Pre-play Communication in Games of Two-Sided Incomplete Information

Mark Fey*
Dept. of Political Science
University of Rochester

Jaehoon Kim†
Korea Development Institute

Lawrence S. Rothenberg
Dept. of Political Science
University of Rochester

March 29, 2007

*Corresponding author. Harkness Hall, University of Rochester, Rochester, NY, 14618. Office: 585-275-5810. Fax: 585-271-1616. Email: markfey@mail.rochester.edu.

†We would like to thank David Austen-Smith, Sandeep Baliga, Eddie Dekel, and Scott Page for their helpful comments. All errors are our own.

Abstract

Communication, even cheap talk in a pre-play stage, is commonly viewed as important for inducing information revelation, coordination, and efficient outcomes. Yet, many current results are based on two assumptions that seem to be inconsistent with many interesting empirical situations: that only one player is privately informed and that actors have no constraints limiting their actions. We remedy both of these deficiencies by specifying a more general model of cheap talk with two-sided incomplete information and bounded action spaces. We find that, typically, full information transmission is possible even with two-sided private information if action spaces are unbounded. On the other hand, imposing bounds on the action spaces of the actors can reduce or even completely prevent information transmission and coordination. The nature of these results, in turn, depend on whether actions are *substitutes* (e.g., public goods provision) or *complements* (e.g., an arms race). Thus, our results emphasize that the resource constraints of players and the nature of the strategic interaction can have a significant effect on the potential gains offered by pre-play communication.

JEL Classification: C72

Keywords: cheap talk, strategic complementarities, strategic substitutes, perfect Bayesian equilibrium

1 Introduction

It is often asserted that one effective way to overcome potential obstacles to good policy outcomes is to create venues for political and economic entities to communicate in a less adversarial fashion (Austen-Smith and Feddersen, 2002; Doraszelski, Gerardi, and Squintani, 2003). Successful communication can induce coordination even between actors with different preferences—be they nation-states, local governments, interest groups, regulators, political parties, firms, or private citizens—to avoid harmful actions or to engage in synergistic activities. For example, nation-states may forestall expensive arms races by committing to nonaggression, local governments may maintain water quality by not polluting, political candidates may preserve their reputations by eschewing negative advertising, or oligopolistic firms may avoid costly investments that will reduce their financial returns. Analogously, an agency may change a regulation if a firm convinces it that it has the required technology to make it successful or an activist group may endorse purchasing a firm's products if the corporation demonstrates environmental and social responsibility.

In this paper, we focus on the idea that pre-play communication, and particularly "cheap talk" (i.e., costless talk with unverifiable claims), can produce improved results via coordination of strategic action. Of course, it is well-known that in certain situations, notably when there is one-sided incomplete information and preference disparity is not too great, a privately informed actor who sends cheap-talk messages to a decision-maker can expand the set of achievable outcomes and produce a more efficient result than would otherwise occur (Austen-Smith, 1994; Crawford and Sobel, 1982; Farrel, 1988, 1993; Farrel and Gibbons, 1989; Krishna and Morgan, 2001 a,b). While encouraging, we worry that such findings do not speak directly to many empirical situations of interest where pre-play communication is an option.

Specifically, there are at least two reasons to question whether these standard results have much to offer in many relevant situations. First, the possibility that both sides in a negotiation are privately informed is not incorporated. The strategic context where each side simultaneously reveals information may be much more complicated than in the standard sender-receiver world; in the multiple sender situation, each player must worry what impact her message will have conditional on the other's private information. Hence, standard results cannot completely capture situations such as nations being privately informed about their ability to develop cost-effective weaponry, local governments having unique data on their citizenry's support for environmental quality, political candidates possessing polling results about the efficacy of negative advertising, firms holding assessments about their potential returns from new technological investments, or activist groups and corporations privately knowing their own commitment to keeping a bargain.

Second, limits on how much or how little players can do in pursuit of their objectives are not incorporated, as an unbounded action space is typically assumed. While unboundedness is a general assumption and allows for maximal information transmission, it is unrealistic for most interesting empirical situations. Indeed, permitting negative actions is often unintuitive, as an actor's decision is commonly thought of as the choice of what, if any, nonnegative action to take, such as how much money to expend or effort to exert. Conversely, the absence of an upper bound assumes no resource constraints (e.g., budgets), technological limits, or institutional restrictions (e.g., laws defining acceptable behavior). Hence, incorporating limits on the choices of actors, particularly in the more complicated world of two-sided private information, would seem imperative for determining the potential benefits of pre-play communication.

In short, extrapolating from models with one-sided incomplete information with an unbounded action space to make claims about what happens when there is two-sided private information with limitations on actions is problematic. Can communication still lead to beneficial information transmission and coordination? To answer this question, we model pre-play communication as cheap talk with two-sided incomplete information and bounded actions. Specifically, we suppose that the two players first costlessly send messages and then simultaneously act. As in the standard cheap talk model, each player has private information about her most preferred action and the actions of each player affect the outcome for the other. A key component in our results is the extent to which each player's action impacts the other player's utility.[1] We first solve the baseline case of the model in which the action spaces of the players

[1] In contrast to standard cheap talk models, the preference discrepancy between the players is not a key factor for information transmission in our model.

are unbounded. We then solve alternative cases given various assumptions about how the action spaces are bounded. In each instance, we classify the extent and direction in which each player's action is impacted by the other's choice by an *externality parameter* measuring a player's evaluation of the other's expectation of her own optimal action on the player's optimal action. As in Baliga and Morris (2002), a negative parameter signifies *strategic complementarities*, by which each player's best response increases in the other player's action, such as when activists and firms coordinate on social responsibility. A positive parameter captures *strategic substitutes*, by which each player's best response decreases in the other player's action, such as classic instances of public goods provision, e.g., when one nation contributes less to collective security when another contributes more.

Broadly, our findings are twofold. First, as in models with one-sided cheap talk, with unbounded action spaces communication can lead to efficient outcomes. Coordination is only precluded in the rare instance when the interaction of the externality parameters is one. However, as the no information transmission situation is knife-edged, slight variations in the interaction of the externality parameters can make full information revelation and coordinated action possible. Second, imposing bounds on the action spaces of the actors can, in some cases, reduce or even completely prevent information transmission and coordination, with the more limited player being disadvantaged relative to the other whether or not there is an informative equilibrium. Furthermore, the effect of such constraints differs notably depending on whether substitutes or complements are being considered. With substitution, requiring that actions be nonnegative precludes full information transmission and, if the externality parameters of both players are not too large, assures no information transmission at all. With strategic complements, full information revelation is still possible when the interaction of the two externality parameters is small, but there is no equilibrium when this interaction is large. If we further restrict the actions of players in the case of strategic complements by assuming limits on the upper bound, information transmission becomes more tenuous and the more constrained player is disadvantaged.

These results have significant implications for the possibility that communication induces better informed actions and greater coordination. Most notably, pre-play communication is likely to be less successful than previous results may have implied.

Additionally, our findings indicate that pre-play communication is likely more efficacious for situations defined by strategic complementarities than by substitutions although, even in the former instance, it is quite likely that effective information transmission will not take place. As for instances of strategic substitutes, such as public goods provision, pre-play communication may simply be ineffectual.

