Is an Ultimatum the Last Word on Crisis Bargaining?

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Abstract

We investigate how the structure of the international bargaining process affects the resolution of crises. Despite the vast diversity in bargaining protocols, we find that there is a remarkably simple structure to how crises may end. Specifically, for any equilibrium outcome of a complex negotiation process, there is a take-it-or-leave-it offer that has precisely the same risk of war and distribution of benefits. In this sense, every equilibrium outcome of a crisis bargaining game is equivalent to the outcome of an offer in the ultimatum game. Consequently, if a state could select the bargaining protocol, it would choose an ultimatum or another protocol with the same result. According to our model, if states are behaving optimally, then we should observe no relationship between the bargaining protocol and the risk of war or distribution of benefits. An empirical analyses of ultimata in international crises supports this claim.

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1 Introduction

One of the main tenets of the international conflict literature is that war is the result of bargaining failure. To understand why states fight each other, we must identify why they were not able to reach a deal at the bargaining table (Fearon 1995). In other words, to understand war we must understand pre-war negotiations. However, the shape of the negotiation process may vary widely from case to case (Snyder and Diesing 1977). Unlike in contexts like voting in legislatures or government formation, where the rules of the game are institutionally fixed, there is no single way for crisis bargaining to take place. Negotiations can take infinitely many forms, from a simple take-it-or-leave-it offer to a lengthy exchange of concessions and counterproposals.[1] Moreover, in real crises, the bargaining procedure is not fixed in advance, as states can choose how they want to negotiate. That is, crisis bargaining procedures are both heterogeneous and endogenous. Understanding the relationship between the bargaining protocol and the outcome of a crisis is therefore critical for both empirical studies and formal models of bargaining and war.

In this paper, we investigate how the structure of the negotiating process affects the resolution of crises. We focus on crises in which states have asymmetric information, since this is the classic context in which war emerges as a bargaining failure (Fearon 1995). States signal their private information through the actions they take at the bargaining table. Consequently, the bargaining protocol essentially sets the limits on how much states can learn about each other.

Given the wide variety of potential bargaining processes, it is useful to focus on the possible outcomes of crisis bargaining. In particular, we consider two dimensions by which outcomes vary. The first is the circumstances under which the states reach a settlement and thus avoid war. For example, some bargaining procedures might result in all states reaching agreements, while others might lead to war between strong states and peace between weak states. The second is the division of benefits under each potential settlement: does a stronger country necessarily receive better terms? Two bargaining protocols can look very different, yet lead to similar or even identical outcomes. The end products of negotiations—the likelihood of war or the distribution of goods between states—are of more interest to researchers and practitioners than the intricacies of the bargaining process itself.

Our main task, therefore, is to characterize how the bargaining protocol affects crisis outcomes. Our main finding identifies a remarkable similarity to the outcomes of a wide variety of potential bargaining protocols. When we strip away the complications in how bargaining plays out and focus solely on how it ends, we find a common structure that

[1] McKibben (2013), in a study of non-crisis international bargaining, also points out this issue.

is surprisingly simple. In particular, we show that any outcome that might result from a complicated negotiation process could also be attained by a simple take-it-or-leave-it offer.

To understand the intuition for this result, consider bargaining between two countries, one of which has private information about its military strength. The stronger the country is, the more it must receive at the bargaining table in order to prefer settling over fighting. Yet if the equilibrium outcome of the negotiation is a peaceful settlement for two "types" of the country, one weaker and one stronger, then the terms of the settlement must be the same for both (Banks 1990). Otherwise, the type receiving the inferior deal could mimic the behavior of the other type in order to receive a better deal. When the country's privately known strength is so great that the terms taken by weaker types are unacceptable, it will choose to fight instead. In equilibrium, we end up with a range of weak types taking the same settlement, while all others fight—exactly the outcome of an ultimatum offer.

We formalize this intuitive argument using Bayesian mechanism design, which allows us to characterize the equilibrium outcomes of a wide class of crisis bargaining games. Previous applications of mechanism design to international conflict have focused on whether war can be averted through bilateral bargaining (Bester and Wärneryd 2006; Fey and Ramsay 2009) or with the help of an outside mediator (Fey and Ramsay 2010; Hörner, Morelli and Squintani 2015). We instead focus on the commonalities across crisis bargaining mechanisms, including those with a positive probability of conflict, building in part on analyses that characterize the relationship between the risk of war and the distribution of goods in the equilibria of crisis bargaining games (Banks 1990; Fey and Ramsay 2011).

In economics, mechanism design techniques have been well developed in the study of monopoly pricing and auctions, and indeed our equivalence result is closely related to famous results on revenue equivalence in auctions and bilateral bargaining (Vickrey 1961; Myerson 1981). Our version of this result demonstrates that the outcomes of two crisis bargaining equilibria will be the same, no matter how different the underlying bargaining protocols are, as long as they result in the same probability of war. This is true even if the equilibria of the two games involve entirely different initial offers, sequences of proposals, exchanges of concessions, and so on.

More broadly, the fact that all outcomes of bargaining procedures are achievable as offers in a ultimatum game allows us better understanding of the deep regularities in bargaining. Complicated bargaining procedures do not allow states to exchange more information than in an ultimatum game, nor do they open up new possibilities for peaceful settlements. Although such procedures seem to offer states these abilities, they do not actually arise in equilibrium play. In this sense, the seemingly endless diversity of bargaining structures is an illusion.

It seems counterintuitive that complex back-and-forth bargaining protocols cannot achieve any more than an ultimatum offer can. How could a take-it-or-leave-it demand, which closes off any possibility of learning more about one's adversary before deciding to fight, have the same result as a strategy that allows for dialogue? It all comes down to the fundamental obstacle to effective diplomacy: incentives to misrepresent private information (Fearon 1995). There is always a temptation for a country to bluff about its strength or its willingness to fight. Although states would like to avoid war when its occurrence is the most costly, making concessions when one's adversary is strong and standing firm when it is weak, this cannot be sustained as equilibrium behavior.

Our equivalence result also enables us to investigate endogenous bargaining protocols. In other words, if a country could choose which bargaining process to use, what would it choose? As all outcomes of a wide variety of bargaining processes are akin to ultimatum offers, the optimal choice is simply to make the best ultimatum offer. In other words, if a country could unilaterally choose how to bargain, it would choose to make an ultimatum.[2] Modeling crisis bargaining as an ultimatum can thus be justified as an optimal choice of the country initiating the crisis. This optimality result is similar to the findings in Leventoğlu (2012), who independently considers optimal crisis bargaining under incomplete information. The key differences between our optimality finding and Leventoğlu's are that we examine a continuous type space and that we do not restrict war payoffs to follow a costly lottery.

Our main results extend to a set of bargaining games with costly delay that includes the alternating offers procedure (Rubinstein 1982), in which bargaining proceeds in a fixed sequence of offers and counteroffers. In all equilibria of these sequential offer crisis bargaining games, the game ends in the first period after the initial offer with weak types accepting the offer and strong types rejecting the offer. Again, this is exactly the outcome of an offer in the one-shot ultimatum game.

There are two important implications of this result. First, the apparent complexity of allowing bargaining to continue over time hides a simple regularity in outcomes. As bargaining ends immediately, the option to continue bargaining is not taken up in equilibrium. Because delay is costly, if a type is going to end up fighting, then it must fight in the first period. But without the risk of fighting in later periods, there is no reason not to settle immediately. This logic severely limits the informativeness of whatever back-and-forth might take place under complex bargaining protocols. This finding mirrors Powell (1996), who shows that the equilibria of alternating-offers bargaining with outside options entail a "bargaining shutdown" with a minimal exchange of offers (see also Leventoğlu and Tarar 2008).

Second, although our earlier results mirror findings in the auction and monopoly pricing

[2] The optimality of the ultimatum game is an analogue of the finding of Riley and Zeckhauser (1983) that the posted price mechanism is best for a seller.

literatures in economics, these results illustrate an important difference between the economic setting and our setting of international conflict. In the economic setting, agents are uncertain about the value of the good, which affects payoffs when agreement is reached and trade occurs. In the international conflict setting, however, uncertainty is about the payoff of war and so this uncertainty affects payoffs only if bargaining fails and war occurs. Therefore, unlike the economic context in which trade occurs at diminishing prices over time and some agreements are delayed, in the international context agreement or fighting occurs immediately and we do not have costly delay. Thus, the differences in the source of uncertainty in the two settings lead to stark differences in equilibrium outcomes.

While our primary concerns are theoretical, we also empirically investigate the relationship between ultimata and the likelihood of war in crises. Whereas earlier work suggests that inflexible bargaining strategies like ultimata should be particularly prone to lead to war (e.g., Lauren 1972; Huth 1988), we find that the issuance of an ultimatum is a negative and statistically insignificant predictor of a crisis escalating to war. Our formal results help explain this surprising empirical finding. We expect a rational actor in a crisis to select an optimal bargaining protocol. The ultimatum game is an optimal choice, but so are countless other bargaining games; what all these optimal choices have in common is that they result in the same probability of war. Therefore, among observed crises, we should expect to see no systematic relationship between ultimatum issuance and war.

The remainder of the paper is organized as follows. In the next section, we define the class of crisis bargaining games that we analyze. In Section 3, we prove our main result—that the equilibria of many crisis bargaining games are outcome equivalent to take-it-or-leave-it offers. Then, in Section 4, we demonstrate the optimality of the ultimatum game. Section 5 extends our results to bargaining games where costly delay is possible. The empirical analysis of ultimata in international crises appears in Section 6. We conclude by discussing the implications of our results for the study of crisis diplomacy.

2 Setup

We begin by laying out the basic ingredients of our analysis. We consider a situation in which two countries are engaged in crisis bargaining over a disputed prize of size 1. If the countries do not come to an agreement to divide the prize peacefully, war will result. We suppose that one side in the conflict has private information about its ability to fight a war. Without loss of generality, we let the country with private information be country 2. Specifically, let the set of possible *types* of country 2 be $T = [\underline{t}, \bar{t}] \subset \mathbb{R}$. The true type of country 2 is drawn from a cumulative distribution function F with density $f(t) > 0$ for all $t \in T$.

The private information of country 2 determines the value of war for both sides. Formally, we let $w_1(t)$ and $w_2(t)$ be the *value of war* for countries 1 and 2 respectively, where each $w_i(t)$ is differentiable. As war is costly, we assume that for all $t \in T$, $w_i(t) < 1$ and $w_1(t) + w_2(t) < 1$. As the type t represents the "strength" of country 2, we also assume that $w_1(t)$ is weakly decreasing in t and $w_2(t)$ is strictly increasing in t. As we impose no further restrictions on the functional form of $w_i(t)$, our results are general and include the two standard views about private information in crisis bargaining, independent values and interdependent values, as special cases.[3] These two special cases describe the relationship between country 2's type and the war payoff of country 1. In the case of independent values, $w_1(t)$ is constant in t. This can arise if the source of uncertainty is country 2's cost of fighting, for example, which does not directly affect country 1's payoff from war. In the case of interdependent values, $w_1(t)$ is decreasing in t. This can arise if country 2 has private information about its own military strength, for example, and country 1 should expect worse war outcomes when facing stronger types of country 2.

We assume that the interaction between the two countries takes the form of a *crisis bargaining game*, a game whose outcome is either a peaceful division of the good or war between the participants. In a crisis bargaining game, a country's payoff is its share of the agreed-upon division if a settlement is reached, or its war payoff $w_i(t)$ otherwise. It is worth emphasizing that this definition encompasses a wide variety of different bargaining procedures, from the simple to the complex. We permit offers to be made in an arbitrary order, offers to be retractable, cheap talk messages to be sent before or during bargaining, etc. To simplify the initial analysis, we begin by assuming settlements are efficient (i.e., the two countries' payoffs sum to 1). We relax this assumption in subsequent sections to show how our main results may extend to models with discounting over time and with bargaining while fighting.

The main substantive restriction on the class of interactions we consider is that we do not allow states' bargaining strategies to affect the players' war payoffs. For example, this rules out models of surprise attack, in which the war outcome depends on the timing and identity of a state cutting off negotiations.[4] This condition also rules out certain models of nuclear brinkmanship, such as Powell (2015), in which states' bargaining strategies influence the destructiveness of an eventual war.[5] This restriction makes the formal analysis more

[3]These two cases emerge from the standard "costly lottery" view of war (Fearon 1995). While our assumptions are consistent with such a functional form, our results do not require that we impose it.

[4]As our focus is on war due to incomplete information, it is natural that we exclude models of surprise attack; Fearon (1995) demonstrates that these constitute commitment problem explanations of war.

[5]Other models treat brinkmanship as a risky move that increases the chance of war, but not the expected payoff to war itself (e.g., Schwarz and Sonin 2007). These models do fall within the class of crisis bargaining games we consider, though not the subset of deterministic models we identify below.

tractable, but it is also in line with our substantive purpose, which is to address the relationship between bargaining protocols and crisis outcomes. By restricting our attention to crisis bargaining games in which states' war payoffs are exogenous, we ensure that any differences across mechanisms reflect differences in how states bargain (the sequence of offers, the possibility of cheap talk, etc.) rather than in the availability or efficacy of other foreign policy tools. Moreover, if war payoffs are endogenous, the main questions we study—whether there exists an outcome-equivalent or utility-improving ultimatum offer—are not well defined, as the ultimatum game assumes a fixed war payoff.

