingent preferences: matching-type independents (or matchers), and nonmatching-type independents (or nonmatchers). Matchers prefer candidate 0 when the state of the world is 0, and they prefer candidate 1 when the state is 1. Nonmatchers prefer candidate 0 in state 1, and they prefer candidate 1 in state 0. Each voter knows the proportion of each type within the population but not the exact number. We refer to this configuration of preferences of the independent voters as adversarial state-contingent preferences.

Incorporating adversarial preferences into a model of elections with private information threatens the intuition behind the swing voter's curse in

that an uninformed voter may be pivotal because other voters simply have different preferences, as well as the possibility that the informed voters are voting against her. Thus, it may be that the results of previous work with congruent preferences are called into question. In this paper, we show that this is partially the case. Specifically, we find that matching-type independents will still face a version of the swing voter's curse. In addition, we show that while elections may have the desirable aggregation properties found in previous work, depending on the parameter values, they may also fail to aggregate the private information of the electorate. In particular, we show that in large elections with adversarial preferences, there are equilibria in which the winner may not be that chosen by a fully informed electorate. In other words, elections may not always aggregate information asymptotically. Such a failure is clearly inefficient from the perspective of social welfare, as the election could elect the *wrong* candidate with probability approaching one in one state of the world. In fact, we show a majority as large as two-thirds of the electorate can fail to elect their preferred candidate.

This paper is organized as follows. In the next section we discuss the literature related to our paper. In section 3 we describe our model and in section 4 we develop our analysis of the model. Section 5 describes the existence of the swing voter's curse for certain types of uninformed voters. In Section 6 we characterize the symmetric Nash equilibria of the model under different parameter values. We use this characterization in section 7 to show that there are equilibria in which the electoral system chooses a minority-preferred candidate in one state of the world. Section 8 describes the impact that such a failure of information aggregation can have on social welfare. Finally, section 9 concludes and discusses our future research agenda. All proofs are contained in the appendix.

2   Related Literature

This paper is related to the growing literature on voting and elections with private information advanced by Austen-Smith and Banks (1995) and Feddersen and Pesendorfer (1996). The key idea in these papers is that since a vote matters only when pivotal, voters should condition their choices on what must be true in the event they are pivotal. Both of these early papers assumed that there were two states of the world and that voters have congruent state-contingent preferences. This first assumption was generalized in

later work by Feddersen and Pesendorfer (1997, 1999). In these papers, there is a one-dimensional state variable and each voter receives a private signal that is correlated with the true state of nature. Note, however, that these papers do not permit adversarial preferences, only heterogeneous congruent preferences. The authors find that, in this model, elections are still able to aggregate information correctly.

Recently, other papers have shown that elections may fail to aggregate information, even with congruent preferences. Razin (2003) shows that this is the case in a model in which voters use their vote as a message to influence the policy of the winning candidate. Martinelli (2005) develops a model of costly information acquisition in which information is aggregated only if it is cheap enough to acquire.

Moving beyond the case of congruent preferences, two recent papers have investigated models which include adversarial preferences. Meirowitz (2005) provides several concrete examples of such preferences. One such example is a setting in which voters have one-dimensional single peaked preferences and face a proposed policy which may be either to the left or right of the status quo policy. Another example is an open primary in which "crossover" voters of the opposing party vote for the least qualified candidate while members of the primary's party vote for the most qualified candidate. Meirowitz analyzes these adversarial preferences in a mechanism design study of deliberation. In addition, Gul and Pesendorfer (2005) consider a model in which different preferences over candidate quality generate adversarial preferences and the state of the world is a strategic choice by one of the candidates. In their benchmark case, information is fully aggregated, which is consistent with our results in the absence of partisans. Their general case, however, differs from our approach because they postulate uncertainty about the proportion of matchers and nonmatchers.

Finally, this paper is connected to a series of papers on committee decision making with strategic voting (Feddersen and Pesendorfer, 1998; Coughlan, 2000; Gerardi, 2000; Duggan and Martinelli, 2001; Li, Rosen and Suen, 2001; Meirowitz, 2002; Persico, 2004). These are distinguished by their focus on committees or juries of fixed size, rather than the large electorates considered in this paper.

3 The Model

We follow the notation introduced by Feddersen and Pesendorfer (1996) with some modifications to accommodate our assumption of adversarial preferences. There are two states of the world, $\mathcal{Z} = \{0, 1\}$, two candidates, $\mathcal{X} = \{0, 1\}$, and four types of agents, $\mathcal{T} = \{0, 1, m, n\}$. Type-0 and type-1 voters are *partisans*: regardless of the state, type-0 voters strictly prefer candidate 0 and type-1 voters strictly prefer candidate 1. Type-m voters, which we call *matchers*, and type-n, which we term *nonmatchers* are *independent voters*. Matchers prefer candidate 0 in state 0 and candidate 1 in state 1 while nonmatchers prefer candidate 1 in state 0 and candidate 0 in state 1. Formally, the utility of a type-m voter for $(x, z) \in \mathcal{X} \times \mathcal{Z}$ is defined as

$$U_m(x, z) = \begin{cases} 0 & \text{if} \quad x = z \\ -1 & \text{if} \quad x \neq z, \end{cases} \tag{1}$$

and the utility of a type-n voter is similarly defined as

$$U_n(x, z) = \begin{cases} 0 & \text{if} \quad x \neq z \\ -1 & \text{if} \quad x = z. \end{cases} \tag{2}$$

Initially, Nature chooses a state $z \in \mathcal{Z}$. State 0 is chosen with probability α, and this is common knowledge among all players. We assume that $\alpha > \frac{1}{2}$. Each of $N+1$ potential voters is assigned a type as follows. With probability p_m she is type-m, with probability p_n she is type-n, with probability p_0 she is type-0, and with probability p_1 she is type-1. Finally, with probability $p_\phi = 1 - p_0 - p_1 - p_m - p_n$, she is a nonvoter.

After the state and the set of voters have been selected, each voter learns the true state of the world with probability q. Formally, each voter independently receives a signal $s \in \mathcal{S} = \{0, \alpha, 1\}$. With probability q, she receives signal $s = z$ and with probability $1 - q$ she receives signal α, regardless of the state. A voter who learns the true state of the world is called *informed*, otherwise she remains *uninformed*. To avoid trivial cases, we assume that $p_\phi > 0$ and $q > 0$. In addition, throughout this paper we assume that $p_m > p_n$ and $p_0 > p_1$.

After the signals are received, an election is held. Formally, each voter chooses an action $a \in \{0, 1, \phi\} \equiv \mathcal{A}$, where 0 or 1 indicates a vote for the respective candidate and ϕ indicates abstention. The winner of the election is decided by plurality rule. If there is a tie, each candidate is chosen with equal probability.

In general, a pure strategy for a voter is a function from $\mathcal{T} \times \mathcal{S}$ to $\mathcal{A}$ and a mixed strategy is a function from $\mathcal{T} \times \mathcal{S}$ to the set of probability distributions on $\mathcal{A}$. In this paper, we restrict our attention to symmetric Bayesian-Nash equilibria, in which voters who are of the same type and receive the same signal choose the same strategy. As in Feddersen and Pesendorfer (1996), because the number of voters is uncertain, there is a strictly positive probability that each voter is pivotal and so all voters except the *uninformed independent voters* (UIVs henceforth) have a strictly dominant strategy to vote for their preferred candidate conditional on the state.[1] Thus, the only strategies that we need to specify in our analysis are those of the uninformed matchers (UIM henceforth) and the uninformed nonmatchers (UIN henceforth). We denote these mixed strategies by τ_m and τ_n, respectively, and a profile of mixed strategies by $\tau = (\tau_m, \tau_n)$. Finally, it will be convenient to use the slightly abusive notation $\tau(a, t)$ for the probability that a type-t voter takes an action a.