By combining two-sided incomplete information with bounded action spaces, our analysis contributes to the extensive literature on cheap talk and communication initiated by Crawford and Sobel (1982). While there have been many important additions and extensions to Crawford and Sobel's work (Farrel and Gibbons, 1989; Rabin, 1990, 1994; Austen-Smith, 1994; Battaglini, 2002; Aumann and Hart, 2003), none have taken our tact of integrating the possibility that both sides who are trying to communicate have incomplete information and face limits on the action space that they have available.[2]

In the main body of our analysis, we first present and solve our model with two-sided incomplete information with no action space restrictions. We then solve for situations with alternative assumptions about the action spaces. The final section concludes by discussing the implications of our results for understanding the usefulness of pre-play communication.

2 Communication with Unbounded Action Spaces

We begin by defining our cheap talk model of two-sided incomplete information. We assume that there are two players, $i = 1, 2$, each with an infinite set of possible actions, Y_i, with $Y = Y_1 \times Y_2$. Each player i is one of an infinite set of possible types, Ω_i, with $\Omega = \Omega_1 \times \Omega_2$. The prior over each player's type space, Ω_i, is $\pi_i \in \Delta(\Omega_i)$, and each player's utility function is $u_i : Y \times \Omega_i \to \mathbb{R}$. To describe the cheap talk stage of the game, we assume that each player i has a continuous message space $\mathcal{M}_i$. Define a messaging strategy for player i by $\mu_i : \Omega_i \to \mathcal{M}_i$ and beliefs for player i as $\lambda_i : \mathcal{M}_j \to \Delta(\Omega_j)$ where $i \neq j$. A pure strategy for player i for a choice of action is a

[2]For a survey, see Farrel and Rabin (1996); for an evolutionary approach, see Kim and Sobel (1992); and to see the effects of a communication stage in particular games, see, for example, Palfrey and Rosenthal (1991), Banks and Calvert (1992), and Austen-Smith and Banks (2000).

function $s_i : \mathcal{M}_i \times \mathcal{M}_j \times \Omega_i \to Y_i$.

For expositional simplicity, we assume throughout the paper that $\Omega_i = [0, 1]$ and that players share a common prior that each ω_i is drawn from a uniform distribution on $[0, 1]$. Moreover, we assume that ω_1 and ω_2 are drawn independently.[3] We also assume that each player's message space is the unit interval, so $\mathcal{M}_i = [0, 1]$. We will analyze the perfect Bayesian equilibria of this game, where $(\mu_1, \mu_2, s_1, s_2, \lambda_1, \lambda_2)$ is a perfect Bayesian equilibrium if each player is playing optimally at all her information sets, given the other player's strategy, and beliefs are updated using Bayes' rule whenever possible.

We first solve this game with no restrictions on the actions spaces of the players. Thus, we assume that each player can take any action on the real line, i.e., $Y_i = \mathbb{R}$ for all i. Our general assumption about utilities is that

$$u_1(y_1, y_2, \omega_1) = g(y_1 + \alpha y_2 - \omega_1) \quad (1)$$

$$u_2(y_1, y_2, \omega_2) = g(y_2 + \beta y_1 - \omega_2), \quad (2)$$

where $\alpha, \beta \in \mathbb{R} \setminus \{0\}$ and the function g is strictly concave and symmetric about 0.[4] As foreshadowed, α and β are externality parameters which capture the effect of one player's chosen action on the other player's utility given her type and action (we exclude the case when both parameters are zero because the game is equivalent to that without cheap talk). Thus, for example, if $\alpha > 1$, a marginal change in player 2's action affects player 1's payoff more than a marginal change in player 1's action; if $\alpha = 1$, these changes are equivalent; and, if $\alpha < 1$, a marginal change in player 1's action has a greater affect on her payoff than a marginal change in player 2's action. In turn, $\alpha\beta$ captures the marginal impact that one player's expectation of the other player's expectation of her optimal action has on her optimal action, i.e., it represents how much each player's optimal action must be adjusted given the changes in that player's expectation of the other player's expectation of her optimal action given the

[3]Independence is not crucial, as our results hold net of types being perfectly correlated. Assuming a uniform distribution on the real line captures the so-called situation of *diffuse views* where a player with some information concerning the state of the world is best off assuming that the likelihood of the other player opting for a particular option is a uniformly distributed random variable over the unit interval (Morris and Shin, 2003).

[4]Thus, g attains its maximum at 0.

equilibrium messaging strategy.

To establish when full communication is possible, we follow Crawford and Sobel by focusing on partitional equilibria, as the underlying logic defining such equilibria is consistent with our analysis. Specifically, concentrating on partitional equilibria captures a four-step process in which each player sends a message representing the subset of the type space containing her type and receives a corresponding message from the other player, infers the subset of the partition that the other player lies in, uses this inference and the message she gives to the other player to form an expectation of optimal actions taken by possible types of the other player, and chooses an optimal action based on her type.[5]

We will show that an infinite number of partitional equilibria exist for any pre-play communication game with $\alpha\beta \neq 1$, ranging from completely uninformative "babbling" equilibria to fully revealing equilibria in which both players are completely informed about the other player's type. Except for the knife-edged situation where the marginal effects of each player's expectation of the other's expectation of her own action on her optimal action is one, any partitional equilibrium with finite size is Pareto-dominated by a fully revealing equilibrium.

We begin with some notation and definitions. For $i = 1, 2$, let $\mathcal{A}^i = \{A^i_\gamma\}_{\gamma\in\Gamma}$ be a partition of the unit interval into subintervals, with index set Γ. That is, for $i = 1, 2$, $\bigcup_{\gamma\in\Gamma} A^i_\gamma = [0, 1]$ and $A^i_\gamma \bigcap A^i_{\gamma'} = \emptyset$, for all $\gamma \neq \gamma' \in \Gamma$, and each A^i_γ is a convex set. We refer to any such $\mathcal{A}^i$ as an *interval partition.* The index set Γ may be finite, countably infinite, or uncountably infinite.[6] Here we permit elements of $\mathcal{A}^i$ to be singletons. All other elements of $\mathcal{A}^i$ are intervals, which may or may not contain their endpoints. Let $d^i(\gamma)$ be the length of the interval A^i_γ. Note that $d^i(\gamma)$ is zero if and only if A^i_γ is a singleton.

These partitions serve as the basis for the messaging strategy for each player. In

[5]Note that each player's messaging strategy should be consistent with her behavioral strategy, the other player's messaging strategy, and her posterior beliefs given the other player's messaging strategy. Also recognize that, as usual in any form of partitional rational expectations equilibrium, the type located at the boundary of two partition subsets must be indifferent between sending messages for the lower or the upper partition subsets and between the corresponding expected optimal actions induced.

[6]It is possible to obtain a more general result for arbitrary measurable partitions of the unit interval at the expense of more complicated notation.

order to define this messaging strategy, for $i = 1, 2$ and for every $\gamma \in \Gamma$, choose a point $a^i(\gamma) \in A^i_\gamma$.[7] Thus, the point $a^i(\gamma)$ is a representative element of A^i_γ for each $\gamma \in \Gamma$. For $x \in [0, 1]$, define $\gamma^i(x)$ to be the value of γ such that $x \in A^i_\gamma$. As $\mathcal{A}^i$ is a partition, a unique such γ must exist and, as such, this function is well-defined. With this, we can define the messaging strategy for player i by

$$\mu_i(\omega_i) = a^i(\gamma^i(\omega_i)). \tag{3}$$

In other words, type ω_i of player i sends the representative element of the interval in $\mathcal{A}^i$ which contains ω_i. It is important to note that this messaging strategy implies that all types in a given interval in $\mathcal{A}^i$ send the same message.

