1 Chapter 28 review exercises

1. (a) To test the null hypothesis that the average of the box is 2, you would use the one-sample z-test.

   (b) To test the null hypothesis that the box is $\boxed{1}$ $\boxed{2}$ $\boxed{3}$, you would use the χ^2-test with a null hypothesis that tells you the contents of the box.

2. The expected frequencies for a sample of 62 from the county population are

Education	Expected frequency
Elementary	$62 \times 28.4\% = 17.6$
Secondary	$62 \times 48.5\% = 30.1$
Some college	$62 \times 11.9\% = 7.4$
College degree	$62 \times 11.2\% = 6.9$

The χ^2 statistic for the observed sample is therefore

$$\begin{aligned}\chi^2 &= \sum \frac{(\text{observed} - \text{expected})^2}{\text{expected}} \\ &= \frac{(1-17.6)^2}{17.6} + \frac{(10-30.1)^2}{30.1} + \frac{(16-7.4)^2}{7.4} + \frac{(35-6.9)^2}{6.9} \\ &= 153.51.\end{aligned}$$

The degrees of freedom for this problem is 3, which is equal to the number of categories minus 1. Looking at the chi-square table in the row for 3 degrees of freedom, our test statistic of 153.51 vastly exceeds the critical value of 11.34 for significance at the 1% level. Therefore, the best answer is (ii), that this distribution of jurors is possible but fantastically unlikely (under the null hypothesis that the jury is a simple random sample from the county population).

5. (a) If $\chi^2 = 15$, the p-value will be bigger with 10 d.f. than with 5 d.f.

   (b) If there are 10 d.f, the p-value will be bigger with $\chi^2 = 15$ than with $\chi^2 = 20$.

6. Under the null hypothesis that the dice are fair, the expected frequencies in 360 rolls are

Sum	Expected frequency
2	10
3	20
4	30
5	40
6	50
7	60
8	50
9	40
10	30
11	20
12	10

The χ^2 statistic for the observed sample of 360 rolls is therefore

$$\chi^2 = \frac{(11-10)^2}{10} + \frac{(18-20)^2}{20} + \frac{(33-30)^2}{30} + \frac{(41-40)^2}{40} + \frac{(47-50)^2}{50} + \frac{(61-60)^2}{60} + \frac{(52-50)^2}{50} + \frac{(43-40)^2}{40} + \frac{(29-30)^2}{30} + \frac{(17-20)^2}{20} + \frac{(8-10)^2}{10} = 2.01.$$

The degrees of freedom for this problem is 10, since the model is fully specified and there are 11 terms summed in the χ^2 statistic. Our statistic of $\chi^2 = 2.01$ corresponds to a very high P-value, even greater than 99%.

This is like Fischer's examination of Mendel's genetics data (see FPP, p. 533–534) — the numbers are too good to be true. Remember that, with 10 degrees of freedom, the χ^2 statistic we would expect if the null hypothesis were true is $\chi^2 = 10$. The observed statistic is much lower, hence the high P-value. Therefore, we conclude that the dice are rigged, and you should not play craps with them.

2 Chapter 29 review exercises

3. The statement is false. Because the sample size of the second investigator is 9 times the sample size of the first, with the same SD, the second investigator's SE will be 1/3 the size of the first's. Recall that the test statistic for a z-test is

$$z = \frac{\text{observed} - \text{expected}}{\text{SE}}.$$

   The larger the z-score is, the lower the P-value. Since the second investigator has a lower SE, he could get a larger z-score (and hence a lower P-value) with a smaller observed deviation from 50 than the first. Therefore, the given statement is false.

6. There are two problems with the investigators' conclusions. First, statistical significance is not equal to practical importance. Although we cannot statistically distinguish it from zero, we can say that "if inflation has an effect, it contributes to the Republican vote."

   Another problem here is that the tests were run on the whole population and not on a sample. There is no box model, so the test statistics are not meaningful.

8. (a) As stated, the question makes sense. The Current Population Survey is a probability sample, so there is an underlying box model.

   (b) We cannot answer it based on the information given, because we need to know the sample sizes and the survey design in order to compute standard errors.

   (c) If the percentages came from independent simple random samples of 50,000, then we can use the usual significance test for a difference in percentages. The null hypothesis is that there is no difference between employment rate between 1985 and 2005; the alternative hypothesis is that there is a difference (two-sided). The SEs of the individual samples are

$$\mathrm{SE}_{1985} = \frac{\sqrt{50000} \times \sqrt{.50 \times .50}}{50000} \times 100\% = 0.224\%$$
$$\mathrm{SE}_{2005} = \frac{\sqrt{50000} \times \sqrt{.59 \times .41}}{50000} \times 100\% = 0.220\%$$

Since the samples are independent, we can use the square root law to calculate the SE of the difference:

$$\mathrm{SE}_{\text{difference}} = \sqrt{(\mathrm{SE}_{1985})^2 + (\mathrm{SE}_{2005})^2} = 0.314\%.$$

The test statistic is

$$\frac{9\% - 0\%}{0.314\%} = 28.7.$$

This test statistic is far beyond the reach of the z-table, and corresponds to a P-value extremely close to 0. Therefore, we conclude that women's employment was indeed greater in 2005 than in 1985.

9. (a) The question makes sense, given that multistage cluster sampling fits a probability model. Most people would agree that the finding is important.

   (b) With two valid sample percentages we can test whether the difference is significant. However, we need to know the sample sizes and the details of the sampling procedure (including sample sizes) in order to be able to calculate test statistics and assess statistical significance.

   (c) With two valid sample percentages calculated from independent simple random samples, we can test whether the difference is statistically significant. To do so with the given information, we use the significance test for a difference in percentages, just as in problem 8c. The null hypothesis is that there is no difference between the percent of students that thought "being very well off financially is very important or essential" in 1970 and in 2000; the alternative hypothesis is that there is a difference (two-sided). The SEs of the individual samples are

$$\mathrm{SE}_{1970} = \frac{\sqrt{1000} \times \sqrt{.36 \times .64}}{1000} \times 100\% = 1.5\%$$
$$\mathrm{SE}_{2000} = \frac{\sqrt{1000} \times \sqrt{.74 \times .26}}{1000} \times 100\% = 1.4\%$$

Since the samples are independent, we can use the square root law to calculate the SE of the difference:

$$\mathrm{SE}_{\text{difference}} = \sqrt{(\mathrm{SE}_{1970})^2 + (\mathrm{SE}_{2000})^2} = 2.05\%.$$

The test statistic is

$$\frac{38\% - 0\%}{2.05\%} = 18.5.$$

This test statistic is far beyond the reach of the z-table, and corresponds to a P-value extremely close to 0. Therefore, we conclude that the percentage of students who thought being well off was important was indeed greater in 2000 than in 1970.