4 Analysis

In this section, we develop the analysis of the model and present two preliminary lemmas that will prove helpful in the succeeding sections.

For a given mixed strategy profile τ, let $\sigma_{z,x}(\tau)$ be the probability that a random draw by nature results in a vote for candidate x if the state is z. As discussed in the previous section, an informed matcher (nonmatcher) votes for x only if $z = x$ ($z \neq x$) while an UIM (UIN) votes for x with probability $\tau(x, m)$ ($\tau(x, n)$) in either state. Therefore, the probability that a draw by nature results in a vote for candidate x in state z is given by

$$\sigma_{z,x}(\tau) = \begin{cases} p_x + p_m(1-q)\tau(x,m) + p_n(1-q)\tau(x,n) + p_n q & \text{if } z \neq x \\ p_x + p_m(1-q)\tau(x,m) + p_n(1-q)\tau(x,n) + p_m q & \text{if } z = x. \end{cases}$$

This leads us to the following lemma, which is easy to verify from the definition of $\sigma_{z,x}(\tau)$.

Lemma 1 *For all strategy profiles τ, $\sigma_{0,0}(\tau) + \sigma_{0,1}(\tau) = \sigma_{1,0}(\tau) + \sigma_{1,1}(\tau)$ and $\sigma_{x,x}(\tau) = \sigma_{y,x}(\tau) + q(p_m - p_n)$, where $x, y \in \{0, 1\}$ with $x \neq y$.*

---

[1] That is, a type-0 voter votes for candidate 0, a type-1 voter votes for candidate 1, and an informed independent voter votes for the candidate which either matches the state of the world or not, depending on her type.

The relationships stated in Lemma 1 are fundamental to our analysis and hold for every strategy profile of the UIVs. The first component of Lemma 1 states that the expected total number of votes is independent of the state. This will be useful in our later discussions of the expected margin of victory. The second and more important component of Lemma 1 shows that the probability of a vote matching the state, $\sigma_{x,x}(\tau)$, is strictly greater than the probability of a vote not matching the state, $\sigma_{y,x}(\tau)$. We term the latter circumstance a "mismatched vote." From the perspective of a UIM, then, the probability of a "correct" vote is greater than the probability of an "incorrect" vote. Similarly, from the perspective of a UIN, the opposite is true.

Turning to abstention, let $\sigma_{z,\phi}(\tau)$ denote the probability that a random draw by nature results in a vote for neither candidate in state z. This nonvoting event can occur either if the agent is assigned the nonvoting type by Nature or if the voter chooses to abstain. The only voters who might have an incentive to abstain are UIVs. Both the probability that nature draws a non-voter, p_ϕ, and the strategy profile of UIVs, τ, do not depend on the state. Hence, $\sigma_{z,\phi}(\tau)$ is independent of the state and so we drop the z term from the expression:

$$\sigma_\phi(\tau) = p_m (1-q) \cdot \tau(\phi, m) + p_n (1-q) \cdot \tau(\phi, n) + p_\phi. \tag{3}$$

In order to determine the optimal strategies for the UIVs, we need to specify the conditions in which a UIV's choice changes the outcome. Specifically, there are three situations in which a voter may be pivotal: (1) an equal number of other agents vote for each candidate; (2) candidate 1 receives one more vote than candidate 0; or (3) candidate 0 receives one more vote than candidate 1. For each voter, the probability of each of these events, given a state z, N other possible agents, and strategy profile τ, is as follows. The probability that an equal number of other voters have voted for each candidate, that is, a tie (denoted by e) occurs, is

$$\pi_e(z,\tau) = \sum_{j=0}^{N/2} \frac{N!}{j!j!(N-2j)!} \sigma_\phi(\tau)^{N-2j} \left(\sigma_{z,0}(\tau)\,\sigma_{z,1}(\tau)\right)^j. \tag{4}$$

Alternatively, the probability that candidate x receives exactly one vote less

than candidate y (the probability that candidate x is down by 1 vote) is

$$\pi_x(z,\tau) = \sum_{j=0}^{(N/2)-1} \frac{N!}{(j+1)!j!\,(N-2j-1)!}\sigma_\phi(\tau)^{N-2j-1}\,\sigma_{z,y}(\tau)\,(\sigma_{z,x}(\tau)\,\sigma_{z,y}(\tau))^j\,. \tag{5}$$

We denote the expected payoff to a type-t UIV of taking action a when the strategy profile used by the UIVs is τ by $EU_t(a,\tau)$ for each $t \in \{m,n\}$. Because both types of UIVs face exactly the same uncertainty about the draws of Nature and because the two types have opposite state-contingent preferences, the relationship between the expected utilities of UIMs and UINs is as follows:

Lemma 2 *For all actions a and strategy profiles τ, $EU_m(a,\tau) = -EU_n(a,\tau)$.*

We use this to give the expected utility difference between each pair of strategies, for each type of UIV:

$$EU_m(1,\tau) - EU_m(\phi,\tau) = -[EU_n(1,\tau) - EU_n(\phi,\tau)]$$
$$= \tfrac{1}{2}\left[(1-\alpha)\left[\pi_e(1,\tau)+\pi_1(1,\tau)\right] - \alpha\left[\pi_e(0,\tau)+\pi_1(0,\tau)\right]\right] \tag{6}$$

$$EU_m(0,\tau) - EU_m(\phi,\tau) = -[EU_n(0,\tau) - EU_n(\phi,\tau)]$$
$$= \tfrac{1}{2}\left[\alpha\left[\pi_e(0,\tau)+\pi_0(0,\tau)\right] - (1-\alpha)\left[\pi_e(1,\tau)+\pi_0(1,\tau)\right]\right] \tag{7}$$

$$EU_m(0,\tau) - EU_m(1,\tau) = -[EU_n(0,\tau) - EU_n(1,\tau)] =$$
$$(1-\alpha)\left[\pi_e(1,\tau)+\tfrac{1}{2}\left(\pi_0(1,\tau)+\pi_1(1,\tau)\right)\right]$$
$$- \alpha\left[\pi_e(0,\tau)+\tfrac{1}{2}\left(\pi_0(0,\tau)+\pi_1(0,\tau)\right)\right] \tag{8}$$

5 The Swing Voter's Curse

In this section we show that, for all sizes of the electorate, a UIM suffers the *swing voter's curse*: whenever a uninformed matcher is indifferent, it is the case that it is more likely that her vote does not match the true state of the world. Accordingly, she is strictly better off by abstaining than rather than voting. In other words, whenever she is indifferent between voting

for candidate 0 and 1, she prefers to abstain. By contrast, a UIN has no such incentive to abstain and, rather, strictly prefers to vote for one of the candidates.

Proposition 1 *Let $N \geq 2$ and N be even. For every symmetric strategy profile τ, if $EU_m(0, \tau) = EU_m(1, \tau)$, then $EU_m(\phi, \tau) > EU_m(1, \tau)$ and $EU_n(1, \tau) > EU_n(\phi, \tau)$.*

This proposition states that a UIM strictly prefers to abstain whenever she is indifferent between voting for candidate 0 and voting for candidate 1. On the other hand, a UIN strictly prefers to vote in such cases. Thus, the swing voter's curse phenomenon identified in Feddersen and Pesendorfer (1996) applies to the type of UIVs with the greater expected fraction in an environment with adversarial state-contingent preferences of independent voters.