Next, we turn to the player's beliefs after observing a message. For $x \in [0, 1]$, a point mass δ_x is the probability measure that puts probability 1 on x and probability 0 on all other points.[8] Integration with respect to δ_x is the same as evaluation at x. For player $j \neq i$, we define j's belief after receiving message m_i as follows. If $d^i(\gamma^i(m_i)) = 0$, then $\lambda_j(m_i) = \delta_{m_i}$. If not, then

$$\lambda_j(\omega_i|m_i) = \begin{cases} 1/d^i(\gamma^i(m_i)) & \text{if } \omega_i \in A^i_{\gamma^i(m_i)} \\ 0 & \text{otherwise.} \end{cases} \tag{4}$$

In this expression, $A^i_{\gamma^i(m_i)}$ is the interval of types that would be expected to send the message m_i and $d^i(\gamma^i(m_i))$ is the length of the interval. It should be clear that this belief is consistent with the messaging strategy in equation (3) via Bayes' Rule. The belief also specifies that a message off the equilibrium path is interpreted as being sent by a type of player i in the interval which contains the message.

Finally, we define the strategies of the players in the second stage, when they choose actions having observed the cheap talk messages of the first stage. For messages m_1 and m_2, let $\lambda_2(\omega_1|m_1)$ and $\lambda_1(\omega_2|m_2)$ be given by equation 4. Define the following strategies:

$$s_1(\omega_1, m_1, m_2) = \omega_1 - \alpha \left[\frac{E_{\lambda_1}\omega_2 - \beta E_{\lambda_2}\omega_1}{1 - \alpha\beta} \right], \tag{5}$$

[7] This is possible as an application of the Axiom of Choice.

[8] That is, the CDF of δ_x is 0 if $\omega < x$ and 1 if $\omega \geq x$.

and

$$s_2(\omega_2, m_1, m_2) = \omega_2 - \beta \left[\frac{E_{\lambda_2}\omega_1 - \alpha E_{\lambda_1}\omega_2}{1 - \alpha\beta} \right], \tag{6}$$

where

$$E_{\lambda_j}\omega_i = \int_{[0,1]} \omega_i \, d\lambda_j = \int_{A^i_{\gamma^i(m_i)}} \omega_i.$$

Note that, in particular, if $\lambda_j = \delta_{m_i}$, then $E_{\lambda_j}\omega_i = m_i$.

Proposition 1 *Suppose that $\alpha\beta \neq 1$. For every pair of interval partitions $(\mathcal{A}^1, \mathcal{A}^2)$, the strategies and beliefs given by equations 3, 4, 5, and 6 form a perfect Bayesian equilibrium. Thus, there exists a continuum of equilibria to this game.*

Proof. Suppose that $\alpha\beta \neq 1$ and fix a pair of interval partitions $(\mathcal{A}^1, \mathcal{A}^2)$. Clearly, the beliefs in equation 4 are consistent with applying Bayes' Rule to the messaging strategies in equation 3. We now show that the strategies given in equations 5 and 6 are sequentially rational. We first consider the choice of action for player 1 with type ω_1, having observed m_1 and m_2 and given $\lambda_1(\omega_2|m_2)$. Player 1's expected utility of playing action y_1 is given by

$$E_{\lambda_1} u_1 = \int g(y_1 + \alpha s_2(\omega_2, m_1, m_2) - \omega_1) \, d\lambda_1.$$

In order to maximize this expression, we use Leibniz's rule as follows:

$$\frac{\partial}{\partial y_1} E_{\lambda_1} u_1 = \int g'(y_1 + \alpha s_2(\omega_2, m_1, m_2) - \omega_1) \, d\lambda_1 = 0.$$

If we let

$$z_2 = -\beta \left[\frac{E_{\lambda_2}\omega_1 - \alpha E_{\lambda_1}\omega_2}{1 - \alpha\beta} \right],$$

then, referring back to equation 6, we can write the first order condition as

$$\frac{\partial}{\partial y_1} E_{\lambda_1} u_1 = \int g'(y_1 + \alpha(z_2 + w_2) - \omega_1) \, d\lambda_1 = 0.$$

As g is strictly concave, it is straightforward to check that the second order condition is satisfied. As g is symmetric around 0, it is easy to see that a solution will always exist and that, at the solution, it must be that

$$y_1 - \omega_1 + \alpha(z_2 + E_{\lambda_1} w_2) = 0.$$

Substituting, we can write the best response of player 1 as

$$y_1^*(\omega_1, m_1, m_2) = \omega_1 - \alpha \left[\frac{(1-\alpha\beta)E_{\lambda_1}\omega_2 - \beta E_{\lambda_2}\omega_1 + \alpha\beta E_{\lambda_1}\omega_2}{1-\alpha\beta} \right],$$

which shows that the strategy given by equation 5 is a best response for player 1. A similar argument shows that player 2's strategy is a best response.

Having thus established that strategies at each information set in the choice game are sequentially rational, we must establish that the (truthful) messaging strategies given by equation 3 are optimal. To show this, suppose that $\omega_1 \in A^1_{\gamma^1(m_1)}$. That is, the type of player 1 is such that she is expected to send message m_1. In order to establish our claim, we will show that player 1 is indifferent across all possible messages, for all possible types of player 2. To show this, consider the difference in utility for player 1 of sending m_1' instead of m_1, generating $\lambda_2(\omega_1|m_1') = \lambda_2'$ and then playing y_1' instead of y_1:

$$\begin{aligned} E_{\lambda_1} u_1(m_1') - E_{\lambda_1} u_1(m_1) = \int g(y_1' + \alpha s_2(\omega_2, m_1', m_2) - \omega_1)\, d\lambda_1 \\ - \int g(y_1 + \alpha s_2(\omega_2, m_1, m_2) - \omega_1)\, d\lambda_1. \end{aligned}$$

From the above, it is clear that the optimal choice for y_1' is given by $y_1' = s_1(\omega_1, m_1', m_2)$ from equation 5. Substituting, we see that

$$\begin{aligned} y_1' + \alpha s_2(\omega_2, m_1', m_2) - \omega_1 &= \omega_1 - \alpha \left[\frac{E_{\lambda_1}\omega_2 - \beta E_{\lambda_2'}\omega_1}{1-\alpha\beta} \right] + \alpha\omega_2 \\ &\quad - \alpha\beta \left[\frac{E_{\lambda_2'}\omega_1 - \alpha E_{\lambda_1}\omega_2}{1-\alpha\beta} \right] - \omega_1 \\ &= \alpha \left[\omega_2 - \frac{E_{\lambda_1}\omega_2 - \beta E_{\lambda_2'}\omega_1}{1-\alpha\beta} - \frac{\beta E_{\lambda_2'}\omega_1 - \alpha\beta E_{\lambda_1}\omega_2}{1-\alpha\beta} \right] \\ &= \alpha \left[\omega_2 - \frac{E_{\lambda_1}\omega_2 - \alpha\beta E_{\lambda_1}\omega_2}{1-\alpha\beta} \right] \end{aligned}$$

$$= \alpha[\omega_2 - E_{\lambda_1}\omega_2].$$

It follows that

$$E_{\lambda_1}u_1(m_1') - E_{\lambda_1}u_1(m_1) = 0,$$

and so player 1 is indifferent over all possible messages. A similar argument applies to player 2. Thus, we have established that the interval partitions $(\mathcal{A}^1, \mathcal{A}^2)$ support an equilibrium. ■