We say a crisis bargaining game is *deterministic* if the only move by Nature is her draw of country 2's type t at the beginning of the game.[6] Thus, in a deterministic bargaining game, the only (exogenous) uncertainty in the game is the uncertainty about the type of country 2. Of course, we permit countries to introduce additional uncertainty through mixed strategies (Meirowitz and Sartori 2008); only additional randomness by Nature is excluded.

While most models studied in the literature are deterministic, there are some exceptions. The most prominent is the alternating offers bargaining model with an exogenous risk of bargaining breakdown (Osborne and Rubinstein 1990; Wagner 2000). Models of brinkmanship in which a player can take an action that induces a lottery over immediate war or continued bargaining are also non-deterministic (Schwarz and Sonin 2007). In the equilibria of bargaining protocols like these, types of country 2 may face an uncertain but positive chance of war, $0 < \pi(t) < 1$, even if both countries employ pure strategies. As such, our equivalence results (Propositions 2 and 3 below) do not apply to non-deterministic bargaining games, though our optimality result (Proposition 4) does apply.

An equilibrium outcome of a crisis bargaining game can be described by the probability that a settlement is reached (as opposed to war) and how the settlement divides the prize between the two countries, as these are the two components that affect the participants' payoffs. Because country 2's negotiation strategy may depend on its private information, the realized outcome will be a function of its type t. The equilibrium outcome is thus given by two functions: the *probability of war*, $\pi(t)$, and the *value of settlement to country 2* if an agreement is reached, $v_2(t)$. Under the assumption of efficient settlements, the value of settlement to country 1 is $v_1(t) = 1 - v_2(t)$.

Given some equilibrium of a particular crisis bargaining game, it is straightforward to derive the associated probability of war and value of settlement as a function of country 2's type. However, as our goal is to characterize how the negotiating protocol influences the

[6]Nature may also move once war has been chosen (e.g., choosing from a distribution over which side wins, as in the costly lottery model of war), as long as the expected value of war for each country and type remains constant throughout the game.

outcomes of international crises, we cannot take the game form as fixed. Instead, we analyze all equilibria of all crisis bargaining games. To make this seemingly impossible task tractable, we follow Banks (1990) and Fey and Ramsay (2011) in relying on the tools of Bayesian mechanism design. The key result in this literature is the *revelation principle*, which states that any equilibrium outcome of a Bayesian game can be attained by an incentive compatible direct mechanism (Myerson 1979). A direct mechanism is simply a Bayesian game in which each player's only action is to "report" her type. Outcomes are then assigned on the basis of the players' reports. An incentive compatible direct mechanism is one in which it is an equilibrium for each player to report her type truthfully.

In the context we consider, namely crisis bargaining games, a *direct mechanism* consists of $\pi(t)$ and $v_2(t)$, the pair of functions that give the probability of war and value of settlement as a function of country 2's type. In order to characterize regularities in the set of crisis bargaining outcomes, we want to identify the set of direct mechanisms that correspond to equilibrium behavior in some crisis bargaining game. A basic requirement of equilibria in these games is that no type of country 2 can strictly increase its payoff by adopting the strategy of some other type (e.g., making more aggressive demands at the bargaining table). Incentive compatibility is the application of this concept to direct mechanisms. In particular, in the context of crisis bargaining games, a direct mechanism is *incentive compatible* if it is an equilibrium for country 2 always to report its type truthfully. According to the revelation principle, if some property is true of all incentive compatible direct mechanisms (π, v_2), then it holds for every equilibrium of every crisis bargaining game. Thus, analyzing the set of incentive compatible direct mechanisms will allow us to identify regularities that hold across all crisis bargaining games.

We now formally define direct mechanisms and incentive compatibility. A direct mechanism is a pair of functions $\pi : T \to [0, 1]$ and $v_2 : T \to \mathbb{R}$ that determine the probability of war and value of settlement, given the type reported by country 2. For a given direct mechanism (π, v_2), it is understood that $v_1(t) = 1 - v_2(t)$. A direct mechanism represents the outcome, in terms of the probability of war and value of settlement, for each type under some strategy profile in some crisis bargaining game. Reporting a type t' in the direct mechanism produces the same outcome as following the strategy employed by type t' under the original strategy profile. The expected utility to type t of country 2 for reporting its type as t' is

$$U_2(t' \mid t) = \pi(t')w_2(t) + (1 - \pi(t'))v_2(t'). \tag{1}$$

By misreporting its type—i.e., following the bargaining strategy of some other type—country 2 may change the probability of war and the settlement value, but not its own war payoff. In equilibrium, of course, each type of country 2 must weakly prefer to follow its own strategy rather than that of some other type. Analogously, a direct mechanism is incentive compatible if and only if country 2 never benefits from misreporting its type: $U_2(t \mid t) \geq U_2(t' \mid t)$ for all $t, t' \in T$. If a direct mechanism is incentive compatible, we can write the utility of type t of country 2 as

$$U_2(t) = U_2(t \mid t) = \pi(t)w_2(t) + (1 - \pi(t))v_2(t). \tag{2}$$

The expected utility of country 1 under this direct mechanism is

$$U_1 = \int_T [\pi(t)w_1(t) + (1 - \pi(t))v_1(t)]\, dF(t). \tag{3}$$

Unlike in most economic situations, in the international arena there is typically no third party able to enforce agreements between disputants. Therefore, peaceful settlements must be voluntary. Following Fey and Ramsay (2011), we say that a crisis bargaining game has *voluntary agreements* if each side has a strategy that yields at least its war payoff with certainty. This condition represents the assumption that a country always retains the option of resorting to war. No country can be forced to accept a settlement that leaves it with less than what it would expect from fighting. For any equilibrium of a game with voluntary agreements, the corresponding direct mechanism must meet the following conditions. Voluntary agreements for country 2 requires

$$v_2(t) \geq w_2(t) \qquad \text{for all } t \in T \text{ such that } \pi(t) < 1. \tag{4}$$

The voluntary agreements condition is slightly different for country 1, because it does not know t directly and thus may not know its own war payoff $w_1(t)$ with certainty. The condition for country 1 is

$$v_1(t) \geq E[w_1(t) \mid \mu(v_1(t), t)] \qquad \text{for all } t \in T \text{ such that } \pi(t) < 1, \tag{5}$$

where $\mu(v_1, t)$ denotes country 1's updated belief about country 2's type after observing that its share of the settlement value is v_1. Thus, country 1 uses the information revealed in the terms of the settlement to evaluate the desirability of war. We use the abbreviation IC for mechanisms that are incentive compatible and ICVA for mechanisms that are incentive compatible and satisfy voluntary agreements.

3 Equivalence

In this section, we consider the class of deterministic crisis bargaining games and find that there are deep underlying regularities in equilibrium outcomes of these games. We begin by illustrating the direct mechanisms that correspond to the equilibria of common crisis bargaining games. We will see that distinct game forms lead to strikingly similar outcomes—a feature not only of our examples, but of deterministic crisis bargaining games more generally.

As an example of the revelation principle and equivalent direct mechanisms in a crisis bargaining game, consider the standard ultimatum game.[7] The timing of the game is as follows: country 1 makes an offer $x \in [0, 1]$, which country 2 may accept or reject.[8] If the offer is accepted, the prize is divided according to the offer, with country 1 receiving the offer value x and country 2 receiving the remainder $1 - x$. If the offer is rejected, war occurs, so each country receives its war payoff: $w_1(t)$ for country 1 and $w_2(t)$ for country 2.

In equilibrium, country 2 must accept any offer that strictly exceeds its (privately known) war payoff and reject any offer that is strictly worse. Formally, this implies accepting any offer x such that $1 - x > w_2(t)$ and rejecting any offer such that $1 - x < w_2(t)$. Let $\tilde{t}(x)$ denote the strongest type of country 2 that weakly prefers accepting an offer of x to fighting (or $\underline{t}$ if all types prefer war), so that

$$\tilde{t}(x) = \begin{cases} \underline{t} & w_2(\underline{t}) > 1 - x, \\ w_2^{-1}(1 - x) & w_2(\underline{t}) \leq 1 - x \leq w_2(\bar{t}), \\ \bar{t} & w_2(\bar{t}) < 1 - x, \end{cases} \tag{6}$$

where $\underline{t}$ is the weakest type of country 2 and $\bar{t}$ the strongest. Assuming country 2 responds optimally to every offer, the expected utility to country 1 of offering x is

$$\xi_1(x) = F(\tilde{t}(x))x + \int_{\tilde{t}(x)}^{\bar{t}} w_1(t)\, dF(t). \tag{7}$$

In equilibrium, country 1 selects its offer x^* so as to maximize $\xi_1(x)$.[9] The resulting equilibrium outcome is that all types of country 2 below $\tilde{t}(x^*)$ settle and receive $1 - x^*$, while all types above $\tilde{t}(x^*)$ fight.

Let us now construct the direct mechanism that corresponds to the equilibrium of the

[7] For another example, see Fey and Ramsay (2011).

[8] In the remainder of the paper, when we refer to the ultimatum game, we mean this variant, in which the uninformed country makes the offer.

[9] In case of multiple equilibria, let x^* be any offer that is optimal for country 1. Fearon (1995) provides sufficient conditions for a unique optimal offer under a costly lottery with independent values.

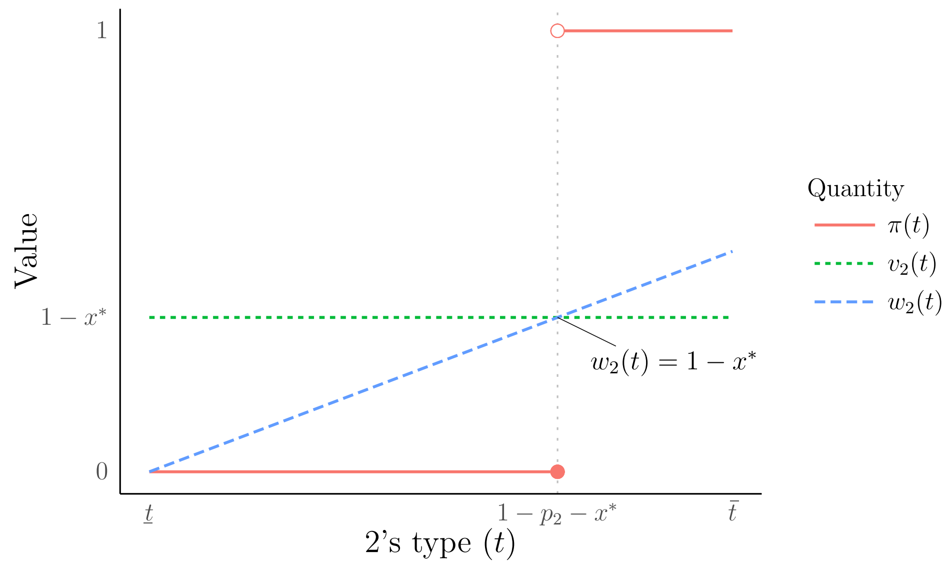

Figure 1: Direct mechanism corresponding to the equilibrium of the ultimatum crisis bargaining game (Fearon 1995). War payoffs pictured here are $w_2(t) = p_2 + t$, with $\underline{t} = -p_2$ and $\bar{t} = 0$.

ultimatum game. Recall that a direct mechanism for a crisis bargaining game consists of two functions of country 2's type: the probability of war, $\pi(t)$, and the value of settlement, $v_2(t)$. In the equilibrium of the ultimatum game, the probability of war is a step function, as the offer is accepted by all types weaker than the indifferent type $\tilde{t}(x^*)$ and rejected by those stronger:

$$\pi(t) = \begin{cases} 0 & t < \tilde{t}(x^*), \\ 1 & t > \tilde{t}(x^*). \end{cases}$$

The value of the settlement to country 2 is simply its share of the offer x^*: $v_2(t) = 1 - x^*$ for all $t \leq \tilde{t}(x^*)$. Figure 1 illustrates this direct mechanism. By the revelation principle, this direct mechanism is incentive compatible, as it corresponds to the outcome of a perfect Bayesian equilibrium. It is clear that this equilibrium gives every type of country 2 at least its war payoff, so voluntary agreements for country 2 is satisfied. As country 1 can demand $x = 1$ and thus be assured of either receiving $w_1(t)$, in case of rejection, or $x = 1 > w_1(t)$, in case of acceptance, this mechanism satisfies voluntary agreements for country 1. Therefore, this direct mechanism is incentive compatible and satisfies voluntary agreements.