To gain further insight into Proposition 1, recall that the informed matchers and informed nonmatchers always vote for different candidates. Because we assume $p_m > p_n$, it is likely that the group of informed nonmatchers will "cancel out" some, but not all, of the votes of the informed matchers. In net, then, the informed voters are likely to vote for the candidate preferred by a UIM. Because of this, it is more likely that the incorrect candidate from the perspective of a UIM is trailing by one vote than that the correct candidate is trailing by one vote in either state. In other words, a UIM is more likely to be pivotal in an incorrect state than in a correct state of the world. Moreover, she knows that if she abstains, the informed matchers are more likely to decide the electoral outcome than the informed nonmatchers. Therefore, a UIM strictly prefers to abstain whenever she is indifferent between voting for candidate 0 and candidate 1. For a UIN, on the other hand, the reverse holds as a direct consequence of Lemma 2.

Proposition 1 has important implications for the characterization of equilibria of the model. Clearly, Proposition 1 implies that there cannot be a mixed strategy equilibrium in which UIMs mix between voting for candidate 0 and voting for candidate 1. Additional constraints on equilibria are available from the following proposition which parallels Proposition 1.

Proposition 2 *Let $N \geq 2$ and N be even. For every symmetric strategy profile τ,*

(a) if $EU_m(0, \tau) = EU_m(\phi, \tau)$, then $EU_m(0, \tau) > EU_m(1, \tau)$ and $EU_n(1, \tau) > EU_n(0, \tau)$, and

9

(b) if $EU_m(1, \tau) = EU_m(\phi, \tau)$, then $EU_m(1, \tau) > EU_m(0, \tau)$ and $EU_n(0, \tau) > EU_n(1, \tau)$.

Obviously this proposition rules out several additional possible mixed strategy equilibria. It is an easy corollary of these two propositions that there cannot be an equilibrium in which both types of UIVs put positive probability on the same action.

6   Turnout in Large Elections

In the previous section we identified the existence of the swing voter's curse in an environment with adversarial preferences. We now turn to a characterization of the equilibria of this model in large elections. Specifically, we define a sequence of games indexed by N and a sequence of strategy profiles for each game as $\{\tau^N\}_{N=1}^{\infty}$.

We start our analysis of the turnout decision of UIVs with a lemma that provides useful facts about the strategic incentives of these voters. Before we present this lemma, we discuss the key role that the *expected margin of victory* in a state has in determining these incentives in large electorates. In particular, we are interested in the expected margin of victory in state 0 versus state 1. Formally, the expected margin of victory in state z is given by $|\sigma_{z,0} - \sigma_{z,1}|$.

Loosely speaking, in a sufficiently large electorate, whichever state has the smaller expected margin of victory will be the most likely true state conditional on making or breaking a tie. This is true by a large deviation argument that a tie or near-tie is exponentially more likely to occur in the state with an expected outcome closest to a tie. Thus, if the expected margin of victory in state 1 is smaller than the expected margin of victory in state 0, then in a sufficiently large electorate, a pivotal voter must infer that the true state of the world is state 1 with probability close to one and should vote accordingly. In sum, if $|\sigma_{0,0} - \sigma_{0,1}| > |\sigma_{1,0} - \sigma_{1,1}|$, then, conditional on being pivotal, the probability that the true state is 1 goes to one as the electorate gets large. Using Lemma 1, this condition can be written as $|\sigma_{1,0} + q(p_m - p_n) - \sigma_{0,1}| > |\sigma_{1,0} - \sigma_{0,1} - q(p_m - p_n)|$, which is equivalent to $(\sigma_{1,0} - \sigma_{0,1} + q(p_m - p_n))^2 > (\sigma_{1,0} - \sigma_{0,1} - q(p_m - p_n))^2$. This inequality simplifies to $4(\sigma_{1,0} - \sigma_{0,1})q(p_m - p_n) > 0$. Given our assumptions, this holds if and only if $\sigma_{1,0} - \sigma_{0,1} > 0$. This line of reasoning is formalized in the following lemma.

Lemma 3 *Consider a sequence of voting games and strategy profiles $\{\tau^N\}_{N=1}^{\infty}$.*

(a) *Fix $\varepsilon > 0$. If $\sigma_{1,0}(\tau^N) - \sigma_{0,1}(\tau^N) > \varepsilon$ for all $N \geq 1$, then there exists $\overline{N}$ such that for each $N' > \overline{N}$, $EU_m(1, \tau^{N'}) > EU_m(\phi, \tau^{N'}) > EU_m(0, \tau^{N'})$ and $EU_n(0, \tau^{N'}) > EU_n(\phi, \tau^{N'}) > EU_n(1, \tau^{N'})$.*

(b) *Fix $\varepsilon > 0$. If $\sigma_{0,1}(\tau^N) - \sigma_{1,0}(\tau^N) > \varepsilon$ for all $N \geq 1$, then there exists $\overline{N}$ such that for each $N' > \overline{N}$, $EU_m(0, \tau^{N'}) > EU_m(\phi, \tau^{N'}) > EU_m(1, \tau^{N'})$ and $EU_n(1, \tau^{N'}) > EU_n(\phi, \tau^{N'}) > EU_n(0, \tau^{N'})$.*

(c) *If for each $N \geq 1$ and for each $t \in \{m, n\}$, $EU_t(0, \tau^N) = EU_t(1, \tau^N)$, then for each $\varepsilon > 0$ there exists $\overline{N}$ such that for each $N' > \overline{N}$, $|\sigma_{1,0}(\tau^{N'}) - \sigma_{0,1}(\tau^{N'})| < \varepsilon$ and $|\sigma_{0,0}(\tau^{N'}) - \sigma_{1,1}(\tau^{N'})| < \varepsilon$.*

Part (a) of the lemma states that if a mismatched vote is more likely in state 1 than state 0, then in a sufficiently large electorate, a UIM prefers to vote for candidate 1 and a UIN prefers to vote for candidate 0. This follows naturally from our argument that this case corresponds to near certainty that the true state is 1, conditional on a UIV being pivotal. Part (b) is the analogous result for the case that a mismatched vote is more likely in state 0 than state 1. Part (c) gives the contrapositive of the first two. That is, if some type is indifferent between voting for the two candidates, then in a sufficiently large electorate, a mismatched vote must be almost equally likely in the two states.

This lemma plays an important role in characterizing the limiting cases of possible equilibria as the size of the electorate gets large. We present this characterization in the following proposition.

Proposition 3 *Suppose $\{\tau^N\}_{N=1}^{\infty}$ is a convergent sequence of equilibria. Then this sequence converges to τ if and only if*

(a) $p_0 - p_1 > (1 - q)(p_m - p_n)$, $\tau(1, m) = 1$, and $\tau(0, n) = 1$.

(b) $p_0 - p_1 \leq (1 - q)(p_m - p_n)$, $\tau(1, m) = \frac{p_0 - p_1 + p_n(1-q)}{p_m(1-q)}$ and $\tau(\phi, m) = 1 - \tau(1, m)$, and $\tau(0, n) = 1$.

(c) $p_0 - p_1 \leq (1 - q)p_n$, $\tau(0, m) = \frac{p_n(1-q) - (p_0 - p_1)}{p_m(1-q)}$ and $\tau(\phi, m) = 1 - \tau(0, m)$, and $\tau(1, n) = 1$.