This Proposition establishes that a a continuum of equilibria exist. The one equilibrium of particular interest included in this continuum is a fully revealing one in which each player truthfully reveals her type.[9] This equilibrium is given by

$$\mu_i(\omega_i) = \omega_i \tag{7}$$

$$s_1(\omega_1, m_1, m_2) = \frac{\omega_1 - \alpha m_2}{1 - \alpha\beta} \tag{8}$$

$$s_2(\omega_2, m_1, m_2) = \frac{\omega_2 - \beta m_1}{1 - \alpha\beta} \tag{9}$$

$$\lambda_j(m_i) = \delta_{m_i}. \tag{10}$$

In addition to being maximally informative, this equilibrium is also first-best in that, regardless of type, the equilibrium payoff of both player is their highest possible utility (namely zero). This is because, with full information, each player can perfectly compensate for the effect of her opponent's actions on her utility. Thus, the players can successfully coordinate their actions voluntarily. Clearly, this equilibrium is Pareto-superior to all other equilibria.

We now show that information transmission and coordination cannot occur when each player's expectation of the other's action leads to behavior that completely offsets. To get a sense of why this is true, note that the equilibrium actions given by equations (8) and (9) go to infinity as $\alpha\beta$ goes to one. Thus, unlike Proposition 1 in which players can always choose extreme enough actions so that they are indifferent

[9] Because any one-to-one messaging strategy is fully revealing, there are actually many fully revealing equilibria that differ only by the messaging strategy. Throughout this paper, we focus on the canonical fully revealing equilibrium in which each player truthfully reports her type.

over their possible messages, if $\alpha\beta = 1$, then the messages of players are not credible because they always want to choose the highest or lowest message possible. Therefore, no information can be transmitted in equilibrium. Moreover, except in the particular case $\alpha = \beta = 1$, if $\alpha\beta = 1$, then the best response of players is to choose ever more extreme actions, and, hence, there is no equilibrium at all. If $\alpha = \beta = 1$, then each player will want to compensate exactly for her action and her own type. This turns out to be possible in a babbling equilibrium in which each player's action is offset be a specific amount. This is expressed in the following Proposition.

Proposition 2 *Suppose that $\alpha\beta = 1$. If $\alpha \neq 1$ or $\beta \neq 1$, then no equilibrium exists. If $\alpha = \beta = 1$, then only babbling equilibria exist. In any such equilibrium, $y_1^*(\omega_1) = \omega_1 - \delta - \frac{1}{2}$ and $y_2^*(\omega_2) = \omega_2 + \delta$ for some $\delta \in \mathbb{R}$.*

Proof. Suppose that $\alpha\beta = 1$ and fix a pair of interval partitions $(\mathcal{A}^1, \mathcal{A}^2)$. Consider the choice of action for player 1 with type ω_1, having observed m_1 and m_2 and given $\lambda_1(\omega_2|m_2)$. Player 1's expected utility of playing action y_1 is given by

$$E_{\lambda_1} u_1 = \int g(y_1 + \alpha s_2(\omega_2, m_1, m_2) - \omega_1)\, d\lambda_1,$$

with first order condition

$$\frac{\partial}{\partial y_1} E_{\lambda_1} u_1 = \int g'(y_1 + \alpha s_2(\omega_2, m_1, m_2) - \omega_1)\, d\lambda_1 = 0.$$

Let z_1 solve

$$\int g'(z_1 + \alpha s_2(\omega_2, m_1, m_2))\, d\lambda_1 = 0.$$

Nothing in this expression depends on ω_1, so z_1 is independent of ω_1. Therefore, player 1's best response must satisfy $y_1 = z_1 + \omega_1$. Likewise, player 2's strategy must satisfy

$$s_2(\omega_2, m_1, m_2) = z_2 + \omega_2$$

for some z_2 that does not depend on ω_2. Thus, player 1's best response solves

$$\frac{\partial}{\partial y_1} E_{\lambda_1} u_1 = \int g'(y_1 - \omega_1 + \alpha(z_2 + \omega_2))\, d\lambda_1 = 0$$

and, as before, must solve

$$y_1 - \omega_1 + \alpha z_2 + \alpha E_{\lambda_1} w_2 = 0.$$

Similarly, player 2's action must solve

$$y_2 - \omega_2 + \beta z_1 + \beta E_{\lambda_2} w_1 = 0.$$

Using the fact that $y_1 = z_1 + \omega_1$ and $y_2 = z_2 + \omega_2$, and substituting, we have

$$z_1 + \alpha(-\beta z_1 - \beta E_{\lambda_2} w_1) + \alpha E_{\lambda_1} w_2 = 0.$$

Because $\alpha\beta = 1$, this simplifies to

$$E_{\lambda_2} w_1 = \alpha E_{\lambda_1} w_2.$$

Making the other substitution and simplifying yields

$$E_{\lambda_1} w_2 = \beta E_{\lambda_2} w_1.$$

The only way that these last two equations can be satisfied for *every* pair of intervals in $(\mathcal{A}^1, \mathcal{A}^2)$ is if $\alpha = 1$ and $\beta = 1$ and both interval partitions are just the trivially one element partition. Otherwise, there is no equilibrium. In the former case, $E_{\lambda_2} w_1 = E_{\lambda_1} w_2 = 1/2$. The first order condition becomes

$$z_1 + z_2 + 1/2 = 0.$$

So setting $z_2 = \delta$, the first order conditions can only be satisfied by $y_1^*(\omega_1) = \omega_1 - \delta - \frac{1}{2}$ and $y_2^*(\omega_2) = \omega_2 + \delta$ for some $\delta \in \mathbb{R}$. This completes the proof. ■

Hence, cheap talk does not help when a player's action is expected to be exactly compensated by the other player's expectation of her own action. The equilibrium obtained from the entire game including the communication stage when $\alpha\beta = 1$ is identical to the underlying Bayesian Nash equilibrium for the game without communication. As each player knows about the possibility for deception, even a truth-telling player will not be believed in equilibrium.

3 Communication with Bounded Action Spaces

The previous results assume that the action spaces of players are not bounded. The absence of bounds signifies that there are no meaningful constraints on what players can do in pursuit of their interests–nations in an arms race have no limits on what they can spend, agencies have no budgetary or statutory restrictions, candidates have infinite campaign funds and no regulatory guidelines, etc. Incorporating limitations on the action spaces of players reduces information transmission, although the extent of this effect varies considerably depending on whether actions are substitutes or complements.

In order to obtain closed form solutions, throughout the remainder of the paper we assume the following functional form for utilities:

$$u_1(y_1, y_2, \omega_1) = -(y_1 + \alpha y_2 - \omega_1)^2 \quad (11)$$
$$u_2(y_1, y_2, \omega_2) = -(y_2 + \beta y_1 - \omega_2)^2. \quad (12)$$

Using the more general functional form for utilities from the previous section would yield qualitatively similar results.