To illustrate the broad similarities in outcomes among distinct bargaining games, we now consider an alternative model in which both sides make demands, which we call the *two-sided demand game*. The model is closely related to the crisis bargaining games studied by Wittman (2009) and Ramsay (2011). In the two-sided demand game, each country i makes

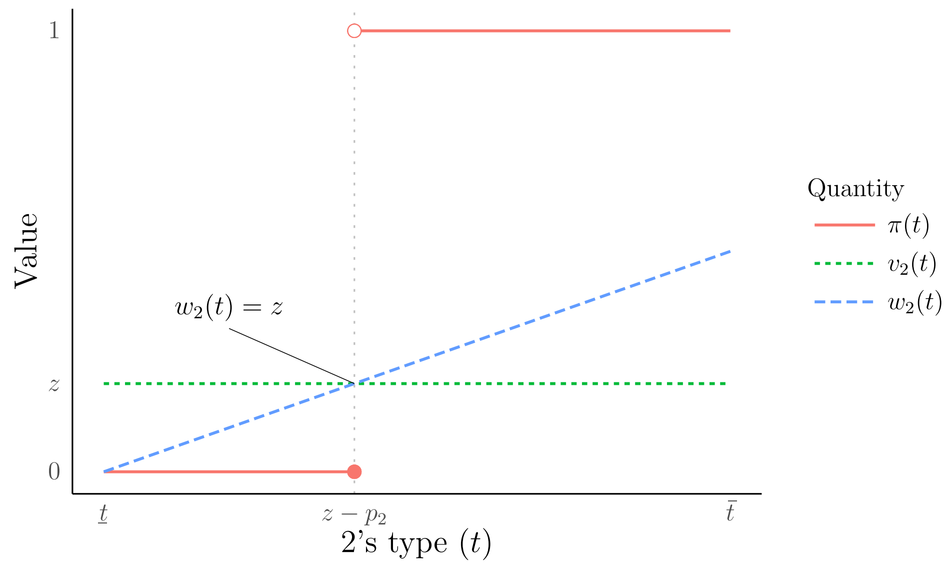

Figure 2: Direct mechanism corresponding to a pure strategy equilibrium of the two-sided demand game. War payoffs and type space are the same as in Figure 1.

a demand $x_i \geq 0$. If the total demands do not exceed the size of the prize, $x_1 + x_2 \leq 1$, then country 1 receives x_1 and country 2 receives $1 - x_1$. Otherwise, if $x_1 + x_2 > 1$, war occurs and each side receives its war payoff, $w_i(t)$. This deterministic crisis bargaining game has many pure strategy equilibria. For ease of exposition, assume $w_1(\bar{t}) > 0$, which means country 1 would always rather fight than accept the worst feasible settlement. Then there is a continuum of equilibria, each characterized by a value $z \in [0, 1 - w_t(\underline{t})]$, in which country 2 either receives z or fights, depending on its type. Strategies in such an equilibrium are $x_1 = 1 - z$ for country 1 and

$$x_2(t) = \begin{cases} z & w_2(t) \leq z, \\ 1 & w_2(t) > z \end{cases}$$

for country 2. Figure 2 plots the direct mechanism that corresponds to this kind of equilibrium.

Although the ultimatum game and the two-sided demand game are distinct in their sequence of play, their equilibrium outcomes are qualitatively similar, as shown in Figures 1 and 2. In the equilibria of both games, all types of country 2 that settle instead of fighting receive the same value of settlement. There is no screening, whereby stronger types of country 2 would receive better terms at the bargaining table. There is a cutoff type below which the dispute ends peacefully for sure and above which war takes place for sure. This cutoff is the type of country 2 that is indifferent between the terms of settlement and fighting. In fact, the outcome of an equilibrium of the two-sided demand game with $x_1 = 1 - z$ is exactly identical to the outcome of offering $x = 1 - z$ in the ultimatum game, assuming a best response by country 2. To see this, notice that country 1's expected payoff in the given equilibrium of the demand game is

$$U_1 = F(\tilde{t}(1-z))(1-z) + \int_{\tilde{t}(1-z)}^{\bar{t}} w_1(t)\, dF(t),$$

which is exactly $\xi_1(1-z)$, its expected payoff from offering $1-z$ in the ultimatum game, as defined by equation (7).

The similarities between the outcomes of these two games are no coincidence. They illustrate broad similarities between the pure strategy equilibrium outcomes of all deterministic crisis bargaining games. We will now show generally that all such equilibria share the following properties. First, there is a cutpoint type below which a settlement is reached for sure and above which war occurs for sure. Second, the value of the bargain that is reached is the same for every type that settles. Third, the terms of settlement are pinned down by the probability of war, so any two equilibria with the same chance of war have the same terms of settlement. Finally, the outcome of any such equilibrium could be attained by a take-it-or-leave-it offer.

Our goal is to characterize properties of all pure strategy equilibria of deterministic crisis bargaining games. To do so, we will characterize the set of direct mechanisms corresponding to such equilibria; by the revelation principle, any property that holds for all such mechanisms also holds for the corresponding equilibria. In the study of bilateral trade, the class of *certainty mechanisms* is particularly useful (Myerson and Satterthwaite 1983; Williams 1987; Ausubel and Deneckere 1993).[10] In our setting, a certainty mechanism is one in which almost every type of country 2 either settles for certain or fights for certain.[11] Formally, a certainty mechanism is a mechanism (π, v_2) such that $\pi(t) = 0$ or 1 for almost all t.

Certainty mechanisms are of particular interest to us because they correspond to pure strategy equilibria of deterministic crisis bargaining games. If each side is using pure strategies, then at each decision node country 1 chooses an action with certainty and almost all types of country 2 are choosing an action with certainty.[12] As a deterministic crisis bargaining game has no chance moves by Nature, this means that, for each type, the probability of war is either zero or one. By the revelation principle, then, we can find an incentive compatible direct mechanism with the same probability of war, which must be a certainty

[10] Certainty mechanisms are also called "0-1 mechanisms."

[11] A property holds for *almost every* type if it holds for all but a measure zero set of types.

[12] As is standard, a pure strategy is one in which almost all types choose an action with certainty: a measure zero set of types is allowed to mix, e.g., a singleton type who is indifferent between accepting an offer and fighting.

mechanism. This argument is the basis of the following proposition:

Proposition 1. *For every pure strategy equilibrium of every deterministic crisis bargaining game, there is an equivalent incentive compatible certainty mechanism.*

In the formal literature on crisis bargaining, models with continuous types are usually solved using pure strategy equilibria.[13] Thus this proposition has a broad reach: since a great deal of the existing literature involves pure strategy equilibria of deterministic bargaining games, the study of incentive compatible certainty mechanisms is widely applicable.

By Proposition 1, any property of all incentive compatible certainty mechanisms is also a property of all pure strategy equilibria of all deterministic bargaining games. We now obtain some preliminary results about incentive compatible mechanisms that will be useful in our later analysis. First, we note that in an incentive compatible mechanism, the probability of war, $\pi(t)$, is weakly increasing with t. Thus, in every equilibrium of every crisis bargaining game—even those that are not deterministic—stronger types of country 2 must run a risk of war at least as high as weaker types. This kind of result is common in mechanism design, and our version is due to Banks (1990, Lemma 1):

Lemma 1. *If* (π, v_2) *is incentive compatible, then* π *is weakly increasing.*

This result implies that every incentive compatible certainty mechanism π is a step function; i.e., there is a cutpoint type $t^W \in T$ such that

$$\pi(t) = \begin{cases} 0 & t < t^W, \\ 1 & t > t^W. \end{cases}$$

Note that we allow t^W to equal the weakest or strongest type, $\underline{t}$ or $\bar{t}$ respectively, so that country 2 may always fight or always settle.

With Lemma 1 in hand, we can show that the *ex ante* probability of war pins down all other aspects of the outcome of a certainty mechanism. In other words, if two certainty mechanisms have the same (nonzero) *ex ante* chance of war, then they also have the same cutpoint type t^W and the same value of settlement for those types that do not fight. We will denote the *ex ante* probability of war of a direct mechanism by Π. As $\pi(t)$ gives the probability of war for each type of country 2 and $F(t)$ is the prior distribution of types, we have

$$\Pi = \int_T \pi(t)\, dF(t).$$

[13]There are exceptions, such as Dal Bó and Powell (2009).

For an incentive compatible certainty mechanism, in which case π is a step function with step t^W, we have $\Pi = 1 - F(t^W)$. Because F is continuous and strictly increasing, this is equivalent to $t^W = F^{-1}(1 - \Pi)$. Therefore, once we know the *ex ante* probability of war of an incentive compatible certainty mechanism, we can pin down the exact form of π. Moreover, every type of country 2 that does not fight ($t < t^W$) must receive the same settlement and, if the expected probability of war is positive, this settlement must be equal to the war payoff of type t^W. As the value of t^W is determined solely by Π, all aspects of the mechanism are determined solely by Π. We thus have the following result. (All proofs are in the Supplemental Appendix.)

Lemma 2. *For every incentive compatible certainty mechanism* (π, v_2)*, the values of* U_1*,* $U_2(t)$*, and* $v_2(t)$ *depend only on* Π *when* $\Pi > 0$ *and depend only on* $v_2(\bar{t})$ *when* $\Pi = 0$*.*

This result shows that all incentive compatible certainty mechanisms with the same positive *ex ante* chance of war are identical in terms of their outcomes and payoffs. Therefore, by Proposition 1, the *ex ante* chance of war also pins down the outcomes and payoffs of any pure strategy equilibrium of a deterministic crisis bargaining game. This result can be described as a Crisis Bargaining Equivalence Theorem, as it closely resembles the Revenue Equivalence Theorem in auction theory (Myerson 1981). As the division of the good under dispute is the same for all types that settle in equilibrium, we refer to this division as the *terms of settlement*. Formally, we identify the terms of settlement with the value $v_2(\underline{t})$, as the weakest type of country 2, $\underline{t}$, will always be among those who settle in equilibrium.[14] The following proposition, which follows directly from Proposition 1 and Lemma 2, states that these terms of settlement, and therefore the equilibrium outcome of crisis bargaining, are equivalent for any two equilibria with the same probability of war.

Proposition 2. *If two pure strategy equilibria of two deterministic crisis bargaining games have the same* ex ante *probability of war,* $\Pi > 0$*, then they have the same terms of settlement and same payoffs.*

This equivalence result reveals deep regularities in equilibrium play in deterministic crisis bargaining games. Even if two of these games have completely different sequences of actions, different order of play by the two sides, different levels of cheap talk, etc., if equilibrium play in these two games has the same expected probability of war, then country 1 and all types of country 2 face exactly the same terms of settlement and incidence of war. Put another way, everything that differs about outcomes across pure strategy equilibrium play in deterministic crisis bargaining games is solely a result of differences in the expected probability of war.

[14]For an equilibrium in which all types go to war, we may assume without loss of generality $v_2(\underline{t}) = w_2(\underline{t})$.

Lemma 2 shows that every type of country 2 that settles receives the same value, namely the war payoff of the strongest type that does not fight. But this means that a take-it-or-leave-it offer could achieve the same result. For any ICVA certainty mechanism (π, v_2), let the *equivalent ultimatum offer* be a strategy profile of the ultimatum game in which country 1 offers the same terms of settlement, $x = 1 - v_2(\underline{t})$, and country 2 employs a best response. The outcome of such a strategy profile is that each type of country 2 with $w_2(t) < 1 - x$ agrees to the division $(x, 1 - x)$, while each type with $w_2(t) > 1 - x$ goes to war. Recall that $\xi_1(x)$ denotes the payoff country 1 receives from making a take-it-or-leave-it offer of x, as defined by equation (7). The following lemma derives the terms of settlement that correspond to a particular *ex ante* probability of war in an ICVA certainty mechanism, and shows that the equivalent ultimatum offer yields the same outcomes and payoffs.

Lemma 3. *Let (π, v_2) be an ICVA certainty mechanism with* ex ante *probability of war Π. The terms of settlement satisfy $v_2(\underline{t}) \geq w_2(\bar{t})$ if $\Pi = 0$ and $v_2(\underline{t}) = w_2(F^{-1}(1 - \Pi))$ if $\Pi > 0$. The equivalent ultimatum offer, with $x = 1 - v_2(\underline{t})$, is outcome-equivalent to (π, v_2). Country 1's* ex ante *expected utility from (π, v_2) is $U_1 = \xi_1(x)$.*

This result shows that ICVA certainty mechanisms are no different in essence than take-it-or-leave-it offers. By Proposition 1, the same is true of all pure strategy equilibria of deterministic crisis bargaining games, giving us the following proposition.

Proposition 3. *For every pure strategy equilibrium of a deterministic crisis bargaining game, there is an equivalent ultimatum offer with the same terms of settlement and probability of war.*

This proposition shows that even though the class of deterministic crisis bargaining games is large and diverse, there is a simple categorization of the equilibrium outcomes of this class of games, namely, the outcomes of ultimatum offers (assuming best responses by the recipient). Thus, we can reduce the seeming complexity of deterministic crisis bargaining games to the analysis of offers in the ultimatum game. To illustrate this surprising result, consider once again the two-sided demand game pictured in Figure 2. The process of negotiation in this game is quite unlike an ultimatum: each side makes demands simultaneously, and war occurs if and only if these demands are incompatible. Yet Proposition 3 shows that any of this game's pure strategy equilibrium outcomes—which types go to war, and the terms of settlement for those that do not—could be achieved by a take-it-or-leave-it offer.