(d) $p_0 - p_1 \leq (1 - q)p_n$, $\tau(\phi, m) = 1$, $\tau(0, n) = \frac{p_n(1-q) - (p_0 - p_1)}{2p_n(1-q)}$ and $\tau(1, n) = 1 - \tau(0, n)$.

11

To help understand this result, recall that $p_0 - p_1$ is the difference in the population share of voters sure to vote for candidate 0 versus the share sure to vote for candidate 1 and $(1-q)(p_m - p_n)$ is the amount by which UIMs outnumber UINs, as a proportion of the population. As the UIMs and UINs have diametrically opposed preferences, the UIMs will always want to "cancel out" the UINs. This leaves a fraction $(1-q)(p_m - p_n)$ of "excess" UIMs. We begin by considering parts (a) and (b). Clearly exactly one of these two equilibria will exist in large electorates. Part (a) describes the case where the expected excess of UIMs is too small to compensate for the difference in partisan advantage enjoyed by candidate 0. In this case, regardless of the strategies used by the UIVs, the expected margin of victory will be smaller in state 1 than in state 0. Therefore, in a sufficiently large electorate, a pivotal voter is almost surely in state 1. Thus, all UIMs should vote for candidate 1 and all UINs should vote for candidate 0, as given in part (a).

Part (b) of the proposition deals with the case in which the expected excess of UIMs is large enough to fully offset the bias introduced by partisans. In this case, the UIMs mix between abstention and voting so as to exactly compensate for the difference in partisan support and the UINs' votes. In this way, the election is left to be decided by the informed voters. As there are likely to be more informed matchers than informed nonmatchers, the UIMs can get their preferred candidate as the electoral outcome in large elections.[2]

In part (b), the UIMs mix between voting for 1 and abstaining. If the share of UINs is greater than the difference in the partisan shares, part (c) of the proposition identifies another equilibrium in which the UIMs mix between voting for 0 and abstaining. Although the specific choice of strategies differs, the logic underlying this additional equilibrium is the same as in part (b). That is, enough UIMs vote in a way that cancels out the votes of the UINs and the remainder abstain, leaving the election to be decided by the excess informed matchers. For this same range of parameters, part (d) describes a final possible equilibrium in which the UIMs abstain and the UINs mix between the two candidates. In this case, the mixing probabilities of the UINs compensate for the difference in partisan support, once again leaving the election to be determined by the informed agents. By the logic of the swing voter's curse, the UIMs best choice is to abstain completely.

---

[2]It should be noted that while the limiting condition for this case is that the probability of a mismatched vote is equal in the two states, in a large but finite election these probabilities are slightly different in the two states. This is to insure that the UIMs are indifferent, as required.

It is worth noting that by setting $p_n = 0$ we get the special case of this model examined by Feddersen and Pesendorfer (1996). In this case, the four potential equilibria identified in Proposition 3 reduce to the two equilibria described in Propositions 2 and 3 of Feddersen and Pesendorfer (1996), as expected.

7   Information Aggregation

We have fully characterized the symmetric Nash equilibria in a large electorate in the previous section. Based on this characterization, we show that under some conditions there are equilibria in which the winning candidate is not the candidate that would win if the electorate were fully informed while, if these conditions do not hold, then the winning candidate is almost surely the one who would win under full information. When the electoral outcome is not the same under incomplete information as it would be under complete information, we say that the equilibrium *fails to aggregate information.* Otherwise, we say that the equilibrium *aggregates information.*

In order to formally define these terms, let W denote the event that the winner of the election (with private information about the state) is the same as the winner of the election when the state of the world is publicly known (by all voters). Clearly, the likelihood of W occurring depends on the strategies τ of the UIVs. We now state the formal definition of asymptotic information aggregation.

Definition 1 *Suppose $\{\tau^N\}_{N=1}^{\infty}$ is a sequence of equilibria that converges to τ. If for every $\varepsilon > 0$ there exists $\overline{N}$ such that for every $N > \overline{N}$, $\Pr[W] > 1-\varepsilon$, we say that τ aggregates information asymptotically. Otherwise, we say that τ fails to aggregate information asymptotically.*

If all equilibria of a voting game aggregate information asymptotically, then we say information is always aggregated asymptotically. Otherwise, we say information is not always aggregated asymptotically.

Consider parameters that satisfy $p_m - p_n < p_0 - p_1 < (1 - q)p_n$ and the equilibrium identified in case (c) of Proposition 3. In this case UIMs vote for candidate 0 with probability $\frac{p_n(1-q)-(p_0-p_1)}{p_m(1-q)}$ and abstain with probability $1 - \frac{p_n(1-q)-(p_0-p_1)}{p_m(1-q)}$, while UINs vote for candidate 1. In state 0, the expected vote share for candidate 0 is $p_1 + p_n + (p_m - p_n)q$, and the expected vote

share for candidate 1 is $p_1 + p_n$. On the other hand, in state 1 the expected vote share for each candidate is reversed. We know that, by the law of large numbers, as the size of the electorate gets large, the actual vote share for each candidate converges to the expected vote share. Thus, under this configuration of parameter values, in state 0 the winner is candidate 0, and in state 1 it is candidate 1. On the other hand, if the true state of the world is 1 and this is publicly known, then a fraction $p_0 + p_n$ prefers candidate 0 and a fraction $p_1 + p_m$ prefers candidate 1. As $p_0 - p_1 > p_m - p_n$, a plurality in state 1, i.e., type-0 partisans and nonmatchers, prefers candidate 0. Thus, we have a situation in which an equilibrium fails to aggregate information asymptotically. The next proposition formalizes this argument about when equilibria aggregate information asymptotically or not.

Proposition 4 *If $p_0 - p_1 < p_m - p_n$ or $p_0 - p_1 > (1-q)p_n$, then information is always aggregated asymptotically. Otherwise, information is not always aggregated asymptotically.*

The two conditions in this proposition are easily understood. As explained above, if $p_0 - p_1 < p_m - p_n$, then with public information, in each state of the world a plurality of voters prefers that the candidate match the state. In this case, all of the equilibria described in Proposition 3 give outcomes that agree with this majority preference with probability approaching one. If this first condition is not satisfied, then in state 1, with public information a plurality of voters would prefer candidate 0. The proof of the proposition shows that this is not a problem for case (a) of Proposition 3. Likewise, the equilibrium condition of case (b) will not be satisfied, leaving cases (c) and (d) in Proposition 3. These are the two cases in which information is not always aggregated asymptotically. On the other hand, the second condition of Proposition 4 is inconsistent with the equilibrium condition of these two cases. Therefore, information is not always aggregated asymptotically if and only if the two conditions of Proposition 4 fail to hold.

It should be emphasized that this failure of information aggregation occurs only in state 1 when the conditions of Proposition 4 do not hold. This is the less likely state, with prior probability $1 - \alpha < 1/2$. One other thing to note about Proposition 4 is that, if we increase the probability of being informed above a certain threshold, the election will always aggregate information asymptotically. This is stated in Corollary 1.

Corollary 1 *If $q > 1 - \frac{p_0 - p_1}{p_n}$, then information is always aggregated asymptotically.*

This corollary is easy to verify by inspection of Proposition 4. Intuitively, information is always aggregated asymptotically if the fraction of informed swing voters is large enough.

Finally, it is once again instructive to consider letting $p_n = 0$, which corresponds to the model of Feddersen and Pesendorfer (1996). Recall that in their model, information is always aggregated asymptotically. This agrees with Proposition 4 above, in that the condition for information aggregation, $p_0 - p_1 > (1 - q)p_n$, is trivially satisfied when $p_n = 0$.