We note for later use that in an unconstrained setting the best response conditions for the players are given by

$$y_1 = \omega_1 - \alpha E_{\lambda_1} y_2 \quad (13)$$
$$y_2 = \omega_2 - \beta E_{\lambda_2} y_1. \quad (14)$$

Thus, all other things being equal, if α is positive (negative), then higher actions by player 2 lead to lower (higher) actions by player 1.

3.1 Nonnegative Action Spaces

We begin by considering the case in which each player's action space is bounded below by zero. Later we will examine the cases in which one or both players' action spaces are also bounded above. Specifically, we assume that $y_1 \geq 0$ and $y_2 \geq 0$. As such, each player's action is interpretable as the amount of money to contribute, arms to build, public goods to provide, effort to make, and the like. We distinguish between strategic complementarities (e.g., an arms race), where α and β are both negative and each player's action is an increasing function of the other player's best response, and strategic substitution (e.g., public goods provision), where α and β are both positive and each player's action is a decreasing function of the other player's best response.[10] For our analysis, it is helpful to define these two strategic situations as follows.

Definition 1 *A Bayesian game has strategic substitutes for both players if* $\alpha > 0$ *and* $\beta > 0$ *and strategic complementarities for both players if* $\alpha < 0$ *and* $\beta < 0$.

Interestingly, and as implied, while limiting actions to be nonnegative significantly reduces the ability of communication to enhance efficiency, the effects differ greatly depending upon whether there is substitution or complementarity. As we show, for substitutes, cheap talk often does not aid coordination on efficient outcomes but, depending upon the externality parameters, it may do so for complements. Hence, for example, using communication for public goods provision may be less useful than when there is a desire to provide more as long as one's partner is cooperating.

3.1.1 Strategic Substitution

Proposition 3 shows how restricting actions to be nonnegative limits the possibility of information transmission for the case of strategic substitutes.

Proposition 3 *Suppose that* $Y_1 = Y_2 = [0, \infty)$. *Suppose also that* y_1 *and* y_2 *are strategically substitutable, i.e.,* $\alpha > 0$ *and* $\beta > 0$.

[10] By applying the logic developed for the case where both α and β are positive, it is easily seen that only babbling equilibria exist when α and β have different signs.

1. If $\alpha \geq 2$, then there exists an equilibrium given by

$$\mu_1(\omega_1) = \omega_1 \textit{ for all } \omega_1$$
$$\mu_2(\omega_2) = \begin{cases} \frac{1}{\alpha} & \textit{if } \omega_2 \in [0, \frac{2}{\alpha}] \\ \omega_2 & \textit{if } \omega_2 \in (\frac{2}{\alpha}, 1] \end{cases}$$
$$s_1(\omega_1, m_1, m_2) = 0$$
$$s_2(\omega_2, m_1, m_2) = \omega_2$$
$$\lambda_1(\omega_2|m_2) = \begin{cases} U[0, \frac{2}{\alpha}] & \textit{if } m_2 \in [0, \frac{2}{\alpha}] \\ \delta_{m_2} & \textit{if } m_2 \in (\frac{2}{\alpha}, 1] \end{cases}$$
$$\lambda_2(\omega_1|m_1) = \delta_{m_1} \textit{ for all } m_1,$$

There is no more informative equilibrium of this kind.

2. If $\beta \geq 2$, then there exists an equilibrium given by

$$\mu_1(\omega_1) = \begin{cases} \frac{1}{\beta} & \textit{if } \omega_1 \in [0, \frac{2}{\beta}] \\ \omega_1 & \textit{if } \omega_1 \in (\frac{2}{\beta}, 1] \end{cases}$$
$$\mu_2(\omega_2) = \omega_2 \textit{ for all } \omega_2$$
$$s_1(\omega_1, m_1, m_2) = \omega_1$$
$$s_2(\omega_2, m_1, m_2) = 0$$
$$\lambda_1(\omega_2|m_2) = \delta_{m_2} \textit{ for all } m_2,$$
$$\lambda_2(\omega_1|m_1) = \begin{cases} U[0, \frac{2}{\beta}] & \textit{if } m_1 \in [0, \frac{2}{\beta}] \\ \delta_{m_1} & \textit{if } m_1 \in (\frac{2}{\beta}, 1] \end{cases}$$

There is no more informative equilibrium of this kind.

3. If $\alpha < 2$ and $\beta < 2$, then only babbling equilibria exist.

Proof. We begin with some preliminary analysis. Suppose that $\alpha > 0$ and $\beta > 0$ and fix a pair of interval partitions $(\mathcal{A}^1, \mathcal{A}^2)$. Consider the choice of action for player 1 with type ω_1, having observed m_1 and m_2 and given $\lambda_1(\omega_2|m_2)$ and an arbitrary strategy $s_2(\omega_2, m_1, m_2)$. Player 1's expected utility of playing action y_1 is given by

$$E_{\lambda_1} u_1 = \int -(y_1 + \alpha s_2(\omega_2, m_1, m_2) - \omega_1)^2 \, d\lambda_1.$$

Maximizing this expression with the constraint $y_1 \geq 0$ gives the following Kuhn-Tucker condition:

$$y_1 = \begin{cases} 0 & \text{if } \omega_1 \leq \alpha E_{\lambda_1} y_2 \\ \omega_1 - \alpha E_{\lambda_1} y_2 & \text{if } \omega_1 > \alpha E_{\lambda_1} y_2. \end{cases}$$

A similar result holds for player 2.

If $\alpha \geq 2$, then it is straightforward to verify that the strategies and beliefs given above form an equilibrium. In order to establish that there is no more informative equilibrium of this kind, suppose not. That is, suppose there is such an equilibrium, with strategies denoted by $\tilde{s_1}$ and $\tilde{s_2}$. In order to be strictly more informative, such an equilibrium must be fully revealing for all ω_1 and for all $\omega_2 \in (2/\alpha, 1]$, and must involve player 2 sending at least two different messages for $\omega_2 \in [0, 2/\alpha]$. Suppose that $\omega_1 = 0$. Then $m_1 = 0$ and, in any equilibrium, $y_1 = 0$ and $y_2 = \omega_2$. In this case, the (*ex ante*) expected utility of player 1 is $E[-(0 + \alpha\omega_2 - 0)^2] = E[-(\alpha\omega_2)^2]$. On the other hand, if $\omega_1 = 1$ and player 1 sends the equilibrium message $m_1 = 1$, then, because player 2 is sending at least two different messages for $\omega_2 \in [0, 2/\alpha]$, there is a message sent with positive probability for which player 1's equilibrium action is positive. Given this, if $\omega_1 = 0$ and player 1 deviates to play $m_1 = 1$, then in the action phase player 2 plays $y_2 = \omega_2 - \beta y_1$ which is either equal to or, with positive probability, strictly less than ω_2. Thus, this deviation gives player 1 higher expected utility, so there can be no more informative equilibrium than that given in the proposition. A similar argument establishes the result for $\beta \geq 2$.