Proposition 3 encapsulates the surprising result that the peculiar features of the take-it-or-leave-it protocol are in fact general lessons about the resolution of international conflict. This is surprising because the ultimatum model makes the stringent assumption that the proposing country is entirely inflexible. The proposer cannot modify an offer that is rejected, retract an offer that is accepted, or listen to any counterproposals from its opponent. Because it excludes the kind of back-and-forth often identified with international diplomacy, the ultimatum model may look like an impoverished representation of crisis bargaining. We find that these looks are deceiving: every pure strategy equilibrium outcome of a richer deterministic crisis bargaining game can be obtained by an equivalent ultimatum offer. Even in games that allow for extensive channels of communication, in equilibrium either states choose not to take advantage of them or their doing so has no meaningful effect on the shape of the outcome. Therefore, the apparent restrictiveness of the ultimatum protocol does not drastically limit its generality. In a similar vein, Fey, Meirowitz and Ramsay (2013) show allowing for retractable offers does not change the equilibrium of the ultimatum game.

To be clear, while the equilibrium outcomes of all deterministic crisis bargaining games have the same structure as take-it-or-leave-it offers, they do not necessarily result in *exactly* the same outcome as the equilibrium of the ultimatum game. In particular, the exact probability of war and terms of settlement may differ across game forms. For example, the aforementioned two-sided demand game may produce equilibria outcome-equivalent to that of the ultimatum game ($z = 1 - x^*$), or with a lower probability of war ($z > 1 - x^*$), or with a higher probability of war ($z < 1 - x^*$).

4 Optimality

In the previous section, we showed that every pure strategy equilibrium outcome of a deterministic crisis bargaining game can be reduced to its terms of settlement, which in turn are equivalent to a particular take-it-or-leave-it offer. This result is particularly useful in finding outcomes that have some desirable property, for example, outcomes that are "best" for country 1, because we must only look at the country's preferences over ultimatum offers to find the right one.

Our aim in this section is to characterize the crisis bargaining game that is optimal from the point of view of country 1. That is, suppose country 1 can initiate a crisis and can choose the bargaining game that will be used to settle the crisis. Which game form should country 1 choose? Formally, we say a direct mechanism (π^*, v_2^*) is an *optimal mechanism* for country 1 if it satisfies incentive compatibility and voluntary agreements, and $U_1^* \geq U_1$ for any other direct mechanism (π, v_2) that satisfies these conditions. By the revelation principle, no equilibrium of a crisis bargaining game with voluntary agreements can give country 1 a greater expected utility than an optimal mechanism.

Here we show that, as suggested above, the ultimatum game is an optimal mechanism.

In other words, when choosing the form of crisis bargaining, country 1 can do no better than choosing the ultimatum game. That is, out of the many complicated bargaining games available, country 1 can achieve its highest expected payoff by making an uncomplicated, inflexible take-it-or-leave-it-offer. Of course, there can also be more complicated game forms with the same equilibrium outcomes as the ultimatum game, as described in the previous section, and so these game forms would be optimal as well. For example, the equilibrium of the demand game with $z = 1 - x^*$, the ultimatum game with retractable offers (Fey, Meirowitz and Ramsay 2013), or the dynamic ultimatum game we describe below in section 5.3 would all be optimal as well.

This optimality result mirrors the finding by Riley and Zeckhauser (1983) that posted prices are the profit-maximizing means of conducting a bilateral trade. We derive our version of this result in a different way, however. We rely on the results of the previous section that identify incentive compatible certainty mechanisms, and thus pure strategy equilibria of deterministic crisis bargaining games, with equivalent ultimatum offers. It follows that making an optimal offer in the ultimatum game is the best possible among incentive compatible certainty mechanisms. Our next result formalizes this argument and, in addition, shows that country 1 cannot do better by choosing an incentive compatible mechanism that is not a certainty mechanism (e.g., a mixed-strategy equilibrium or a game with non-deterministic play).

Proposition 4. *The ultimatum game is an optimal mechanism for country 1.*

This result has striking substantive implications. It allows us to say that if country 1 could unilaterally select the bargaining protocol, it would choose to make a take-it-or-leave-it offer. When the ultimatum game is embedded in a model of war, it can be thought of as the reduced-form outcome of a process by which one country selects the bargaining protocol endogenously. In this sense, the use of a take-it-or-leave-it offer may be justified substantively rather than being seen as an artificial modeling choice made for mathematical convenience.

The optimality of the ultimatum game is particularly remarkable when one considers that its equilibrium entails a positive probability of war under many conditions (Fearon 1995, Claim 2). It means that a country may be best off under a bargaining protocol that sometimes leads to war, despite the existence of a range of settlements that are Pareto superior to war. The logic of this result is the familiar risk-reward tradeoff. Under any incentive compatible mechanism with no chance of war, every type of country 2 must receive the same payoff as the strongest type. By our equivalence results, then, the optimal crisis bargaining game with zero probability of war is equivalent to the outcome of country 1 making an offer $x \leq 1 - w_2(\bar{t})$ in the ultimatum game. As such, the reason why country 1 would prefer the ultimatum game to a peaceful mechanism is *identical* to the reason it prefers the equilibrium offer within the ultimatum game to one that would be accepted by all types of country 2. By accepting some risk of war with the strongest types of its opponent, as in the equilibrium of the ultimatum game, country 1 increases its payoff from negotiations with the weaker types.

5 Inefficient Settlements

Up to this point, we have focused on cases in which all peaceful settlements from bargaining are efficient—they divide the entire resource so that one side receives a certain share and the other side receives the remainder. This is typically true of one-shot bargaining games in which there is immediate agreement or war. However, many interesting models of bargaining studied in the literature allow for inefficient outcomes short of full-scale war. Most notably, these includes models of bargaining over time: if players discount the future, then reaching a settlement after some delay is inefficient, as both players would be better off reaching the same settlement immediately. Potential inefficiencies also arise in models in which players incur costs in order to signal their strength in negotiations, such as in games with bargaining while fighting (e.g., Filson and Werner 2002) or burning money (e.g., Fearon 1997).

In this section, we extend our equivalence and optimality results to games with the possibility of inefficient settlements. First, we examine the alternating offers game (Rubinstein 1982) and closely related bargaining protocols in which negotiations take place over time and players discount the future. We find that the equilibria of these games entail either immediate fighting or an immediate settlement; it follows that their equilibrium outcomes, like those of the deterministic bargaining games without delay we examined above, could be attained by a simple take-it-or-leave-it offer. Second, we show that our original optimality result continues to hold when we relax the assumption that peaceful outcomes are always efficient. Specifically, no crisis bargaining game with inefficient settlements can yield a better outcome for country 1 than the equilibrium of the one-shot ultimatum game. Finally, to address whether a country can credibly commit to making an ultimatum, we examine a particular dynamic bargaining game in which the offering country can make a new offer after an initial offer is rejected. The equilibrium outcome of this game is exactly the same as that of the one-shot ultimatum game.

5.1 Equivalence in Games with Costly Delay

We now consider bargaining games with discounting in which there is an offer made in each period which can either be accepted, ending the game, or rejected, leading to a new offer in the next period.[15] In addition, in keeping with our focus on voluntary agreements, we suppose that either side can choose to fight instead of bargain.[16] More formally, we suppose there is an infinite number of periods $\tau = 1, 2, \ldots$ and a common discount factor $\delta \in (0, 1)$.[17] In each period τ, one side makes an offer to divide the resource, $(x^\tau, 1 - x^\tau)$, or choose to fight. If the offer is made, the other side can either accept the offer, ending the game with payoffs $(\delta^{\tau-1}x^\tau, \delta^{\tau-1}(1-x^\tau))$, reject the offer and move to the next period, or fight. If either side chooses to fight, the game ends with each side receiving their war payoff, appropriately discounted. Other than assuming that country 1 makes the first offer in period 1, we do not specify the order that offers are made. This class of games includes the alternating offers model and similar games, such as the one in which country 1 always makes the offer. We refer to this class of games as *sequential offer crisis bargaining games*.

The information structure remains as before. Country 1 is uncertain about the war payoff of country 2, $w_2(t)$. On the other hand, country 1's war payoff w_1 is commonly known and does not depend on t. Because war is inefficient, we suppose $w_1 + w_2(t) < 1$ for all $t \in T$. Country 1's belief is updated according to Bayes' Rule.

Our main result, which is proved in the Supplementary Appendix, is that our earlier equivalence result continues to apply to bargaining with costly delay. In particular, we have the same result as Proposition 3, that outcomes of sequential offer crisis bargaining games correspond to offers in the ultimatum game.

Proposition 5. *For every pure strategy equilibrium of every sequential offer crisis bargaining game, there is an equivalent ultimatum offer with the same terms of settlement and probability of war.*

Once again, we have an equivalence between bargaining with costly delay and the ultimatum game. Importantly, delay cannot occur in this context and therefore there is no inefficiency in equilibrium. Fighting cannot occur in equilibrium later than the first period because a type that is fighting strictly prefers to do so immediately. But without fighting, agreement is immediate, as in the standard alternating offers game without fighting.

This result also contrasts with economic models in which price offers serve to "screen" buyers with different valuations of a good. In such models, the seller drops her price over

[15]This is similar to the model in Powell (1996), though we do not consider flow payoffs.
[16]This specification is advocated in Leventoğlu and Tarar (2008).
[17]Our results would not change if the discount factors of the two sides differed.

time, and low value buyers must wait until the price drops in order to reach an agreement. This effect does not occur in our context because the private information of country 2 only affects its payoff to war, not its payoff to agreement. In other words, the value of agreement to all types is equal. This forms the basis of our result that delay cannot occur and shows the important differences between bargaining in economics and in international crises.

Here we have assumed that delay is inefficient, that there is no exogenous source of information revelation over time, and that players' uncertainty concerns their war payoffs. Relaxing any of these assumptions may change our conclusion of no delay in equilibrium, as previous studies of crisis bargaining demonstrate. Leventoğlu and Tarar (2008) analyze a dynamic bargaining model with flow payoffs, in which players receive status quo payoffs until an agreement is reached and delay therefore is not inefficient; they identify equilibria with delayed settlements. Slantchev (2003) considers a model of bargaining while fighting, in which battle outcomes signal information about states' types. In this setting, because stronger types have greater continuation values for rejecting an offer, it is possible to sustain equilibria with delay in which weaker types are screened out early on. Similar costly signaling dynamics might allow for equilibrium delay if country 2's incomplete information concerned her discount factor or her being a behavioral "crazy type" (see Acharya and Grillo 2015) rather than her war payoff, as we assume here.[18]

As an example of our result, consider the standard alternating offers model. In this case, if the second period were reached, agreement would occur immediately on the division $(\delta/(1+\delta), 1/(1+\delta))$, as usual. This means that every type of country 2 will accept an initial offer with $1 - x \geq \delta/(1+\delta)$. Now let $d^* = 1 - \delta/(1+\delta)$ and let x^* be the equilibrium offer for a one-shot ultimatum game, which we assume is unique. Then it is easy to check that the unique equilibrium of the dynamic alternating offers crisis bargaining game is for country 1 to choose to fight if $d^* < w_1$, and, if $d^* > w_1$, for country 1 to make an offer equal to $\min\{d^*, x^*\}$. All types of country 2 respond to this offer by accepting or choosing to fight. For some parameter values, the outcome of this game is identical to the equilibrium offer of an ultimatum game, while for other parameters country 1 makes a more generous offer.

5.2 Optimality with Inefficiency

Above in Proposition 4, we showed that no bargaining protocol can give country 1 a greater payoff than making an ultimatum offer. We now consider whether this result continues to hold when we allow for inefficient settlement outcomes, such as arise in models with delay, bargaining while fighting, or other forms of costly signaling. It is conceivable that the

[18] We thank an anonymous reviewer for suggesting this possibility.

possibility of inefficient settlements might change the answer we derived before, as country 1 might use the threat of inefficient outcomes (e.g., delay) to force country 2 to accept offers that would it not otherwise. However, we show that this is not the case. Across all crisis bargaining games, with or without efficient settlements, it is optimal for country 1 to choose the one-shot ultimatum game.

Recall that the revelation principle allows us to identify an optimal bargaining protocol with an optimal ICVA direct mechanism. In the earlier discussion of optimality, we only considered those mechanisms with efficient settlements, in which $v_1(t) + v_2(t) = 1$, the total size of the prize, for all t. We now allow for a type-dependent amount of inefficiency, writing $v_1(t) + v_2(t) = 1 - d(t)$, where $d(t) \in [0, 1]$ represents the amount of inefficiency in the corresponding crisis bargaining equilibrium.[19] We find that an optimal mechanism must entail no realized inefficiency, so $d(t) = 0$ for all t. From there it follows that the equilibrium of the one-shot ultimatum game, which we have already seen is optimal among efficient mechanisms, is also optimal in this wider class.

Proposition 6. *An optimal bargaining game with discounting for country 1 is to make a take-it-or-leave-it offer to country 2.*

5.3 Credibility of an Ultimatum

We have shown that the optimal choice of crisis bargaining game for country 1 is the most inflexible option, the ultimatum game. But perhaps we should be concerned that this result assumes an unrealistic ability for country 1 to commit to this game form. After all, it requires a rejected offer to lead to immediate war, but wouldn't there be a temptation for country 1 to make a last-ditch, more generous offer to avoid destructive fighting? The answer is, somewhat surprisingly, that relaxing the commitment assumption on the part of country 1 does not change our results. Even if country 1 has the ability to make new offers in the face of rejection, it chooses not to and the outcome is the same as the one-shot ultimatum game. In this way, country 1's threat to fight after a rejected offer is credible.