8 Welfare Analysis

In the previous section we characterized the conditions under which an election either aggregates information or fails to aggregate information asymptotically. In fact, this section showed that under some conditions, an environment with adversarial preferences has equilibria in which plurality rule almost surely chooses a minority-preferred candidate. Now the question is how much impact such a situation has on the electorate's welfare.

Clearly, the worst case from the point of view of voter welfare is a situation in which a candidate favored by the smallest minority wins the election. Suppose the state z equals 1. By Proposition 4, if $p_m - p_n \leq p_0 - p_1 \leq (1 - q)p_n$, then there is an equilibrium that fails to aggregate information asymptotically. Specifically, candidate 1 is elected even though a majority of voters prefer candidate 0. In this section, we show that in this case, the share of voters who prefer candidate 0, $p_0 + p_n$, is bounded above by $2/3$ and below by $1/2$. In other words, the size of the majority when the election chooses a minority-preferred candidate can be anywhere from 50% to 66 2/3%, depending on the values of the model's parameters.

To verify the above claim, note that in state 1, $p_0 + p_n$ of the electorate prefers candidate 0 and $p_1 + p_m$ of them wants to have candidate 1 in office. We must find values of the parameters that maximize $p_0 + p_n$ subject to $p_m > p_n$ and $p_m - p_n \leq p_0 - p_1 \leq (1 - q)p_n$. The last inequality suggests that we should set $p_1 = 0$ and $p_0 = p_n - \varepsilon$. Then from $p_m > p_n$ and $p_0 + p_m + p_n = 1 - p_\phi$, it easily follows that p_n is bounded above by $1/3$ and thus $p_0 + p_n$ is bounded above by $2/3$.

Tables 1 reports numerical examples of this situation. The table supposes that state 1 is realized and gives the proportion of voters who prefer candidate 0 and vote shares of the candidates under incomplete information.

Table 1: Failure of Information Aggregation when $z = 1$

p_0	p_1	p_m	p_n	q	p_0+p_n	p_1+p_m	$\tau(0,m)$	$\tau(\phi,m)$	VS0	VS1
.3	.01	.35	.34	.1	**.64**	.36	.051	.949	.4993	**.5007**
.2	.01	.4	.39	.1	**.59**	.41	.447	.553	.4994	**.5006**
.1	.01	.45	.44	.1	**.54**	.46	.756	.244	.4995	**.5005**

9 Conclusion

In this paper we have considered the behavior of voters in a large electorate with have adversarial preferences. We have shown that the swing voter's curse applies to one type of voter and that unlike settings with non-adversarial preferences, there are equilibria which fail to correctly aggregate information. In other words, the winner in a large electorate under incomplete information need not coincide with that chosen by a sincere and fully informed electorate.

These results call into question the arguments that date back to Condorcet that majority rule elections can arrive at a correct decision even with dispersed private information. With adversarial preferences, we can no longer be certain that election outcomes can be justified on the grounds that they reflect majority preferences and are legitimately imposed on the remaining minority. Moreover, our results speak to the empirical findings of Bartels (1996) for the six U.S. presidential elections from 1972 to 1992 that the outcome of these elections differs from what would be expected from a fully informed electorate.[3] We also view the richer description of the strategic behavior of independents presented here to be empirically valuable.

That being said, we have shown that the conditions for the possible selection of a minority-preferred candidate require that the information level of voters about politics be low enough and that the partisan support of the two candidates is not balanced. This suggests at least two possible ways to

---

[3]See also Althaus (1998) and Lau and Redlawsk (1997).

increase the effectiveness of elections. One is to improve voters' information level and the other is to foster a competitive and responsive two-party system.

Appendix

PROOF of Proposition 1: Let $N = 2\eta$ where $\eta \in \mathbb{N}$ and define the following:

$$L_j = \frac{(2\eta)!}{j!j!(2\eta - 2j)!}\sigma_\phi(\tau)^{2\eta - 2j}, \qquad M_j = \frac{(2\eta)!}{j!j!t(2\eta - 2j - 1)!}\sigma_\phi(\tau)^{2\eta - 2j - 1}$$

$$\Phi = \sum_{j=0}^{\eta} L_j(\sigma_{1,0}(\tau)\sigma_{1,1}(\tau))^j, \qquad \Psi = \sum_{j=0}^{\eta} L_j(\sigma_{0,0}(\tau)\sigma_{0,1}(\tau))^j$$

$$\phi = \sum_{j=0}^{\eta-1} M_j(\sigma_{1,0}(\tau)\sigma_{1,1}(\tau))^j, \qquad \psi = \sum_{j=0}^{\eta-1} M_j(\sigma_{0,0}(\tau)\sigma_{0,1}(\tau))^j.$$

We begin with the indifference condition $EU_m(1,\tau) = EU_m(0,\tau)$. From equation (8), this can be written as

$$(1 - \alpha)\left[\pi_e(1,\tau) + 1/2(\pi_0(1,\tau) + \pi_1(1,\tau))\right] =$$
$$\alpha\left[\pi_e(0,\tau) + 1/2(\pi_0(0,\tau) + \pi_1(0,\tau))\right],$$

and from the definitions of Φ, ϕ, Ψ, and ψ, we get

$$(1 - \alpha)\left[\Phi + 1/2(\sigma_{1,0}(\tau) + \sigma_{1,1}(\tau))\phi\right] = \alpha\left[\Psi + 1/2(\sigma_{0,0}(\tau) + \sigma_{0,1}(\tau))\psi\right].$$

Using the fact that $\sigma_{1,0}(\tau) + \sigma_{1,1}(\tau) = \sigma_{0,0}(\tau) + \sigma_{0,1}(\tau)$, we can solve for α and find that

$$\alpha = \frac{\Phi + \frac{1}{2}\left[\sigma_{1,0}(\tau) + \sigma_{1,1}(\tau)\right]\phi}{\Phi + \Psi + \frac{1}{2}\left[\sigma_{0,0}(\tau) + \sigma_{0,1}(\tau)\right](\phi + \psi)}. \tag{9}$$

Next, we consider the relative value of voting versus abstaining given the indifference condition above. Using equation (6), we have

$$
\begin{aligned}
EU_m(1,\tau) - EU_m(\phi,\tau) &= \frac{1}{2}[(1 - \alpha)[\pi_e(1,\tau) + \pi_1(1,\tau)] - \alpha[\pi_e(0,\tau) + \pi_1(0,\tau)]] \\
&= \frac{1}{2}[(1 - \alpha)[\Phi + \sigma_{1,0}(\tau)\phi] - \alpha[\Psi + \sigma_{0,0}(\tau)\psi]] \\
&= \frac{1}{2}\left[\Phi + \sigma_{1,0}(\tau)\phi - \alpha[\Phi + \Psi + \frac{1}{2}(\sigma_{0,0}(\tau) + \sigma_{0,1}(\tau))(\phi + \psi)]\right] \\
&\quad + \frac{\alpha}{2}\left[\frac{1}{2}(\sigma_{0,0}(\tau) + \sigma_{0,1}(\tau))(\phi + \psi) - (\sigma_{1,0}(\tau)\phi + \sigma_{0,0}(\tau)\psi)\right].
\end{aligned}
$$