Finally, we must show that no non-babbling equilibrium exists if $\alpha < 2$ and $\beta < 2$. So, suppose that $\alpha < 2$ and $\beta < 2$ and that a non-babbling equilibrium exists. It is easy to show, based on the argument above, that there is no equilibrium in which a player always plays action $y_i = 0$. It is also true that there is no equilibrium in which a player never strictly prefers the zero action. Therefore, in any equilibrium, each players must play the zero action with positive probability and positive actions with positive probability. In particular, we can suppose, without loss of generality, that m_1^0 is the equilibrium message for type $\omega_1 = 0$ of player 1 and there is some other message m_1' sent in equilibrium by some other type ω_1 such that $E_{\lambda_2(m_1')}y_1 > E_{\lambda_2(m_1^0)}y_1$. It follows that in any equilibrium, for all ω_2,

$$y_2(\omega_2, m_1') < y_2(\omega_2, m_1^0).$$

But the expected equilibrium payoff for type $\omega_1 = 0$ of player 1 is $E[-(0+\alpha y_2(\omega_2, m_1^0)-0)^2] = E[-(\alpha y_2(\omega_2, m_1^0))^2]$, while the expected payoff of sending message m_1' is $E[-(\alpha y_2(\omega_2, m_1'))^2]$. Therefore, this is a profitable deviation, and so no non-babbling equilibria exist. ■

Thus, if the externality parameters of both players are not too large, there are only babbling equilibria.[11] If the externality parameter of a player is large enough, though, there is an equilibrium in which that player fully reveals her type and always chooses the zero action. This equilibrium is consistent with a major firm opening up a new market and inducing a rival firm to forgo entry or, in a non-market context, a situation in which one interest group lobbies a regulator and gains a preferential decision while the other does not participate in lobbying and accepts the resulting rules.

3.1.2 Strategic Complementarity

Unlike substitutability, fully informative cheap talk is possible in some cases with strategic complementarity. In particular, our results depend on how much the optimal actions of each player are influenced by the other, i.e., whether $\alpha\beta \geq 1$ (large effect) or $\alpha\beta < 1$ (small effect). While fully revealing cheap talk is possible when $\alpha\beta < 1$, no equilibrium exists when $\alpha\beta \geq 1$. Put differently, full information revelation is possible if players' actions are less than completely offset by the best response of the

[11] As characterizing the babbling equilibria in this case involves solving a fourth-order polynomial, it is not presented here.

other players. Otherwise, there is no information revelation and a regressive process induces ever more extreme choices, so no equilibrium exists.

Proposition 4 *Suppose that* $Y_1 = Y_2 = [0, \infty)$*. Suppose also that* y_1 *and* y_2 *are strategic complements, i.e.,* $\alpha < 0$ *and* $\beta < 0$*.*

1. *If* $\alpha\beta < 1$*, then for every pair of interval partitions* $(\mathcal{A}^1, \mathcal{A}^2)$*, the strategies and beliefs given by equations 3, 4, 5, and 6 form a perfect Bayesian equilibrium.*
2. *If* $\alpha\beta \geq 1$*, then no equilibrium exists.*

Proof. Suppose $\alpha < 0$ and $\beta < 0$. If $\alpha\beta < 1$, then it is clear that equations 5 and 6 are always positive and, thus, the arguments in Proposition 1 go through. Consequently, the equilibria identified in that Proposition exist here.

If $\alpha\beta \geq 1$, then the argument is similar to the proof of Proposition 2. Given that both players' actions are nonnegative, inspection of the utility functions leads to the obvious implication that $y_i \geq \omega_i$ for $i = 1, 2$. Writing $y_i = z_i + \omega_i$, as in the proof of Proposition 2, this implies that $z_i \geq 0$ for $i = 1, 2$. Following the argument there, we have the equilibrium condition

$$z_1 - \alpha\beta z_1 - \alpha\beta E_{\lambda_2} w_1 + \alpha E_{\lambda_1} w_2 = 0,$$

which simplifies to

$$(1 - \alpha\beta) z_1 - \alpha\beta E_{\lambda_2} w_1 + \alpha E_{\lambda_1} w_2 = 0.$$

As $\alpha < 0$, $\beta < 0$, $z_1 \geq 0$, and $\alpha\beta \geq 1$, the first term in this equation is less than or equal to zero and the second and third terms are both negative. Thus, the equilibrium condition does not hold, so no equilibrium exists. ■

To gain some intuition for this case, recall the fully revealing equilibrium from the unbounded case given by equations (8) and (9). The numerator of each of these expressions is positive, as $\alpha < 0$ and $\beta < 0$, and the sign of the denominator depends on whether $\alpha\beta$ is greater or less than one. If $\alpha\beta < 1$, then both of these expressions are always positive and so the constraints on the actions spaces of players are not binding. If $\alpha\beta > 1$, however, the constraints are binding and no equilibrium is possible. Indeed, in this case there is a multiplier effect by which one choice of optimal action yields a larger choice for the opponent in monotonic fashion (for a similar result, see Baliga and Sjöström (2004)). Thus, no equilibrium exists. Each player wants to outdo the other player's action and, as a result, always has an incentive to deviate and, due to this multiplier effect, chooses an always higher action. Interestingly, such a spiral effect of each player's expectation of the other player's action on her own action would seem to characterize the arms race engaged in by the U.S. and the Soviet Union during the Cold War.

3.2 Nonnegative Action Space with Upper Bounds

To build more realism into our analysis, we next incorporate the possibility that actors face constraints that restrict how much they can contribute in pursuit of their objective. We do this by further restricting the action space for strategic complements so that is bounded above as well as below.[12] Specifically, we focus on (1) how our results about information revelation are affected; and (2) how, given the effects on expected actions, a player with an upper bounded action space is advantaged or disadvantaged vis-à-vis a player with an unbounded or less severely constrained action space. Once again, the key factor for our analysis is how much each player's optimal action is influenced by the other, i.e., whether the combined effect is greater than or less than one.

3.2.1 Small Combined Effect

We first analyze the situation where $\alpha\beta < 1$, so that the combined effect of each player's evaluation of the other player's expectation of her own optimal action on the player's optimal action is less than 1. We begin by restricting the first player's action space at zero and $\overline{y}_1$, so that $y_1 \in [0, \overline{y}_1]$, and the second player's action space only at zero. When we do this, a fully revealing equilibrium is possible but, in contrast to the situation with no upper bound, no longer assured. Full revelation depends on whether the upper bound restricts the believability of player 1's messaging. When it

[12]For strategic substitution, we do not need to discuss the impact of imposing upper bounds on the actions spaces because the lower bounds already significantly hinder informational efficiency.

does not, full coordination through communication and a fully revealing equilibrium will exist; if the restricted upper bound is small enough to impact the believability of player 1's message, the most informative equilibrium is only partially informative. Furthermore, when a partially informative equilibrium is most informative, player 1 is disadvantaged relative to player 2 because she always reveals her type to player 2 but only some types of player 2 reveal their type, with the others sending a pooling message. Thus, depending on the benefits of partial information revelation, heavily resource constrained actors may be loathe to engage in pre-play communication to try and avert bad outcomes, whether it be an arms race, negative campaigning, or a technology ramp-up between firms given the possible distributional implications.