We examine a dynamic ultimatum bargaining game in which, in each period, country 1 can make an offer or choose to fight, and country 2 can accept the offer, reject it, or choose to fight. If an offer is made and country 2 rejects, play moves to the next period with a new offer from country 1, and otherwise the game ends. Therefore, country 1 may always make a new offer instead of fighting. Full details of the model appear in the Appendix.

We show that every equilibrium of this dynamic ultimatum game has the same outcome,

[19] Proposition 5 above shows that $d(t) = 0$ in the direct mechanisms corresponding to the equilibria of sequential offer crisis bargaining games, a special case of the class of games with inefficient settlements.

which is identical to the outcome of the standard one-shot ultimatum game. In other words, even though country 2 has the ability to reject an offer and continue bargaining, country 1 makes the same offer as it would in a one-shot ultimatum game and country 2 responds in the same manner. This resolves any concern about the credibility of choosing a one-shot ultimatum game.

Proposition 7. *The dynamic ultimatum game has a unique equilibrium outcome, which is the same outcome as the one-shot ultimatum game.*

It may seem surprising that after its initial offer is rejected, country 1 would choose to fight instead of increasing the offer, but a simple intuition drives the result. If country 1 were to make a more generous offer after rejection, then all types of country 2 would reject the initial offer—not just the types for which the initial offer is truly worse than fighting. But then, because rejection is not informative, the risk-reward tradeoff is the same for country 1 after rejection as before, meaning it has no incentive to make a more generous offer. In other words, incentive compatibility precludes an equilibrium in which country 1 avoids war by ratcheting up its offer over time. Then, because of discounting, the types of country 2 that would eventually fight are best off doing so in the first period. Consequently, the equilibrium path of play is identical to that of the ordinary ultimatum game.

6 Empirical Analysis

Our formal results suggest that numerous bargaining protocols produce outcomes qualitatively similar to those of take-it-or-leave-it offers. Yet previous scholarship has portrayed ultimata as a particularly dangerous tool of crisis diplomacy. In a classic study of ultimata in coercive diplomacy, Lauren (1972, 163) predicts that an ultimatum increases the chance that a crisis will escalate into war: "An ultimatum readily excites rather than inhibits tensions, and only decreases bargaining flexibility. As such, it only increases the possibility of exceeding an opponent's threshold of tolerance, provoking an imprudent and unfavorable response and precipitating the critical eruption of a crisis." Similarly, in a canonical study of deterrence, Huth (1988, 51) predicts that a "firm and unyielding" diplomatic strategy in which a country "does not reciprocate accommodative initiatives taken by the potential attacker" will result in a greater chance of deterrence failure, and therefore conflict, than a "firm but flexible" strategy in which the country reciprocates concessions. According to these accounts, we should observe a greater probability of war in crises with an ultimatum than those without one.

In this section, we empirically analyze the relationship between ultimata and international crisis outcomes. Contrary to conventional expectations, we find no evidence that making an ultimatum increases the chance of war: the point estimate is negative and statistically insignificant. As we describe below, our theoretical results provide an explanation for this otherwise puzzling absence of a correlation between ultimata and war. As such, while the primary contribution of our formal analysis is theoretical, we find that it can help make sense of a surprising empirical finding as well.

6.1 Expectations about the Effect of an Ultimatum

We now explain why the absence of a correlation between ultimata and the risk of war is consistent with our formal analysis. In section 4, we showed that the equilibrium of the ultimatum game is optimal, as in it maximizes a country's *ex ante* expected utility, within a broad class of crisis bargaining protocols. A naïve reading of this result might lead us to predict that every crisis should end with an ultimatum, which clearly is not true. However, while we showed that making an ultimatum is an optimal protocol, it is not the only one. In fact, we have already characterized two alternative protocols (the two-sided demand game we used to illustrate Proposition 3 and the dynamic game in section 5.3) with equilibria outcome-equivalent to that of the one-shot ultimatum game. Many others might exist. Therefore, our observation of non-ultimatum bargaining is not inconsistent with utility-maximizing behavior on the part of states in crisis. What our theory predicts is that whatever bargaining protocol is chosen be outcome-equivalent to the equilibrium of the ultimatum game.

We derive empirical hypotheses from a simple model of an endogenously selected bargaining protocol. As in our baseline model, there are two countries in dispute over a valuable prize. At the outset of the interaction, Nature randomly gives one of the two disputants the option to select the bargaining protocol. After that, play continues according to the terms of the selected protocol, resulting either in war or a peaceful division of the prize. The involvement of Nature reflects the possibility that either country may have the initiative to determine the form of negotiations, though which country has this initiative may not be observable to external analysts.[20] By the revelation principle (Myerson 1979), selecting a bargaining protocol is equivalent to selecting a direct mechanism as we describe above—a mapping from the countries' types into the chance of war and the terms of settlement. Therefore, in every equilibrium of the game with the endogenous selection of bargaining protocols, the country that gets to choose the protocol will choose an optimal mechanism.

[20] In this context, it may be more natural to assume both countries have private information. We show in separate work that the optimality of making an ultimatum holds up under two-sided incomplete information about the costs of fighting. Details are available from the authors.

We have shown (Proposition 4) that the equilibrium of the ultimatum game is an optimal mechanism. Consequently, one equilibrium of the game with the endogenous selection of bargaining protocols is for the country with initiative to choose to make an ultimatum. However, choosing any alternative protocol that is outcome-equivalent to the equilibrium of the ultimatum game is optimal as well. In our theoretical analysis, we call the equilibrium outcomes of two bargaining games equivalent if they meet two conditions. First, the probability of war must be the same in each. Second, the distribution of goods between states in case of a settlement must also be the same. Any differences in how the outcome is achieved—for example, the exact sequence of offers—is immaterial as long as these conditions hold. According to our theory, states should either make an ultimatum or use some other bargaining protocol that is outcome-equivalent. Since we do not necessarily observe which state has the initiative to set the bargaining protocol, we formulate our hypotheses at the crisis level rather than at the state level.

Hypothesis 1. *All else equal, the probability of war is equal in crises in which an ultimatum is issued and those in which an ultimatum is not issued.*

Hypothesis 2. *All else equal, the terms of settlement in case of peace are equal in crises in which an ultimatum is issued and those in which an ultimatum is not issued.*

It is important to recognize that these hypotheses refer to the population of observed international crises. Our model does not imply that exogenous manipulation of the bargaining protocol would lead to no change in the probability of war or the terms of settlement. Rather, we claim there is no difference within the set of *optimal* bargaining protocols, which includes the equilibrium of the ultimatum game. Because the crisis bargaining procedure is endogenous to state decisions, rational behavior should lead to an optimal choice of bargaining protocol, and thus the probability of war and the distribution of benefits should not differ across otherwise similar crises.

6.2 Data and Methods

Our sample is the set of international crises included in the International Crisis Behavior dataset, version 10 (Brecher and Wilkenfeld 2000). The unit of observation is the international crisis. As our interest is in negotiations that may lead to war, we exclude crises that occur between states that are already at war with one another, leaving a sample of $N = 406$ crises between 1918 and 2007.[21]

[21] We drop six cases due to missing data on covariates; the results do not change substantially if we impute the missing cases.

The independent variable of primary interest is whether any of the states or organizations involved in the crisis issued an ultimatum.[22] In the formal account of ultimata we draw from (Fearon 1995), one state makes a demand with no opportunity for counterproposals: either the recipient of the demand accepts it, or else war occurs. Accordingly, we consider an ultimatum to have been issued if the following conditions, which also draw from Lauren's (1972) definition of an ultimatum, are met:

1. A state (or group of states) makes a specific demand of another state (or group of states).

2. The demand is backed by an implicit or explicit threat that force will be used if the recipient does not agree to or comply with the demand.

3. There is a deadline by which the recipient must reply to the demand or comply with its terms.

For each crisis, we code *Ultimatum* as 1 if all three of these conditions are met, and as 0 otherwise. To determine which crises meet these conditions, we rely on the crisis descriptions provided by the ICB Project, which are available online through the ICB Data Viewer.[23] A list of these cases is provided in Table 5 in the Appendix.

To best estimate the causal effect of issuing an ultimatum on crisis outcomes, we control for potentially confounding variables. Our source for all variables is the International Crisis Behavior system-level dataset. First, we control for the type of issue that is at stake in the crisis. The variable *Military-Security Issue* is an indicator for whether the crisis pertains, in whole or in part, to military force or national security.[24] We include this variable because both the manner of negotiations and the likelihood of war may be systematically different in controversies over military or security issues than in other crises. Second, we include a set of variables concerning the characteristics of the states involved in the crisis. We focus on characteristics that are known to be important determinants of war—namely, major power status, socio-political similarity, military capability parity, and geographical proximity (Bremer 1992). The variable *Major Power Involvement* is an indicator for whether any major powers (which, after 1945, are only the United States and the Soviet Union/Russia) play a role in the crisis. Our measure of socio-political similarity is *Heterogeneity*, the maximal number of differences in four state attributes tracked by ICB—"military capability, political

[22] A state or international organization may issue an ultimatum, per our coding, even if it does not meet the ICB definition of a "crisis actor."

[23] http://www.cidcm.umd.edu/icb/dataviewer/.

[24] All variables other than *Ultimatum* are constructed from variables in the ICB system-level dataset. For details, see the Appendix.

regime, economic development, and culture" (Brecher and Wilkenfeld 2000, p. 53)—between any pair of adversaries in the crisis. To measure military capability parity, we include an indicator for whether there is a substantial difference between adversarial states (or coalitions) in terms of their overall size, force size, economic strength, and nuclear capability, denoted *Power Disparity*. Finally, we control for the geographical relationship among crisis actors with *Proximity*, a categorical variable for whether the actors are contiguous, near one another, or distant.

Hypothesis 1 concerns the (lack of) effect of an ultimatum on the likelihood of war. The corresponding dependent variable in our dataset is *War*, an indicator for whether full-scale war occurs. Hypothesis 2 concerns the relationship between ultimata and the distributive outcome of the crisis—the terms of settlement. Ideally we would measure the value of the underlying stakes of the crisis and how this value is distributed between each crisis participant. In practice, however, no straightforward measure of the distribution of benefits is available. As a proxy we use *Satisfaction*, an index of crisis actors' perceived satisfaction with the ultimate outcome ranging from 1 (unanimous dissatisfaction) to 5 (unanimous satisfaction).[25] As long as lopsided settlements produce more dissatisfaction than those with a more equal distribution of benefits, the *Satisfaction* measure should be closely related to the terms of settlement, which is the underlying variable of interest in Hypothesis 2.

6.3 Results

We begin with a simple examination of the bivariate relationship between ultimatum issuance and the outbreak of war. A cross-tabulation of these variables appears in Table 1. The weakness of the bivariate relationship is evident. War occurs in 8 of the 62 crises in which an ultimatum is issued (13%), compared to 49 of the 344 crises with no ultimatum (14%). Contrary to traditional accounts of crisis diplomacy, war occurs less frequently following an ultimatum than in other crises. However, the difference in the likelihood of war is so slight that we cannot reject the null hypothesis of independence ($\chi^2 = 0.0066, p = 0.94$). Therefore, the bivariate analysis provides virtually no evidence to contravene Hypothesis 1.

		War	
		0	1
Ultimatum	0	295 (86%)	49 (14%)
	1	54 (87%)	8 (13%)

Table 1: Cross-tabulation of *Ultimatum* and *War*. Row percentages in parentheses.

[25] The Appendix reports auxiliary analyses of two other dependent variables; the results are similar.

	Dependent Variable	
	War (H1)	Satisfaction (H2)
Ultimatum	−0.62 (0.45)	−0.24 (0.21)
Military-Security Issue	0.71 (0.50)	−0.36 (0.19)
Major Power Involvement	1.20* (0.33)	−0.09 (0.16)
Heterogeneity	0.07 (0.13)	0.04 (0.06)
Proximity: Near	−1.00 (0.63)	−0.10 (0.23)
Proximity: Distant	−0.89 (0.47)	0.16 (0.22)
Power Disparity	−0.13 (0.36)	0.20 (0.18)
Intercept	−2.93 (0.61)	3.92 (0.25)
Method	Logit	Linear
Log-Likelihood	−150.88	−709.79
Observations	400	400

Table 2: Regression analyses of Hypothesis 1 and Hypothesis 2. $*$: $p < 0.05$.

Our regression analyses, the results of which appear in Table 2, provide further support for our hypotheses. The regression with *War* as the dependent variable, which pertains to Hypothesis 1, appears in the first column. Consistent with our hypothesis, the coefficient on *Ultimatum* is statistically indiscernible from zero at any conventional significance level ($p = 0.17$). The null result is consistent with the idea that states choose optimal bargaining protocols, and that the risk of war under an optimal protocol is equivalent to the risk of war engendered by the best ultimatum offer. To be clear, our results are not so strong that we can conclusively rule out any meaningful effect (see Rainey 2014). But, by the same token, we find no conclusive evidence against the hypothesis of no effect, and the results accord better with our hypothesis than with the standard view that ultimata increase the risk of war.