18

Applying equation (9) to the first α in this expression and using Lemma 1 yields

$$EU_m(1,\tau) - EU_m(\phi,\tau) = \frac{1}{2}\left[\Phi + \sigma_{1,0}(\tau)\phi - \left[\Phi + \frac{1}{2}\left[\sigma_{1,0}(\tau) + \sigma_{1,1}(\tau)\right]\phi\right]\right]$$
$$+ \frac{\alpha}{2}\left[\frac{1}{2}(\sigma_{0,0}(\tau) + \sigma_{0,1}(\tau))(\phi + \psi) - (\sigma_{1,0}(\tau)\phi + \sigma_{0,0}(\tau)\psi)\right]$$
$$= \frac{1}{4}[\sigma_{1,0}(\tau)\phi - \sigma_{1,1}(\tau)\phi + \alpha[((p_m - p_n)q + \sigma_{0,1}(\tau)$$
$$- \sigma_{1,0}(\tau))(\phi + \psi) - 2(p_m - p_n)q\psi]].$$

A final application of Lemma 1 gives

$$EU_m(1,\tau) - EU_m(\phi,\tau) =$$
$$\frac{1}{4}[(\sigma_{1,0}(\tau) - \sigma_{0,1}(\tau))(\phi - \alpha(\phi + \psi)) - (p_m - p_n)q((1 - \alpha)\phi + \alpha\psi)].$$

As $(p_m - p_n)q((1-\alpha)\phi + \alpha\psi)$ is positive, if $(\sigma_{1,0}(\tau) - \sigma_{0,1}(\tau))(\phi - \alpha(\phi+\psi))$ is less than or equal to zero, then $EU_m(1,\tau) < EU_m(\phi,\tau)$. To show this, we again apply equation (9) and simplify. Omitting some straightforward manipulations, we arrive at

$$(\sigma_{1,0}(\tau) - \sigma_{0,1}(\tau))(\phi - \alpha(\phi + \psi)) = \frac{(\sigma_{1,0}(\tau) - \sigma_{0,1}(\tau))(\Psi\phi - \Phi\psi)}{\Phi + \Psi + 1/2[\sigma_{0,0}(\tau) + \sigma_{0,1}(\tau)](\phi + \psi)}.$$

The denominator of this expression is clearly positive. For the numerator, a straightforward modification of the proof of Lemma 1 in Fey and Kim (2002) establishes that if $\sigma_{1,0}(\tau) < (>)\sigma_{0,1}(\tau)$, then $\Psi\phi > (<)\Phi\psi$ and if $\sigma_{1,0}(\tau) = \sigma_{0,1}(\tau)$, then $\Psi\phi = \Phi\psi$. In each case, then, the numerator is less than or equal to zero, and so $EU_m(1,\tau) < EU_m(\phi,\tau)$ This establishes the first part of the proposition. The second part of the proposition is an immediate consequence of Lemma 2. ∎

PROOF of Proposition 2.: The proof of this proposition parallels the proof given above. Therefore, we present a shortened version here, with only those details that differ from the earlier proof.

For the first part of the proposition, the indifference condition $EU_m(0,\tau) = EU_m(\phi,\tau)$ can be written as

$$\alpha[\pi_e(0,\tau) + \pi_0(0,\tau)] = (1 - \alpha)[\pi_e(1,\tau) + \pi_0(1,\tau)],$$

19

which can be written as

$$\alpha[\Psi + \sigma_{0,1}(\tau)\psi] = (1 - \alpha)[\Phi + \sigma_{1,1}(\tau)\phi].$$

Solving for α, we obtain

$$\alpha = \frac{\Phi + \sigma_{1,1}(\tau)\phi}{\Psi + \sigma_{0,1}(\tau)\psi + \Phi + \sigma_{1,1}(\tau)\phi}. \tag{10}$$

From the proof of Proposition 1, we have $EU_m(1, \tau) - EU_m(\phi, \tau) = \frac{1}{2}[(1 - \alpha)[\Phi + \sigma_{1,0}(\tau)\phi] - \alpha[\Psi + \sigma_{0,0}(\tau)\psi]]$. Using equation (10) here and doing some algebra yields the following expression for the utility difference:

$$EU_m(1, \tau) - EU_m(\phi, \tau) = \Big[(\sigma_{1,0}(\tau) - \sigma_{0,1}(\tau))(\Psi\phi - \Phi\psi) - (p_m - p_n)q[\Psi\phi + \Phi\psi$$
$$+ (\sigma_{1,0}(\tau) + \sigma_{1,1}(\tau))\phi\psi]\Big]/2\Big[\Psi + \sigma_{0,1}(\tau)\psi + \Phi + \sigma_{1,1}(\tau)\phi\Big].$$

The denominator and the second term in the numerator are clearly positive. By the same logic as in the previous proof, the first term is less than or equal to zero. We conclude that $EU_m(1, \tau) < EU_m(\phi, \tau)$. The other parts of the proposition can be proved in a similar fashion. ∎

PROOF of Lemma 3.: To establish part (a), fix an $\varepsilon > 0$ such that $\sigma_{1,0}(\tau^N) - \sigma_{0,1}(\tau^N) > \varepsilon$ for all $N \geq 1$. Then $(\sigma_{1,0}(\tau^N) - \sigma_{0,1}(\tau^N))(p_m - p_n)q > 0$ holds. Rewriting the left hand side, we get

$$\sigma_{1,0}(\tau^N)(p_m - p_n)q + \sigma_{1,0}(\tau^N)\sigma_{0,1}(\tau^N)$$
$$-\sigma_{1,0}(\tau^N)\sigma_{0,1}(\tau^N) - \sigma_{0,1}(\tau^N)(p_m - p_n)q > 0$$
$$\sigma_{1,0}(\tau^N)((p_m - p_n)q + \sigma_{0,1}(\tau^N)) -$$
$$\sigma_{0,1}(\tau^N)((p_m - p_n)q + \sigma_{1,0}(\tau^N)) > 0$$
$$\sigma_{1,0}(\tau^N)\sigma_{1,1}(\tau^N) - \sigma_{0,1}(\tau^N)\sigma_{0,0}(\tau^N)) > 0.$$

This last expression allows us to find a $\xi > 0$ such that for all $N \geq 1$, $\sigma_{1,0}(\tau^N) > \xi$, $\sigma_{1,1}(\tau^N)\sigma_{1,0}(\tau^N) - \sigma_{0,0}(\tau^N)\sigma_{0,1}(\tau^N) > \xi$, and $\sigma_\phi(\tau^N) > \xi$. From equations (6), (7), and (8), $EU_m(1, \tau^N) > EU_m(\phi, \tau^N)$ and $EU_m(\phi, \tau^N) > EU_m(0, \tau^N)$ if the following conditions hold:
(i) $(1 - \alpha)\pi_e(1, \tau^N) - \alpha\pi_e(0, \tau^N) > 0$,
(ii) $(1 - \alpha)\pi_1(1, \tau^N) - \alpha\pi_1(0, \tau^N) > 0$,
(iii) $(1 - \alpha)\pi_0(1, \tau^N) - \alpha\pi_0(0, \tau^N) > 0$.