Proposition 5 *Suppose that $Y_1 = [0, \overline{y}_1]$ and $Y_2 = [0, \infty)$. Suppose also that that y_1 and y_2 are strategic complements, i.e., $\alpha < 0$ and $\beta < 0$, and that $\alpha\beta < 1$.*

1. *If $\overline{y}_1 \geq \frac{1-\alpha}{1-\alpha\beta}$, then there exists a fully revealing equilibrium given by*

$$\begin{aligned}
\mu_i(\omega_i) &= \omega_i \\
s_1(\omega_1, m_1, m_2) &= \frac{\omega_1 - \alpha m_2}{1 - \alpha\beta} \\
s_2(\omega_2, m_1, m_2) &= \frac{\omega_2 - \beta m_1}{1 - \alpha\beta} \\
\lambda_j(m_i) &= \delta_{m_i}.
\end{aligned}$$

2. *If $\frac{2-\alpha}{2-2\alpha\beta} < \overline{y}_1 < \frac{1-\alpha}{1-\alpha\beta}$, then there exists a maximally informative equilibrium given by*

$$\mu_1(\omega_1) = \omega_1 \text{ for all } \omega_1$$

$$\mu_2(\omega_2) = \begin{cases} \omega_2 & \text{if } \omega_2 \in [0, \tau) \\ \frac{\tau+1}{2} & \text{if } \omega_2 \in [\tau, 1] \end{cases}$$

$$s_1(\omega_1, m_1, m_2) = \begin{cases} \frac{\omega_1 - \alpha m_2}{1-\alpha\beta} & \text{if } m_2 \in [0, \tau) \\ \omega_1 + \frac{2\alpha\beta\omega_1 - \alpha(1+\tau)}{2-2\alpha\beta} & \text{if } m_2 \in [\tau, 1] \end{cases}$$

$$s_2(\omega_2, m_1, m_2) = \begin{cases} \frac{\omega_2 - \beta m_1}{1-\alpha\beta} & \textit{if } m_2 \in [0,\tau) \\ \omega_2 + \frac{\alpha\beta(1+\tau)-2\beta m_1}{2-2\alpha\beta} & \textit{if } m_2 \in [\tau, 1] \end{cases}$$

$$\lambda_1(\omega_2|m_2) = \begin{cases} \delta_{m_2} & \textit{if } m_2 \in [0,\tau) \\ U[\tau,1] & \textit{if } m_2 \in [\tau,1] \end{cases}$$

$$\lambda_2(\omega_1|m_1) = \delta_{m_1} \textit{ for all } m_1,$$

where $\tau = \frac{2-\alpha+2(\alpha\beta-1)\overline{y}_1}{\alpha}$.

Proof. It is straightforward to verify that the strategies and beliefs given above form equilibria for the two cases. In order to establish that the second equilibrium is maximally informative, we must show that there is no more informative equilibrium than the one specified above. Suppose there is such an equilibrium, with strategies denoted by $\tilde{s_1}$ and $\tilde{s_2}$. In order to be strictly more informative, such an equilibrium must be fully revealing for all ω_1 and for all $\omega_2 \in [0,\tau)$, and must involve player 2 sending at least two different messages for $\omega_2 \in [\tau, 1]$. So, without loss of generality, assume that the messaging strategies in this equilibrium satisfy $\tilde{\mu_1}(\omega_1) = \omega_1$ for all ω_1 and $\tilde{\mu_2}(\omega_2) = \omega_2$ for all $\omega_2 \in [0,\tau)$. Then it must be the case that for all $m_2 < \tau$, $\tilde{s_1} = (\omega_1 - \alpha m_2)/(1-\alpha\beta)$ and $\tilde{s_2} = (\omega_2 - \beta m_1)(1-\alpha\beta)$.

If $m_2 > \tau$, then it is more difficult to give the equilibrium strategies. As player 2 sends at least two different messages from the interval $[\tau, 1]$, the belief of player 1, $\lambda_1(\omega_2|m_2)$, will depend on the message $m_2 \in [\tau, 1]$. On the other hand, as player 1 is fully revealing her type, player 2 will know player 1's equilibrium action y_1 with certainty. Thus, player 2 will play her first-best action $y_2 = \omega_2 - \beta y_1$. From this, player 1 will know that player 2's expected action is $E_{\lambda_1} y_2 = E_{\lambda_1}\omega_2 - \beta y_1$. Thus, player 1's equilibrium action must be

$$\tilde{s_1}(\omega_1, m_1, m_2) = \min\left(\frac{w_1 - \alpha E_{\lambda_1}\omega_2}{1-\alpha\beta}, \overline{y}_1\right).$$

Some simple algebra shows that the first term in this expression is the minimum when

$$\frac{1}{\alpha}(\omega_1 - (1-\alpha\beta)\overline{y}_1) \geq E_{\lambda_1}\omega_2. \tag{15}$$

When this condition holds, the (*ex ante*) expected utility of player 1 can be calculated to be $E[-(\alpha\omega_2 - \alpha E_{\lambda_1}\omega_2)^2] = -\alpha^2 \operatorname{Var}[\lambda_1(m_2)]$. If this condition does not hold (and so player 1 plays $\overline{y}_1$), then the expected utility of player 1 is $E[-(\alpha\omega_2 - (\omega_1 - (1-\alpha\beta)\overline{y}_1))^2]$. In particular, when $\omega_1 = 1$, condition (15) simplifies to $(\tau+1)/2 \geq E_{\lambda_1}\omega_2$ and the expected utility of player 1 when her equilibrium action is $\overline{y}_1$ simplifies to $E[-\alpha^2(\omega_2 - (\tau+1)/2)^2]$.

In order to show that this is not an equilibrium, we will show that a type $\omega_1 = 1$ of player 1 strictly prefers to send the lower message $m_1 = \alpha + (1-\alpha\beta)\overline{y}_1$. In this case, it is easy to check that player 2, after observing this message, will know that condition (15) is satisfied. Therefore, player 2 will play action $y_2 = \omega_2 - \beta\overline{y}_1 - \alpha\beta(1 - E_{\lambda_1}\omega_2)/(1-\alpha\beta)$. Knowing this, the optimal action of type $\omega_1 = 1$ of player 1 after sending $m_1 = \alpha + (1-\alpha\beta)\overline{y}_1$ is

$$\min\left(1 + \alpha\beta\overline{y}_1 - \alpha\frac{\alpha E_{\lambda_1}\omega_2 - \alpha\beta}{1-\alpha\beta}, \overline{y}_1\right).$$

If m_2 is such that the first term in this expression is smaller, then the expected utility of player 1 is $-\alpha^2 \operatorname{Var}[\lambda_1(m_2)]$. If the second term in the expression is smaller, then the expected utility of player 1 is $E[-\alpha^2(\omega_2 - (\tau+1)/2 - \alpha\beta(1-E_{\lambda_1}\omega_2)/(1-\alpha\beta))^2]$. This is strictly better than the expected utility after sending $m_1 = 1$. So, for some m_2, player 1 receives the same expected utility and, for some m_2, she does strictly better. We thus conclude that player 1 can gain by deviating from fully revealing her type, so there is no more informative equilibrium than that given in the Proposition. ■

Having seen that putting an upper bound on one player's action space both limits the possibility of information transmission and disadvantages the restricted player when the constraint is binding, it is intuitive that putting limits on both players further inhibits the informational usefulness of pre-play communication. Indeed, if the upper bound of the action spaces of both players is low enough, then no coordination via information transmission is possible and there are only babbling equilibria. This is expressed in Corollary 1.