Similarly, our results with *Satisfaction* as the dependent variable are consistent with Hypothesis 2. Remember that *Satisfaction* is our rough proxy for the distribution of benefits after the crisis. If countries in crises employed bargaining protocols that were not outcome-equivalent to making an ultimatum, then issuing an ultimatum would predict a different distribution of benefits—and thus potentially different levels of satisfaction with that distribution. On the contrary, we find that the coefficient on *Ultimatum* is statistically indiscernible from zero ($p = 0.24$). We cannot detect an effect of issuing an ultimatum on crisis participants' evaluation of the outcome, which is consistent with the idea that the distribution of outcomes does not differ across the set of bargaining protocols that countries would choose to employ.

7 Conclusion

We have shown that many complex bargaining protocols cannot achieve any more than a simple take-it-or-leave-it offer. Similarly, we have proven that if a country could unilaterally choose the protocol, it cannot do better than making an ultimatum. For applied formal theorists, the upshot of these findings is that crisis bargaining can be modeled as a simple ultimatum game with surprisingly little loss of generality. As an optimal mechanism, the ultimatum game can be motivated as the reduced form of a game in which the bargaining protocol is chosen endogenously. Moreover, more complex bargaining protocols in the class we study would not produce qualitatively different results. In an empirical extension, we find no evidence of a correlation between the issuance of an ultimatum and the probability that a crisis escalates into war, and we show how our formal analysis can explain this otherwise puzzling result.

According to our findings, scholars who are primarily interested in crisis outcomes need not worry about the multiplicity of potential bargaining protocols. The ultimatum protocol is relatively tractable to work with, yet the range of outcomes attainable by take-it-or-leave-it bargaining includes all possible equilibrium outcomes of a wide class of crisis bargaining games. Moreover, the ultimatum game is what a state would choose if it could set the bargaining protocol unilaterally. Being both simple in setup and general in applicability, the ultimatum game is in fact an ideal workhorse model for bargaining theories of conflict.

Our findings also suggest directions for further empirical study. The logic behind our hypotheses extends beyond ultimata: if the bargaining protocol is selected endogenously, then *no* observable aspect of that protocol should be related to the risk of war or terms of settlement, as every optimal mechanism is outcome-equivalent. For example, whether states begin bargaining by making concessions or by listing demands should not systematically predict the risk of war. Nor should the choice of whether to tackle all issues at once or to go one issue at a time. These predictions hold only insofar as the choice of protocol is endogenous to crisis participants, not if it is imposed by a third party such as a mediator.

References

Acharya, Avidit and Edoardo Grillo. 2015. "War with Crazy Types." *Political Science Research and Methods* 3(2):281–307.

Ausubel, Lawrence M and Raymond J Deneckere. 1993. "Efficient Sequential Bargaining." *The Review of Economic Studies* 60(2):435–461.

Banks, Jeffrey S. 1990. "Equilibrium Behavior in Crisis Bargaining Games." *American Journal of Political Science* 34(3):599–614.

Bester, Helmut and Karl Wärneryd. 2006. "Conflict and the Social Contract." *The Scandinavian Journal of Economics* 108(2):231–249.

Brecher, Michael and Jonathan Wilkenfeld. 2000. *A Study of Crisis*. Ann Arbor: University of Michigan Press.

Bremer, Stuart A. 1992. "Dangerous Dyads: Conditions Affecting the Likelihood of Interstate War, 1816-1965." *The Journal of Conflict Resolution* 36(2):309–341.

Dal Bó, Ernesto and Robert Powell. 2009. "A Model of Spoils Politics." *American Journal of Political Science* 53(1):207–222.

Fearon, James D. 1995. "Rationalist Explanations for War." *International Organization* 49(3):379–414.

Fearon, James D. 1997. "Signaling Foreign Policy Interests: Tying Hands versus Sinking Costs." *The Journal of Conflict Resolution* 41(1):68–90.

Fey, Mark, Adam Meirowitz and Kristopher W Ramsay. 2013. "Credibility and Commitment in Crisis Bargaining." *Political Science Research and Methods* 1(1):27–52.

Fey, Mark and Kristopher W. Ramsay. 2009. "Mechanism Design Goes to War: Peaceful Outcomes with Interdependent and Correlated Types." *Review of Economic Design* 13(3):233.

Fey, Mark and Kristopher W. Ramsay. 2010. "When Is Shuttle Diplomacy Worth the Commute? Information Sharing through Mediation." *World Politics* 62(4):529–560.

Fey, Mark and Kristopher W Ramsay. 2011. "Uncertainty and Incentives in Crisis Bargaining: Game-Free Analysis of International Conflict." *American Journal of Political Science* 55(1):149–169.

Filson, Darren and Suzanne Werner. 2002. "A Bargaining Model of War and Peace: Anticipating the Onset, Duration, and Outcome of War." *American Journal of Political Science* pp. 819–837.

Hörner, Johannes, Massimo Morelli and Francesco Squintani. 2015. "Mediation and Peace." *The Review of Economic Studies* 82(4):1483–1501.

Huth, Paul K. 1988. *Extended Deterrence and the Prevention of War.* New Haven: Yale University Press.

Lauren, Paul Gordon. 1972. "Ultimata and Coercive Diplomacy." *International Studies Quarterly* 16(2):131–165.

Leventoğlu, Bahar. 2012. "Crisis Bargaining without Mediation." Typescript, Duke University.
URL: *http://people.duke.edu/ bl38/articles/CrisisBargaingWithoutMediation.pdf*

Leventoğlu, Bahar and Ahmer Tarar. 2008. "Does Private Information Lead to Delay or War in Crisis Bargaining?" *International Studies Quarterly* 52(3):533–553.

McKibben, Heather Elko. 2013. "The Effects of Structures and Power on State Bargaining Strategies." *American Journal of Political Science* 57(2):411–427.

Meirowitz, Adam and Anne E Sartori. 2008. "Strategic Uncertainty as a Cause of War." *Quarterly Journal of Political Science* 3(4):327–352.

Merlo, Antonio and Charles Wilson. 1995. "A Stochastic Model of Sequential Bargaining with Complete Information." *Econometrica* pp. 371–399.

Myerson, Roger B. 1979. "Incentive Compatibility and the Bargaining Problem." *Econometrica* 47(1):61–73.

Myerson, Roger B. 1981. "Optimal Auction Design." *Mathematics of Operations Research* 6(1):58–73.

Myerson, Roger B. and Mark A. Satterthwaite. 1983. "Efficient Mechansims for Bilateral Trading." *Journal of Economic Theory* 29(2):265–281.

Osborne, Martin J and Ariel Rubinstein. 1990. *Bargaining and Markets.* San Diego: Academic Press.

Powell, Robert. 1996. "Bargaining in the Shadow of Power." *Games and Economic Behavior* 15(2):255–289.

Powell, Robert. 2015. "Nuclear Brinkmanship, Limited War, and Military Power." *International Organization* 69(3):589–626.

Rainey, Carlisle. 2014. "Arguing for a Negligible Effect." *American Journal of Political Science* 58(4):1083–1091.

Ramsay, Kristopher W. 2011. "Cheap Talk Diplomacy, Voluntary Negotiations, and Variable Bargaining Power." *International Studies Quarterly* 55(4):1003–1023.

Riley, John and Richard Zeckhauser. 1983. "Optimal Selling Strategies: When to Haggle, When to Hold Firm." *The Quarterly Journal of Economics* 98(2):267–289.

Rubinstein, Ariel. 1982. "Perfect Equilibrium in a Bargaining Model." *Econometrica* 50(1):97.

Schelling, Thomas C. 1966. *Arms and Influence*. New Haven and London: Yale University Press.

Schwarz, Michael and Konstantin Sonin. 2007. "A Theory of Brinkmanship, Conflicts, and Commitments." *The Journal of Law, Economics, & Organization* 24(1):163–183.

Slantchev, Branislav L. 2003. "The Principle of Convergence in Wartime Negotiations." *American Political Science Review* 97(4):621–632.

Snyder, Glenn Herald and Paul Diesing. 1977. *Conflict Among Nations*. Princeton: Princeton University Press.

Vickrey, William. 1961. "Counterspeculation, Auctions, and Competitive Sealed Tenders." *The Journal of Finance* 16(1):8–37.

Wagner, R Harrison. 2000. "Bargaining and War." *American Journal of Political Science* pp. 469–484.

Wilkenfeld, Jonathan, Kathleen Young, David M Quinn and Victor Asal. 2005. *Mediating International Crises*. New York: Routledge.

Williams, Steven R. 1987. "Efficient Performance in Two Agent Bargaining." *Journal of Economic Theory* 41(1):154–172.

Wittman, Donald. 2009. "Bargaining in the Shadow of War: When Is a Peaceful Resolution Most Likely?" *American Journal of Political Science* 53(3):588–602.

A Online Appendix

A.1 Equivalence

A.1.1 Proof of Lemma 2

We first state a helpful lemma that allows us to further break down the problem. We follow Banks (1990) and show that an incentive compatible direct mechanism can be fully described by π and the expected utility of country 2's strongest type, $U_2(\bar{t})$. In other words, once we know π and $U_2(\bar{t})$, we can ascertain the outcome of the direct mechanism for every type of country 2, as the settlement values $v_2(t)$ for types that do not fight for certain are pinned down by these two components.

Lemma 4. *Suppose (π, v_2) satisfies incentive compatibility. For all $t \in T$,*

$$(1 - \pi(t))v_2(t) = U_2(\bar{t}) - \pi(t)w_2(t) - \int_t^{\bar{t}} w_2'(s)\pi(s)\,ds. \tag{8}$$

Proof. By the envelope theorem (Banks 1990, Lemma 6), $dU_2(t)/dt = (dw_2(t)/dt)\pi(t)$. Integrating both sides from t to $\bar{t}$ gives

$$U_2(\bar{t}) - U_2(t) = \int_t^{\bar{t}} w_2'(s)\pi(s)\,ds.$$

Substituting in the definition of $U_2(t)$ gives

$$U_2(\bar{t}) - [\pi(t)w_2(t) + (1 - \pi(t))v_2(t)] = \int_t^{\bar{t}} w_2'(s)\pi(s)\,ds.$$

After rearranging terms, this yields (8). □

We can now prove Lemma 2.

Lemma 2. *For every incentive compatible certainty mechanism (π, v_2), the values of U_1, $U_2(t)$, and $v_2(t)$ depend only on Π when $\Pi > 0$ and depend only on $v_2(\bar{t})$ when $\Pi = 0$.*

Proof. Let (π, v_2) be an incentive compatible certainty mechanism with $\Pi > 0$. Then π is a step function with step $t^W = F^{-1}(1 - \Pi)$, and $U_2(\bar{t}) = w_2(\bar{t})$. The value of settlement to types of country 2 that do not fight is

$$v_2(t) = w_2(\bar{t}) - \int_{F^{-1}(1-\Pi)}^{\bar{t}} w_2'(s)\,ds = w_2(F^{-1}(1 - \Pi))$$

by Lemma 4. Country 1's expected utility is thus

$$U_1 = (1 - \Pi)(1 - w_2(F^{-1}(1 - \Pi))) + \int_{F^{-1}(1-\Pi)}^{\bar{t}} w_1(t)\, dF(t), \tag{9}$$

and country 2's payoff is

$$U_2(t) = \begin{cases} w_2(F^{-1}(1 - \Pi)) & t \leq F^{-1}(1 - \Pi), \\ w_2(t) & t > F^{-1}(1 - \Pi). \end{cases}$$

Examining each of these expressions, we see that the direct mechanism (π, v_2) enters into U_1 and $U_2(t)$ only through Π.

Now let (π, v_2) be an incentive compatible certainty mechanism in which $\Pi = 0$. Since war never occurs, $\pi(t) = 0$ for all t. The value of settlement to each type of country 2 is

$$v_2(t) = U_2(\bar{t}) = v_2(\bar{t})$$

by Lemma 4. It is immediate that $U_2(t) = v_2(\bar{t})$ for all t and that $U_1 = 1 - v_2(\bar{t})$. Therefore, the direct mechanism (π, v_2) enters into the payoffs only through $v_2(\bar{t})$. □

A.1.2 Proof of Lemma 3

As defined in the text, given an ICVA certainty mechanism (π, v_2), the equivalent ultimatum offer is a strategy profile of the ultimatum game in which country 1 offers the same terms of settlement and country 2 best responds. Because each type of country 2 is best-responding, the equivalent ultimatum outcome is itself an incentive compatible direct mechanism. The following proof relies on this fact in invoking Lemma 2.

Lemma 3. *Let* (π, v_2) *be an ICVA certainty mechanism with* ex ante *probability of war* Π. *The terms of settlement satisfy* $v_2(\underline{t}) \geq w_2(\bar{t})$ *if* $\Pi = 0$ *and* $v_2(\underline{t}) = w_2(F^{-1}(1 - \Pi))$ *if* $\Pi > 0$. *The equivalent ultimatum offer, with* $x = 1 - v_2(\underline{t})$, *is outcome-equivalent to* (π, v_2). *Country 1's* ex ante *expected utility from* (π, v_2) *is* $U_1 = \xi_1(x)$.