To establish these three conditions, we use the following result, which is Lemma 0 in Feddersen and Pesendorfer (1996). Let $(a_N, b_N, c_N)_{N=1}^{\infty}$ be a sequence that satisfies $(a_N, b_N, c_N) \in [0,1]^3$, $a_N < b_N - \delta$ and $\delta < c_N$, for all N and for some $\delta > 0$. Then for $i = 0, 1$

$$\frac{\displaystyle\sum_{j=0}^{(N/2)-i} \frac{N!}{(j+i)!j!\,(N-2j-i)!} c_N^{N-2j-i} a_N^j}{\displaystyle\sum_{j=0}^{(N/2)-i} \frac{N!}{(j+i)!j!\,(N-2j-i)!} c_N^{N-2j-i} b_N^j} \to 0 \text{ as } N \to \infty.$$

Combining this result with the fact that $\sigma_{1,1}(\tau^N)\sigma_{1,0}(\tau^N) - \sigma_{0,0}(\tau^N)\sigma_{0,1}(\tau^N) > \xi$ and $\sigma_\phi(\tau^N) > \xi$, we have

$$\frac{\alpha \pi_e(0, \tau^N)}{(1-\alpha)\,\pi_e(1, \tau^N)} =$$

$$\frac{\alpha \displaystyle\sum_{j=0}^{N/2} \frac{N!}{j!j!(N-2j)!} \sigma_\phi(\tau^N)^{N-2j} (\sigma_{0,0}(\tau^N)\sigma_{0,1}(\tau^N))^j}{(1-\alpha) \displaystyle\sum_{j=0}^{N/2} \frac{N!}{j!j!(N-2j)!} \sigma_\phi(\tau^N)^{N-2j} (\sigma_{1,0}(\tau^N)\sigma_{1,1}(\tau^N))^j} \to 0 \text{ as } N \to \infty.$$

Therefore, condition (i) is satisfied for sufficiently large N. Similarly, $\sigma_{1,1}(\tau^N)\sigma_{1,0}(\tau^N) - \sigma_{0,0}(\tau^N)\sigma_{0,1}(\tau^N) > \xi$, $\sigma_\phi(\tau^N) > \xi$, $\sigma_{0,0}(\tau^N) > \xi$, and $\sigma_{0,1}(\tau^N) > \xi$ imply that

$$\frac{\alpha \pi_1(0, \tau^N)}{(1-\alpha)\pi_1(1, \tau^N)} =$$

$$\frac{\alpha\sigma_{0,0}(\tau^N) \displaystyle\sum_{j=0}^{N/2} \frac{N!}{(j+1)!j!(N-2j-1)!} \sigma_\phi(\tau^N)^{N-2j-1} (\sigma_{0,0}(\tau^N)\sigma_{0,1}(\tau^N))^j}{(1-\alpha)\sigma_{1,0}(\tau^N) \displaystyle\sum_{j=0}^{N/2} \frac{N!}{(j+1)!j!(N-2j-1)!} \sigma_\phi(\tau^N)^{N-2j-1} (\sigma_{1,0}(\tau^N)\sigma_{1,1}(\tau^N))^j} \to 0$$

$$\text{as } N \to \infty.$$

21

and

$$\frac{\alpha\pi_0(0,\tau^N)}{(1-\alpha)\pi_0(1,\tau^N)} =$$

$$\frac{\alpha\sigma_{0,1}(\tau^N)\sum_{j=0}^{N/2}\frac{N!}{(j+1)!j!(N-2j-1)!}\sigma_\phi(\tau^N)^{N-2j-1}(\sigma_{0,0}(\tau^N)\sigma_{0,1}(\tau^N))^j}{(1-\alpha)\sigma_{1,1}(\tau^N)\sum_{j=0}^{N/2}\frac{N!}{(j+1)!j!(N-2j-1)!}\sigma_\phi(\tau^N)^{N-2j-1}(\sigma_{1,0}(\tau^N)\sigma_{1,1}(\tau^N))^j} \to 0$$

$$\text{as } N \to \infty.$$

Hence, conditions (ii) and (iii) are also satisfied.

Using an analogous argument we can show that if there exists an $\epsilon > 0$ such that $\sigma_{0,1}(\tau^N) - \sigma_{1,0}(\tau^N) > \epsilon$ for all $N \geq 0$, then $EU_m(0,\tau^N) > EU_m(\phi,\tau^N) > EU_m(1,\tau^N)$. This is part (b). Finally, part (c) is simply the contrapositive of part (a). ∎

PROOF of Proposition 3.: Let $\{\tau^N\}_{N=1}^\infty$ be a convergent sequence of equilibria. There are 49 theoretically possible combinations of various mixed and pure strategies for the UIMs and UINs. Using Propositions 1 and 2 and the corollary of these two propositions that there cannot be an equilibrium in which the two types of UIVs put positive probability on the same action, it is possible to rule out 40 of these combinations. This leaves 9 possible combinations to investigate.

Now consider the limit τ of the convergent sequence. Recall that $\sigma_{1,0}(\tau) = p_0 + p_m(1-q)\tau(0,m) + p_n(1-q)\tau(0,n) + p_n q$ and $\sigma_{0,1}(\tau) = p_1 + p_m(1-q)\tau(1,m) + p_n(1-q)\tau(1,n) + p_n q$. We will analyze three cases corresponding to $\sigma_{1,0}(\tau)$ being greater than, less than, or equal to $\sigma_{0,1}(\tau)$. First, if $\sigma_{1,0}(\tau) > \sigma_{0,1}(\tau)$, then there exists an $\varepsilon > 0$ such that $\sigma_{1,0}(\tau^N) - \sigma_{0,1}(\tau^N) > \varepsilon$, for sufficiently large N. By part (a) of lemma 3, for sufficiently large N, the best response for UIMs is to vote for 1 while the best response for UINs is to vote for 0. These strategies imply that $\sigma_{1,0}(\tau) = p_0 + p_n(1-q) + p_n q = p_0 + p_n$ and $\sigma_{0,1}(\tau) = p_1 + p_m(1-q) + p_n q$. Therefore, we must have $p_0 + p_n - (p_1 + p_m(1-q) + p_n q) > 0$. These equations are all consistent as long as $p_0 - p_1 > (1-q)(p_m - p_n)$ in the limit τ. This establishes part (a).

Alternatively, if $\sigma_{0,1}(\tau) > \sigma_{1,0}(\tau)$, then there exists an $\varepsilon > 0$ such that $\sigma_{0,1}(\tau^N) - \sigma_{1,0}(\tau^N) > \varepsilon$, for sufficiently large N. By part (b) of lemma 3, for sufficiently large N, the best response for UIMs is to vote for 0 while

the best response for UINs is to vote for 1. These strategies imply that $\sigma_{1,0}(\tau) = p_0 + p_m(1-q) + p_n q$ and $\sigma_{0,1}(\tau) = p_1 + p_n(1-q) + p_n q = p_1 + p_n$. Therefore it must be that $p_1 + p_n - (p_0 + p_m(1-q) + p_n q) > 0$ or, more simply, $p_1 - p_0 > (1-q)(p_m - p_n)$. But this is impossible as the left hand side is negative and the right hand side is positive.

So we are left with the case that, in the limit, $\sigma_{0,1}(\tau) = \sigma_{1,0}(\tau)$. By part (c) of lemma 3, this is the only case in which mixed strategies are possible. There are only three such potential equilibria that cannot be eliminated by the arguments in the first paragraph. The first possible mixed strategy equilibrium involves the UIMs mixing between voting for 1 and abstaining and the UINs voting for 0, as required by Proposition 2. With these strategies, $\sigma_{1,0}(\tau) = p_0 + p_n(1-q) + p_n q$ and $\sigma_{0,1}(\tau) = p_1 + p_m(1-q)\tau(1,m) + p_n q$. For these to be equal, we must have $\tau(1,m) = p_0 - p_1 + p_n(1-q)/p_m(1-q)$. Clearly, this is always positive. As it is a probability, it must also satisfy $p_0 - p_1 + p_n(1-q)/p_m(1-q) \le 1$, which holds if $p_0 - p_1 \le (p_m - p_n)(1-q)$. This establishes part (b).