Corollary 1 *Suppose that* y_1 *and* y_2 *are strategically complementary and that* $\alpha\beta < 1$, $\overline{y}_1 \leq \frac{2-\alpha}{2-2\alpha\beta}$, *and* $\overline{y}_2 \leq \frac{2-\beta}{2-2\alpha\beta}$. *Then only babbling equilibria exist.*

Thus, with a small combined effect on optimal actions, a sufficiently restricted positive action space with complementarities leads to the conclusion that pre-play communication will not only not help achieve a more efficient outcome but will aid the bargaining position of the player with less strict constraints. Thus, the more restricted actor—be it a nation-state, politician, firm, etc.—will be considerably disadvantaged.

3.2.2 Large Combined Effect

We now assume that $\alpha\beta \geq 1$, with both players' action spaces again bounded below by zero and player 1's action space bounded above by $\overline{y}_1$. As we have already established that there is no informative equilibrium when $\alpha\beta \geq 1$ and action spaces are nonnegative, it must be that there is no information revelation when an upper bound is added. The major impact of bounding the action spaces is the existence of a unique babbling equilibrium.

Specifically, when player 1's upper bound is $\overline{y}_1$, there is a unique equilibrium in which no information is transmitted and player 1 always chooses this upper bound for her equilibrium action. Player 2, in turn, optimizes against this expected action, allowing each type of player 2 to attain a first-best outcome with utility zero. Thus, bounding the action space of player 1 advantages player 2.

Proposition 6 *Suppose that* $Y_1 = [0, \overline{y}_1]$ *and* $Y_2 = [0, \infty)$. *Suppose also that* y_1 *and* y_2 *are strategic complements, i.e.,* $\alpha < 0$ *and* $\beta < 0$, *and that* $\alpha\beta \geq 1$. *Then there exists a unique equilibrium. This equilibrium is a babbling equilibrium in which* $s_1(\omega_1, m_1, m_2) = \overline{y}_1$ *for all* ω_1 *and* $s_2(\omega_2, m_1, m_2) = \omega_2 - \beta\overline{y}_1$.

Proof. We know from Proposition 2 that only babbling equilibria are possible is this case. It is straightforward to verify that the strategies given above form a babbling equilibrium. The only thing that remains to be shown is that there are no other babbling equilibria. The first order condition for player 2 gives $y_2 = \omega_2 - \beta E y_1$. On the other hand, the Kuhn-Tucker condition for player 1 requires that

$$y_1 = \min\left(\omega_1 - \alpha E y_2, \overline{y}_1\right) = \min\left(\omega_1 - (\alpha/2) + \alpha\beta E y_1, \overline{y}_1\right).$$

But $\alpha < 0$ and $\alpha\beta \geq 1$ imply that $\omega_1 - (\alpha/2) + \alpha\beta E y_1 > E y_1$ for all ω_1. As $y_1 > E y_1$ cannot hold for all ω_1, the only possible equilibrium strategy for player 1 is $y_1 = \overline{y}_1$ for all ω_1. But this is exactly the equilibrium strategy identified in the Proposition. Therefore, this equilibrium is unique. ■

When both players have upper bounds, the same logic applies.

4 Conclusion

All in all, our theoretical results indicate that we must reexamine claims that pre-play communication will lead to better outcomes. While full information revelation is possible when both sides have private information and the action spaces of players are unbounded, this becomes much less likely when action spaces are bounded. In many instances, either only babbling equilibria exist or no equilibria exist at all.

Empirically, the relevance of our findings depends upon whether multiple players have private information and whether action spaces are sufficiently bounded to impact information revelation and limit the actions of the players. Although these parameters are difficult to measure, casual observation of an array of empirical situations of interest would suggest that both of these conditions are met. As such, our results would be largely a tale of caution to those who believe that deliberation can completely solve many vexing problems that currently exist.

In the future, we would like to extend our model to incorporate more players. This would, for example, be a way to incorporate a biased or unbiased mediator, to address issues of coalition formation in the pre-play stage, or to examine how cheap talk in moderately sized deliberative bodies such as committees might be relevant. Such extensions would, in turn, aid our understanding of how these phenomenon play out in the real world.

References

AUMANN, ROBERT J., AND SERGIU HART. 2003. "Long Cheap Talk," *Econometrica* 71: 1619–1660.

AUSTEN-SMITH, DAVID. 1994. "Strategic Transmission of Costly Information," *Econometrica* 62: 955–964.

AUSTEN-SMITH, DAVID, AND JEFFREY S. BANKS 2000. "Cheap Talk and Burned Money," *Journal of Economic Theory* 91: 1–16.

AUSTEN-SMITH, DAVID, AND TIMOTHY FEDDERSEN 2002. "Deliberation and Voting Rules," *Mimeo* Northwestern University, Center for Mathematical Studies in Economics and Management Science.

BALIGA, SANDEEP, AND STEPHEN MORRIS 2002. "Co-ordination, Spillovers, and Cheap Talk," *Journal of Economic Theory* 105: 450–468.

BALIGA, SANDEEP, AND TOMAS SJÖSTRÖM 2004. "Arms Races and Negotiations," *Review of Economic Studies* 71: 351–369.

BANKS, JEFFREY S., AND RANDALL L. CALVERT 1992. "A Battle-of-Sexes Game with Incomplete Information," *Games and Economic Behavior* 4: 347–372.

BATTAGLINI, MARCO. 2002. "Multiple Referrals and Multidimensional Cheap Talk," *Econometrica* 70: 1379–1401.

CRAWFORD, VINCENT P., AND JOEL SOBEL. 1982. "Strategic Information Transmission," *Econometrica* 50: 1431–1451.

DORASZELSKI, ULRICH, DINO GERARDI, AND FRANCESCO SQUINTANI. 2003. "Communication and Voting with Double-Sided Information," *Contributions to Theoretical Economics* 3: 1084–1124.

FARREL, JOSEPH. 1988. "Communication, Coordination, and Nash Equilibrium," *Economic Letters* 27: 209–214.

Farrel, Joseph. 1993. "Meaning and Credibility in Cheap Talk Games," *Games and Economic Behavior* 5: 514–531.

Farrel, Joseph, and Robert Gibbons. 1989. "Cheap talk Can Matter in Bargaining," *Journal of Economic Theory* 48: 221–237.

Farrel, Joseph, and Matthew Rabin. 1996. "Cheap Talk," *Journal of Economic Perspectives* 10:3 103–118.

Yong-Gwan, Kim, and Joel Sobel. 1992. "An Evolutionary Approach to Preplay Communication," *Econometrica* 63: 1181–1193.

Krishna, Vijay, and John Morgan. 2001. "A Model of Expertise," *Quarterly Journal of Economics* 116: 747–775.

Krishna, Vijay, and John Morgan. 2001. "Asymmetric Information and Legislative Rules: Some Amendments," *American Political Science Review* 95: 435–452.

Morris, Stephen, and Hyun Song Shin. 2003. "Global Games: Theory and Applications," in *Advances in Economics and Econometrics (Proceedings of the Eighth World Congress of the Econometric Society),* edited by M. Dewatripont, L. Hansen and S. Turnovsky. Cambridge: Cambridge University Press.

Palfrey, Thomas R., and Howard Rosenthal 1991. "Testing for Effects of Cheap Talk in a Public Goods Game with Private Information," *Games and Economic Behavior* 3: 183–220.

Rabin, Matthew. 1990. "Communication between Rational Agents," *Journal of Economic Theory* 51: 144–170.

Rabin, Matthew. 1994. "A Model of Pre-game Communication," *Journal of Economic Theory* 63: 370–391.