Proof. Let (π, v_2) be an ICVA certainty mechanism. Suppose $\Pi > 0$, so π is a step function with step $t^W = F^{-1}(1 - \Pi)$. Consider a strategy profile of the ultimatum game where $x = 1 - w_2(t^W)$ and country 2 employs a best response. The probability of war under this strategy profile is

$$\Pr(w_2(t) > 1 - x) = \Pr(t > t^W) = 1 - F(t^W) = \Pi.$$

Therefore, by Lemma 2, this strategy profile is equivalent to the original mechanism. Equation (9) simplifies to $U_1 = \xi_1(x)$ in this case.

Alternatively, suppose $\Pi = 0$ under the original mechanism, and consider a strategy profile of the ultimatum game where $x = 1 - v_2(\bar{t})$ and country 2 employs a best response. Voluntary agreements implies $v_2(\bar{t}) \geq w_2(\bar{t}) \geq w_2(t)$ for all t, so the offer will be accepted with probability 1. The strategy profile is therefore outcome-equivalent to the original mechanism, again by Lemma 2. As $x \leq 1 - w_2(\bar{t})$, $U_1 = x = \xi_1(x)$. □

A.2 Optimality

A.2.1 Proof of Proposition 4

We begin with two intermediate results. The first builds on Lemma 4: because π and $U_2(\bar{t})$ fully determine the outcomes of an incentive compatible direct mechanism, it is immediate that they also determine each country's utility under such a mechanism. This lemma expresses the expected utility of country 1 in a convenient form.

Lemma 5. *Suppose* (π, v_2) *satisfies incentive compatibility. Country 1's expected utility is*

$$U_1 = 1 - U_2(\bar{t}) + \int_T \pi(t)[w_2'(t)F(t) - (1 - w_1(t) - w_2(t))f(t)]\,dt. \tag{10}$$

Proof. Because $v_1(t) + v_2(t) = 1$, it follows that $(1-\pi(t))v_1(t) = (1-\pi(t)) - (1-\pi(t))v_2(t)$. Therefore, using Lemma 4 and substituting into (3), we have

$$\begin{aligned} U_1 &= \int_T \left[\pi(t)w_1(t) + (1-\pi(t)) - U_2(\bar{t}) + \pi(t)w_2(t) + \int_t^{\bar{t}} w_2'(s)\pi(s)\,ds\right] dF(t) \\ &= 1 - U_2(\bar{t}) - \int_T (1 - w_1(t) - w_2(t))\pi(t)f(t)\,dt + \int_T \int_t^{\bar{t}} w_2'(s)\pi(s)f(t)\,ds\,dt. \end{aligned}$$

Examining the last term in this expression, we see that

$$\begin{aligned} \int_T \int_t^{\bar{t}} w_2'(s)\pi(s)f(t)\,ds\,dt &= \int_T \int_{\underline{t}}^{s} w_2'(s)\pi(s)f(t)\,dt\,ds \\ &= \int_T w_2'(s)\pi(s)\left[\int_{\underline{t}}^{s} f(t)\,dt\right] ds \\ &= \int_T w_2'(s)\pi(s)F(s)\,ds. \end{aligned}$$

The expected utility of country 1 can therefore be written as (10). □

The next intermediate result draws from the Second Mean Value Theorem for Integrals, which we restate here:

Theorem 1. *Suppose $g : [a,b] \to \mathbb{R}$ is weakly increasing and $h : [a,b] \to \mathbb{R}$ is integrable. Then there exists $z \in [a,b]$ such that*

$$\int_a^b g(x)h(x)\,dx = g(a)\int_a^z h(x)\,dx + g(b)\int_z^b h(x)\,dx.$$

In what follows, we suppose that $T = [\underline{t}, \bar{t}] \subset \mathbb{R}$ and F is a cumulative distribution function on T with density $f(t)$.

Lemma 6. *Suppose $\pi : T \to [0,1]$ is weakly increasing and $h : T \to \mathbb{R}$ is integrable. Then there exists $t^* \in T$ such that the function*

$$\pi^*(t) = \begin{cases} 0 & t < t^*, \\ 1 & t > t^* \end{cases}$$

satisfies

$$\int_T \pi(t)h(t)\,dF(t) \leq \int_T \pi^*(t)h(t)\,dF(t).$$

Proof. To begin, define a function π' by $\pi'(\underline{t}) = 0$, $\pi'(\bar{t}) = 1$, and $\pi'(t) = \pi(t)$ for all $t \in (\underline{t}, \bar{t})$. Clearly, this function is weakly increasing and

$$\int_T \pi(t)h(t)\,dF(t) = \int_T \pi'(t)h(t)\,dF(t).$$

We can write

$$\int_T \pi'(t)h(t)\,dF(t) = \int_T \pi'(t)h(t)f(t)\,dt$$

and $h(t)f(t)$ is integrable, so we can use Theorem 1 to write

$$\begin{aligned}\int_T \pi'(t)h(t)\,dF(t) &= \pi'(\underline{t})\int_{\underline{t}}^z h(t)f(t)\,dt + \pi'(\bar{t})\int_z^{\bar{t}} h(t)f(t)\,dt \\ &= \int_z^{\bar{t}} h(t)f(t)\,dt.\end{aligned}$$

for some $z \in T$.

There are two cases. If $\int_z^{\bar{t}} h(t)f(t)\,dt < 0$, then setting $t^* = \bar{t}$ we have

$$\int_T \pi(t)h(t)\,dF(t) = \int_z^{\bar{t}} h(t)f(t)\,dt < 0 = \int_T \pi^*(t)h(t)\,dF(t).$$

On the other hand, if $\int_z^{\bar{t}} h(t)f(t)\,dt \geq 0$, then setting $t^* = z$ we have

$$\int_T \pi(t)h(t)\,dF(t) = \int_z^{\bar{t}} h(t)f(t)\,dt = \int_T \pi^*(t)h(t)f(t)\,dt = \int_T \pi^*(t)h(t)\,dF(t).$$

This completes the proof. □

Proposition 4. *The ultimatum game is an optimal mechanism for country 1.*

Proof. We begin by showing there exists an ICVA certainty mechanism that gives country 1 the highest payoff among all ICVA certainty mechanisms. By Lemma 3, every ICVA certainty mechanism is equivalent to a strategy profile of the ultimatum game in which country 1 offers x, country 2 employs a best response, and country 1 receives a utility of $\xi_1(x)$. Now pick an equilibrium offer x^* of the ultimatum game and denote the corresponding ICVA mechanism by (π^*, v_2^*). As x^* is a maximizer of $\xi_1(x)$, it follows (π^*, v_2^*) gives country 1 the highest possible payoff among all ICVA certainty mechanisms.

To complete the proof, we must show that (π^*, v_2^*) is optimal among all ICVA mechanisms. Suppose not. That is, suppose (π', v_2') is an ICVA mechanism with utility U_1' to country 1 with $U_1' > \xi_1(x^*)$. As this mechanism is not certainty, it must have expected probability of war $\Pi' > 0$. By Lemma 5, we have $U_1' = 1 - U_2'(\bar{t}) + L(\pi')$, where

$$L(\pi) = \int_T \pi(t)[w_2'(t)F(t) - (1 - w_1(t) - w_2(t))f(t)]\,dt.$$

Voluntary agreements implies $U_2'(\bar{t}) \geq w_2(\bar{t})$. Because $h(t) = w_2'(t)F(t) - (1 - w_1(t) - w_2(t))f(t)$ is integrable, by Lemma 6 there exists certainty mechanism π'' with $L(\pi'') \geq L(\pi')$. Assigning $v_2'' = w_2(F^{-1}(1 - \Pi''))$, it is clear that this a IC certainty mechanism and that $U_2''(\bar{t}) = w_2(\bar{t})$. Moreover, by Lemma 3, $U_1'' = \xi_1(x'')$ for some x''. Putting these things together, we have

$$U_1' = 1 - U_2'(\bar{t}) + L(\pi') \leq 1 - w_2(\bar{t}) + L(\pi') \leq 1 - U_2''(\bar{t}) + L(\pi'') = U_1''.$$

But this gives $\xi_1(x'') = U_1'' \geq U_1' > \xi_1(x^*)$, which contradicts the fact that x^* is a maximizer of $\xi_1(x)$. This proves that (π^*, v_2^*) is an optimal ICVA mechanism. □

A.3 Bargaining with Costly Delay

A.3.1 Proof of Proposition 5

We consider a sequential offer crisis bargaining game as defined in the text. We denote such a sequential offer crisis bargaining game by G. A public history of length τ is simply a list of offers that were made in periods $1, \ldots, \tau$ as well as the response of the recipient. Here, we include a history of length 0, which is simply the empty set. To prove Proposition 5, we obtain three helpful lemmas on dynamic crisis bargaining. We first identify some general properties of equilibria in the class of sequential offer crisis bargaining games. Our first lemma indicates that after every possible history, both on and off the equilibrium path, fighting can only occur in the initial period of the continuation game.

Lemma 7. *In every pure strategy equilibrium of every sequential offer crisis bargaining game, for every $\tau \geq 1$ and every history of length $\tau - 1$, in the continuation game beginning with period τ, fighting can occur with positive probability only in period τ.*

Proof. Suppose there is a pure strategy equilibrium and a history of length $\tau - 1$ such that fighting occurs in the continuation game at some period $\tau' > \tau$. Depending of whether country 1 or country 2 makes the offer in this period, this means that for some type t of country 2 either this type is choosing to fight in response to an offer or making an offer that is resulting in fighting. In either case, the continuation value in this period is no greater than $w_2(t)$. So in period $\tau' - 1$, reaching the next period is worth no more than $\delta w_2(t)$. But as type t can choose to fight in period $\tau' - 1 \geq \tau$, which gives a payoff of $w_2(t)$, it cannot be optimal to for this type to reach period τ'. This contradicts the supposition that period τ' is reached by the equilibrium strategies. □

This lemma means that, on the equilibrium path, fighting can only occur in the initial period and not in any later period. The reasoning is simply that because delay is costly, a type of country 2 that knows it will fight later can be strictly better off by fighting in the initial period.

This result has several useful implications. First, it implies that after every history, the continuation value for all types of continuing the game must be equal. This follows from the fact that the types only differ in their war payoff. Therefore, as fighting does not occur on the path of play later in the game, a standard incentive compatibility argument that a given type can mimic the actions of another type and receive that type's payoff yields the desired implication. The second implication of the lemma is especially useful because it implies that if the initial offer of country 1 in period 1 is rejected and play moves to the second period, the game has a unique outcome that is independent of the offer made in the first period.

Lemma 8. *In every pure strategy equilibrium of every sequential offer crisis bargaining game, for every initial offer $(x, 1 - x)$, if the game does not end with probability one in the first period, then it ends in the second period with agreement on the division $(d, 1 - d)$, which depends does not depend on the initial offer $(x, 1 - x)$.*

Proof. Fix a sequential offer crisis bargaining game G. Suppose there is a pure strategy equilibrium and an initial offer $(x, 1-x)$ such that there is a positive probability that this offer is rejected and play moves to the second period. By the previous lemma, we know that fighting cannot occur on the equilibrium path in this period or some later period.

Now consider a complete information Γ with the same sequence of proposers as G, the same (discounted) payoffs from agreement, and the same actions except that neither side can ever choose war. In other words, Γ is the same game as G but in each period a proposer makes an offer which can be accepted, ending the game, or rejected, in which case the game moves to the next period. Now consider the subgame of Γ beginning at the second period after the offer $(x, 1-x)$ has been rejected. It follows from Merlo and Wilson (1995) that the unique equilibrium outcome of this subgame is immediate agreement on a division $(d, 1-d)$ that depends on only the discount factor δ and the order in which offers are made. As these do not depend on the initial offer $(x, 1-x)$, the value of d does not depend on this value. Now return to the game G and consider the second period after $(x, 1-x)$ is rejected in the first period. As fighting does not occur on the equilibrium path from this period onward in G, it follows that the equilibrium strategies given by the unique equilibrium of Γ also form an equilibrium to G. Moreover, every equilibrium strategy profile for starting in period 2 will correspond to an equilibrium of Γ because fighting is never chosen on the equilibrium path and so these same strategies will be optimal in a game in which fighting is not an option. In other words, equilibrium play from period 2 onwards in G must be identical to a game in which fighting is not an option. Finally, because there is a unique equilibrium in Γ, there must be a unique equilibrium for this subgame in G. In other words, the unique equilibrium outcome in G in period 2 after $(x, 1-x)$ is rejected is immediate agreement on $(d, 1-d)$, which does not depend on the value of x. $\square$

This characterization of equilibrium play allows us to prove our main result for sequential offer crisis bargaining games. This result states that, even allowing for discounting and delay, all pure strategy equilibria of sequential offer crisis bargaining games are equivalent to certainty mechanisms. To be clear, a certainty mechanism in this context is one in which almost all types either accept the offer made in period 1 or fight in period 1. In particular, a certainty mechanism is "no delay" in that the bargaining ends with probability one in the first period.

Lemma 9. *Every pure strategy equilibrium of every sequential offer crisis bargaining game corresponds to a certainty mechanism.*

Proof. Fix a pure strategy equilibrium of a sequential offer crisis bargaining game and let x^* be the offer made in period 1 in this equilibrium. Let T_A be the set of types of country 2 that accept the offer x^*, let T_R be the set of types of that reject the offer without fighting, and let T_F be the set of types who fight in the first period. It is sufficient for our result to prove that the set T_R has measure zero according to F. To prove this, suppose not. That is, assume the set T_R has positive probability according to F.