The second possible mixed strategy equilibrium specifies mixing by the UIMs over voting for 0 and abstaining, and the UINs voting for 1, again as required by Proposition 2. These strategies imply $\sigma_{1,0}(\tau) = p_0 + p_m(1-q)\tau(0,m) + p_n q$ and $\sigma_{0,1}(\tau) = p_1 + p_n(1-q) + p_n q$. Setting these two terms equal gives $\tau(0,m) = p_1 - p_0 + p_n(1-q)/p_m(1-q)$. As $p_1 - p_0$ is negative and $p_n < p_m$, this expression is less than 1. It is nonnegative when $p_0 - p_1 \le p_n(1-q)$. This establishes part (c). The last potential equilibrium has the UIMs abstaining and the UINs mixing between voting for 0 and 1. This gives $\sigma_{1,0}(\tau) = p_0 + p_n(1-q)\tau(0,n) + p_n q$ and $\sigma_{0,1}(\tau) = p_1 + p_n(1-q)(1-\tau(0,n)) + p_n q$. These expressions are equal if $\tau(0,n) = p_1 - p_0 + p_n(1-q)/2p_n(1-q)$. As before, this is clearly less than 1 and is nonnegative if $p_0 - p_1 \le p_n(1-q)$, which establishes part (d).

This accounts for all possible mixed strategy equilibria. It is not difficult to show that the remaining pure strategy equilibria are either impossible or special cases of the mixed strategy equilibria analyzed above. ∎

PROOF of Proposition 4.: To begin, if the state of the world is publicly known, then in state 0, the expected vote share for candidate 0 is $p_0 + p_m$ and the expected vote share for candidate 1 is $p_1 + p_n$. Clearly, in a sufficiently large electorate, candidate 0 will win the election with probability approaching 1. On the other hand, in state 1, the expected vote share for candidate 0 is $p_0 + p_n$ and the expected vote share for candidate 1 is $p_1 + p_m$. Thus,

if $p_0 - p_1 > p_m - p_n$, then candidate 0 will win the election with probability going to 1 as N gets large and if $p_0 - p_1 < p_m - p_n$, then candidate 1 will win the election with probability going to 1.

Now suppose $\{\tau^N\}_{N=1}^\infty$ is a sequence of equilibria that converges to τ. By Proposition 3, it suffices to check the four cases listed in the proposition. Starting with part (a), in state 0 the expected vote share for candidate 0 is $p_0 + qp_m + (1-q)p_n$ and the expected vote share for candidate 1 is $p_1 + qp_n + (1-q)p_m$. Therefore candidate 0 will win the election with probability close to 1 in a large electorate if $p_0 - p_1 > (1-q)(p_m - p_n) - q(p_m - p_n)$. As part (a) requires $p_0 - p_1 > (1-q)(p_m - p_n)$, the previous inequality must be satisfied. So in state 0 the elected candidate is the same as in the case of public information. In state 1, the expected vote share for candidate 0 is $p_0 + p_n$ and the expected vote share for candidate 1 is $p_1 + p_m$. These expected vote shares are exactly the same as with public information, so again the elected candidate will be the same as with public information.

For part (b), as is discussed in the text, it is easy to show that informed type-m voters decide the election. Therefore, in a sufficiently large election, candidate 0 is elected in state 0 and candidate 1 is elected in state 1, with probability approaching 1. This is, with probability approaching 1, the same outcome as with public information occurs in state 0 and it will be the same in state 1 if $p_0 - p_1 < p_m - p_n$. But the equilibrium condition for part (b) is $p_0 - p_1 \leq (1-q)(p_m - p_n)$, so this equilibrium aggregates information.

Lastly, we consider parts (c) and (d) of Proposition 3. The equilibrium condition for these two cases is $p_0 - p_1 \leq (1-q)p_n$ In both parts, informed matchers determine the election outcome, so the candidate elected will match the state with probability close to 1 in a large electorate. As with part (b), in state 1 these outcomes will be the same as with public information if $p_0 - p_1 < p_m - p_n$. But if $p_0 - p_1 \geq p_m - p_n$, this outcome will differ from the outcome under public information. Thus, for parameters satisfying $p_m - p_n \leq p_0 - p_1 \leq (1-q)p_n$, information is not always aggregated while outside this range, information is always aggregated. This completes the proof. ∎

References

S. ALTHAUS, Information Effects in Collective Preferences, *Amer. Polit. Sci. Rev.* **92** (1998) 545–558.

D. AUSTEN-SMITH AND J. BANKS, Information Aggregation, Rationality and the Condorcet Jury Theorem, *Amer. Polit. Sci. Rev.* **90** (1995) 34–45.

L. BARTELS, Uninformed Votes: Information Effects in Presidential Elections, *Amer. J. Polit. Sci.* **40** (1996) 194–230.

P. COUGHLAN, In Defense of Unanimous Jury Verdicts: Mistrials, Communication, and Strategic Voting, *Amer. Polit. Sci. Rev.* **94** (2000) 375–393.

J. DUGGAN AND C. MARTINELLI, A Bayesian Model of Voting in Juries, *Games Econ. Behav.* **37** (2001) 259–294.

T. FEDDERSEN AND W. PESENDORFER, The Swing Voter's Curse, *Amer. Econ. Rev.* **86** (1996) 408–424.

T. FEDDERSEN AND W. PESENDORFER, Voting Behavior and Information Aggregation in Elections with Private Information, *Econometrica* **65** (1997) 1029–1058.

T. FEDDERSEN AND W. PESENDORFER, Convicting the Innocent: The Inferiority of Unanimous Jury Verdicts under Strategic Voting, *Amer. Polit. Sci. Rev.*, **92** (1998) 23–35.

T. FEDDERSEN AND W. PESENDORFER, Abstention in Elections with Asymmetric Information and Diverse Preferences, *Amer. Polit. Sci. Rev.*, **93** (1999) 381–398.

M. FEY AND J. KIM, The Swing Voter's Curse: Comment, *Amer. Econ. Rev.*, **92** (2002) 1264–1268.

D. GERARDI, Jury Verdicts and Preference Diversity, *Amer. Polit. Sci. Rev.* **94** (2000) 395–406.

F. GUL AND W. PESENDORFER, Partisan Politics and Aggregation Failure with Ignorant Voters, *Mimeo, Princeton University* (2005).

R. Lau and D. Redlawsk, Voting Correctly, *Amer. Polit. Sci. Rev.* **91** (1997) 585–598.

H. Li, S. Rosen, and W. Suen, Conflicts and Common Interests in Committees, *Amer. Econ. Rev.*, **91** (2001) 1478–1497.

C. Martinelli, Would Rational Voters Acquire Costly Information?, *J. Econ. Theory*, forthcoming

A. Meirowitz, Informative Voting and Condorcet Jury Theorems with a Continuum of Types, *Social Choice and Welfare* **19** (2002) 219–236.

A. Meirowitz, Deliberative democracy or market democracy: Designing institutions to aggregate preferences and information, *Mimeo, Princeton University* (2005).

N. Persico, Voter Participation and Strategic Uncertainty, *Rev. Econ. Stud.* **71** (2004) 165–191.

R. Razin, Signaling and Election Motivations in a Voting Model with Common Values and Responsive Candidates, *Econometrica*, **71** (2003) 1083–1119.