From the previous lemma, we know that every type in T_R will agree to the division $(d, 1-d)$ in the second period, giving all of these types an expected payoff of $\delta(1-d)$ in the first period. In order to make this optimal, it must be that $1 - x^* \leq \delta(1-d)$, which is equivalent to $x^* \geq 1 - \delta(1-d)$. Note as well that if $x^* > 1 - \delta(1-d)$, then no types of country 2 will accept the offer x^*. In other words, T_A has positive probability only if $x^* = 1 - \delta(1-d)$. Therefore, we can write the equilibrium utility of country 1 as

$$u_1(x^*) = \delta d P(T_R) + (1 - \delta(1-d))P(T_A) + w_1 P(T_F).$$

In order for it be optimal for country 1 to offer x^* instead of choosing fight, it must be that $u_1(x^*) \geq w_1$. After simplification, this is equivalent to

$$(\delta d - w_1)P(T_R) + (1 - \delta + \delta d - w_1)P(T_A) \geq 0.$$

Clearly, then, it must be that $1 - \delta(1-d) > w_1$.

Now consider an arbitrary $x' \in (w_1, 1 - \delta(1-d))$. If country 1 were to deviate to offer $(x', 1-x')$ in the first period, then all types in T_A and T_R would accept this offer for sure and all types in T_F would either accept this offer or continue to choose to fight. We thus have that

$$u_1(x') \geq x'(P(T_R) + P(T_A)) + w_1 P(T_F).$$

As $P(T_R) > 0$, we can choose x' such that

$$x'(P(T_R) + P(T_A)) > \delta d P(T_R) + (1 - \delta(1-d))P(T_A).$$

But this implies that $u_1(x') > u_1(x^*)$, which contradicts the assumption that x^* is the equilibrium offer. Therefore it must be that the set T_R has probability zero, so that all equilibria correspond to certainty mechanisms. □

This lemma is all that we need to show that our earlier equivalence result continues to apply to bargaining with costly delay, which gives Proposition 5.

A.3.2 Optimality with Inefficiency

Here we prove the optimality of the one-shot ultimatum game even when we allow for inefficient settlement outcomes.

Proposition 6. *An optimal bargaining game with discounting for country 1 is to make a take-it-or-leave-it offer to country 2.*

Proof. The argument is follows very similar lines to that presented in the previous section. In particular, it is easy to derive the following expression for country 1's expected utility for an incentive compatible direct mechanism:

$$U_1 = 1 - U_2(\bar{t}) + \int_T \pi(t)[w_2'(t)F(t) - (1 - w_1(t) - w_2(t))f(t)]\,dt \\ - \int_T d(t)[1 - \pi(t)]f(t)\,dt. \quad (11)$$

From the arguments in the previous section, we know that the top line of this expression is maximized by choosing a step function for π that corresponds to the one-shot ultimatum game. But it is easy to see that the bottom line in this expression is always nonpositive, so it is maximized by setting $d(t) = 0$, for example. So the one-shot ultimatum game maximizes the overall expression. □

A.3.3 Credibility of an Ultimatum

We define the dynamic ultimatum game to be a sequential offer crisis bargaining game, as previously defined, in which country 1 makes the offer in every period. To be clear, in each period country 1 makes an offer which country 2 can accept, reject, or choose to fight. If country 2 rejects an offer, then play moves to the next period. We also give country 1 the ability to choose to fight, in line with the logic of voluntary agreements. If country 2 accepts the offer made in a period, the game ends and if either side chooses to fight, the game ends.

We make the same assumptions as before about uncertainty. That is, country 1's war payoff w_1 is common knowledge and country 2's war payoff is given by $w_2(t)$, with $w_1+w_2(t) < 1$ for all t. In addition, we assume a common discount factor $\delta \in (0, 1)$. Finally, we assume that the equilibrium offer x^* of the one-shot ultimatum game is unique.

Proposition 7. *The dynamic ultimatum game has a unique equilibrium outcome, which is the same outcome as the one-shot ultimatum game.*

Proof. Consider an arbitrary offer $(x, 1 - x)$ in the first round. By Lemma 8, if the game does not end with probability one after this offer, then agreement is reached immediately on the division $(1, 0)$ in the second period, as this is the unique equilibrium agreement of the game without fighting. But no type would prefer to reject $1 - x$ in favor of receiving a payoff of zero in the next period. So for all offers, the game ends with probability one in the first period. Therefore, in equilibrium, every type of country 2 plays the same response to x in this game as it would in response to offer x in the one-shot ultimatum game. This implies that the expected utility of country 1 for a given offer is the same as in the one-shot ultimatum game. Therefore, the equilibrium behavior of both sides in this game is the same as in the ultimatum game. □

A.4 Empirical Analysis

A.4.1 Variable Definitions and Descriptive Statistics

All variables are coded using version 10.0 of the International Crisis Behavior project's system-level dataset.

Military-Security Issue Coded as 1 if `ISSUES` equals 3 (military-security issue alone), 4 (two issues, including military-security), or 5 (three or more issues); and as 0 otherwise.

Major Power Involvement Coded as 1 if any of `GPINV` (Great Power Involvement in Crisis), `GPINVTP` (Content of Great Power Activity as Third Parties), or `POWINV` (US and USSR/Russia Joint Involvement in Crisis) exceeds 2; and as 0 otherwise.

Heterogeneity Coded as `HETERO` − 1, so as to equal the absolute number of attribute differences between adversarial actors.

Proximity Identical to `GEOGREL` (Geographic Proximity of Principal Adversaries).

Power Disparity Coded as 1 if `POWDISSY` (Power Discrepancy) exceeds 1, and as 0 otherwise, as in Wilkenfeld et al. (2005, p. 111).

War Coded as 1 if `VIOL` (Violence) equals 4 (full-scale war), and as 0 otherwise.

Intensity Identical to `VIOL` (Violence).

Satisfaction Coded as 6 − `EXSAT` (Extent of Satisfaction about Outcome), so that higher values represent greater satisfaction. Cases where `EXSAT` equals 6 (single-actor case) or 7 (no adversarial actor) are coded using `OUTEVL` (Extent of Satisfaction about Outcome) in the actor-level data.

Recurrence Coded as 1 if `OUTESR` (Escalation or Reduction of Tension) equals 1 (tension escalation) and as 0 if `OUTESR` equals 2 (tension reduction). Cases where `OUTESR` equals 3 (recent case) are dropped.

Variable	Min	Max	Mean	SD
Independent Variables				
Ultimatum	0	1	0.15	0.36
Military-Security Issue	0	1	0.83	0.38
Major Power Involvement	0	1	0.51	0.50
Heterogeneity	0	4	2.48	1.36
Proximity: Contiguous	0	1	0.72	0.45
Proximity: Near	0	1	0.11	0.31
Proximity: Distant	0	1	0.17	0.37
Power Disparity	0	1	0.76	0.43
Dependent Variables				
War	0	1	0.14	0.35
Intensity	1	4	2.25	1.02
Satisfaction	1	5	3.80	1.44
Recurrence	0	1	0.44	0.50

Table 3: Summary statistics for all variables used in the analysis of crisis data.

A.4.2 Auxiliary Analyses

As an auxiliary analysis, we examine two more dependent variables pertaining to crisis outcomes. The first of these is *Recurrence*, an indicator for whether another crisis between the same actors occurred within five years of the end of the crisis in question.[26] The second is *Intensity*, which measures the level of violence in the crisis on a scale from 1 (no violence) to 4 (full-scale war). We use the same covariates as before.

Table 4 reports the results of these auxiliary analyses. The coefficient on *Ultimatum* in the *Recurrence* equation is, once again, statistically indiscernible from zero ($p = 0.65$). Perhaps more surprising are the results for *Intensity*, which measures the greatest level of violence used in a crisis. We estimate that crises in which an ultimatum is issued score about 0.4 points (approximately 0.4 standard deviations) less than comparable crises without ultimata on the four-point scale of the level of violence. This finding, in combination with our results for full-scale war, suggests that there is less low-level violence in crises with ultimata. We suspect that this may be because issuing an ultimatum brings a crisis to a head, forcing

[26] We lose 18 cases when crisis recurrence is the dependent variable because the measure is unavailable for crises after 2002.

	Dependent Variable	
	Recurrence	Intensity
Ultimatum	0.14 (0.30)	-0.39^* (0.14)
Military-Security Issue	0.36 (0.29)	0.16 (0.13)
Major Power Involvement	-0.11 (0.23)	0.26^* (0.11)
Heterogeneity	0.26^* (0.09)	0.07 (0.04)
Proximity: Near	-0.88^* (0.37)	-0.50^* (0.16)
Proximity: Distant	-0.30 (0.31)	-0.38^* (0.15)
Power Disparity	0.12 (0.26)	-0.19 (0.12)
Intercept	-1.12 (0.37)	2.13 (0.17)
Method	Logit	Linear
Log-Likelihood	-252.42	-559.23
Observations	382	400

Table 4: Results of auxiliary analyses. $*$: $p < 0.05$.

the states involved to reach an agreement quickly or else face war.[27] A state that issues an ultimatum is putting all of its cards on the table—there is neither time nor reason to "sink costs" through low-level military action (Schelling 1966; Fearon 1997).

[27]Crises with ultimata are indeed resolved more quickly: they last 134 days on average, compared to 168 days for crises without ultimata.

A.4.3 Crises with Ultimata

Number	Name	Years	Issued By	Issued To
6	Hungarian War	1919	Supreme Council	HUN
10	Bessarabia	1919–1920	RUS	ROM
19	Costa Rica-Panama Border	1921	USA	COS, PAN
20	German Reparations	1921	UKG, FRN	GMY
21	Karl's Return To Hungary	1921	CZE, YUG	HUN
28	Corfu Incident	1923	ITA	GRC
31	Mosul Land Dispute	1924	UKG	TUR
35	Shantung	1927–1929	JPN	CHN
38	Chinese Eastern Railway	1929	RUS	CHN
40	Shanghai	1932	JPN	CHN
43	Jehol Campaign	1933	JPN	CHN
45	Austria Putsch	1934	ITA	GMY
53	Alexandretta	1936–1939	TUR	FRN
57	Postage Stamp Crisis	1937	HON	NIC
60	Anschluss	1938	GMY	AUS
61	Polish Ultimatum	1938	POL	LIT
62	Czechoslovakia May Crisis	1938	UKG, FRN, CZE	GMY
64	Munich	1938	RUS	POL
68	Czechoslovakia's Annexation	1939	GMY	CZE
69	Memel	1939	GMY	LIT
70	Danzig	1939	GMY	POL
71	Invasion Of Albania	1939	CZE	ALB
74	Entry Into Wwii	1939	GMY	POL
75	Soviet Occupation Of The Baltic	1939	RUS	LAT, EST
80	Romanian Territory	1940	RUS	ROM
102	Communism In Romania	1945	RUS	ROM
104	Trieste 1	1945	UKG, USA	YUG
105	French Forces-Syria	1945	UKG	FRN
106	Kars-Ardahan	1945–1946	RUS	TUR
115	Marshall Plan	1947	RUS	CZE
131	Soviet Bloc-Yugoslavia	1949–1951	RUS	YUG
151	Goa 1	1955	POR	IND
152	Suez Nationalization-War	1956–1957	RUS	UKG, FRN, ISR
184	Bizerta	1961	TUN	FRN
190	Goa 1	1961	IND	POR
198	Dominican Republic-Haiti 1	1963	DOM	HAI

227	Prague Spring	1968	RUS, POL, HUN, BUL	CZE
234	Shatt-Al-Arab 1	1969	LIB	IRQ
239	Cienfuegos Submarine Base	1970	USA	RUS
250	Zambia Raid	1973	ZAM	Rhodesia
259	Mayaguez	1975	USA	CAM
266	Uganda Claims	1976	KEN	UGA
274	Poplar Tree	1976	USA	PRK
315	Solidarity	1980–1981	RUS	POL
323	Mozambique Raid	1981	RUS	African National Congress
330	Gulf Of Syrte 1	1981	LIB	USA
362	Chad-Libya 1	1986	FRN	LIB
367	Mozambique Ultimatum	1986	MZM	MAW
376	Aegean Sea 1	1987	GRC	TUR
393	Gulf War	1990–1991	USA	IRQ
403	Yugoslavia Ii: Bosnia	1992–1995	NATO	Bosnian Serbs
404	Papua New Guinea-Solomon	1992	SOL	PNG
409	Operation Accountability	1993	ISR	SYR
411	Haiti Military Regime	1994	USA	HAI
412	Iraq Troop Deployment-Kuwait	1994	USA, UKG	IRQ
423	Cyprus-Turkey Missile Crisis	1998	TUR	CYP
427	Us Embassy Bombings	1998	USA	AFG
428	Syria-Turkey	1998	TUR	SYR
429	Unscom Ii Operation Desert Fox	1998	USA	IRQ
430	Kosovo	1999	NATO	YUG
437	Myanmar-Thailand	2002	THI	MYA
440	Iraq Regime Change	2002–2003	USA	IRQ

Table 5: The set of International Crisis Behavior cases in which an ultimatum is issued, according to our coding rule.