Risky but Rational:
War as an Institutionally-Induced Gamble*

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Abstract

We present and process-trace a complete information model of diversionary war. In our model, leaders must retain the support of some fraction of a selectorate whose response in turn depends on the outcome of an international conflict. The need to retain the loyalty of a segment of the selectorate generates *institutionally-induced risk preferences* in leaders. Under specified conditions, this in turn results in the leaders’ choice of risky options and war emerges as a *rational gamble*. We analyze when the leader prefers to impose such a gamble, what the optimal gamble would be, and its effect on crisis bargaining between two leaders. We find that when leaders have institutionally-induced risk preferences, whether leaders rationally choose to initiate or continue a war can depend on which selectorate cares most about the outcome of the conflict. A reexamination of the bargaining over a settlement to end the First World War in December 1916–January 1917 and Germany’s contemporaneous consideration of unrestricted submarine warfare allows us to demonstrate the empirical relevance of the model.
Risk-taking is a central topic in both formal and more traditional studies of international conflict and war. Indeed, as the prospects of victory in war are never certain, the study of war is necessarily the study of risk-taking. However, how and why states’ or leaders’ incentives to take risk vary and how they interact with other incentives remain understudied questions. To broaden the current literature’s perspective on risk-taking and throw new light on the diversionary use of force, in this paper we present a formal model of international bargaining and conflict with domestic audiences. Our goal is to examine the origins and consequences of leaders’ and states’ attitudes towards risk and international conflict.

The model we present has three distinctive features. First, we include domestic political institutions in which leaders must retain the support of some fraction of the selectorate in order to remain in power and investigate the effect of these institutions on the incentives of leaders to take risky actions. Thus, a distinctive feature of our model is the *institutionally-induced risk preferences* of leaders. Specifically, if leaders prefer to stay in power and the outcome of the crisis can affect their chances to stay in power, we argue, then for some range of outcomes—where a small improvement or worsening has a large effect on whether a leader stays in office—leaders face increasing marginal returns and will therefore have institutionally-induced risk-seeking preferences; for other ranges of outcomes leaders are risk-averse. As the leaders of the two sides only care about remaining in power, but their selectorates care about the outcome of the crisis, the risk preferences of the leaders are induced by domestic political institutions at work. Notably, we do not assume that leaders prefer or avoid risk for psychological reasons; rather the risk preferences of leaders are institutionally-induced.\(^1\)

The second distinctive feature of our model is that we allow both leaders the option of imposing a *rational gamble*. By this, we mean that both leaders can choose actions to deliberately introduce risk into the outcome of a crisis, an idea that dates back to Schelling (1960, 1966). For instance, a leader could introduce a new weapon or strategy or concentrate his forces for an attack on one front, leaving another front vulnerable to counterattack. We formalize this idea in our model by allowing leaders to choose a random

\(^1\)In international relations, risk-taking has most often been analyzed from the perspective of psychological theories. Thus, psychologists have relied on various theories to examine how risk tolerance and risk aversion can influence leaders’ decisions (Hermann and Hagan, 1998; Satterfield, 1998). Within this literature, prospect theory—which hypothesizes that individuals (leaders) are risk-averse in the ‘domain of gains’ and risk-seeking in the ‘domain of losses’ (Levy, 1996; Taliaferro, 2004)—has made the most inroads into the study of international relations. Unfortunately, international relations scholars have had difficulty with a fundamental attribute of prospect theory: *ex ante* specification of the ‘reference point’, the point where the domain of losses ends and the domain of gains begins. As a result, prospect theory has proven difficult to rigorously test.
variable whose outcome might be significantly worse or significantly better than the known outcome. For simplicity, we suppose such rational gambles have a fixed mean, but our results would easily extend to cases in which leaders faced a mean-variance tradeoff in choosing among rational gambles.

The third distinctive feature of our model is that it is a complete information game. Although our model is a principal-agent problem in the broadest sense because the selectorate and the leader have different interests, it does not focus on incomplete information as the cause of the problem. In this way our model differs from the formal models of diversionary war by Richards et al. (1993), Downs and Rocke (1994), Hess and Orphanides (1995), Smith (1996), Bueno de Mesquita et al. (1999) and Tarar (2006). Instead, the tension between the selectorate and the leader in our model lies in the fact that the former group cannot commit to not remove leaders that do poorly in wartime or not reward those who are victorious. In our view, this is a reasonable assumption about political behavior, particularly given the close connection between the military and the selectorate in many countries.

Our model combines these three features with a simple crisis bargaining model. The first side can either offer a settlement or choose a rational gamble. If a settlement was offered, the second side can accept the offer or reject the offer and impose a rational gamble. If the first side chose to gamble, on the other hand, the second side has the option of making the resulting gamble more risky, but not less risky. In solving the game, we characterize the optimal gamble for each side and show that conflict results when an international opponent—even an institutionally-induced risk-averse opponent—is better off choosing the optimal gamble than paying the bargaining premium required to avoid war. Crucial to our findings is a measure of how important the war is to the selectorate of each country. We show that in some circumstances, whether or not war ensues depends on whether or not the war outcome is more important to the selectorate of the leader who can make an offer than it is to his opponent’s selectorate.

The best-known work in formal theory on leaders’ incentives to take risks is Downs and Rocke (1994) and thus it is useful to briefly contrast our model with their approach. Their model (as well as Richards et al. (1993)) is driven by the incomplete information of citizens which leads to monitoring problems in a classic principal-agent setup. Specifically, the leader has private information about the outcome of a potential international conflict and his type, adventurous or timid, which can be conceptualized as the leader’s attitude toward risk. Voters respond to this monitoring problem in a way that creates incentives for certain types of leaders to make risky choices. Our model differs from this approach in several important respects. First, our model has complete information and, as a result, we
demonstrate that incomplete information is not necessary to explain risk-taking in crises. Second, our model is placed in an international context and thus explores the connections between domestic politics and international bargaining. Finally, our results do not depend on the “timidness” of leaders. In our model, the preferences of leaders are induced by domestic political institutions rather than assumed to be shaped by some exogenous “taste” parameter.

Our model is also richer than the straightforward observation that a sufficiently highly risk-seeking leader prefers a high variance conflict to a certain peace (Fearon, 1995). First, our model goes beyond this observation by deriving the induced risk preferences of leaders, rather than assuming them. Second, this observation refers to leaders that are sufficiently risk-seeking over the entire range of outcomes. In our model, the institutionally-induced risk preferences of leaders are risk-seeking over some outcomes and risk-averse over others. Third, our model moves beyond the risk preferences of leaders to show how their choices depend on the importance of the war to their selectorates and the location of the status quo.

Our model is also theoretically distinct from the canonical rationalist explanations of war as the result of private information (Fearon, 1995) or commitment problems (Powell, 2006) because it is a complete information model with a commitment problem between the selectorate and the leader, rather than between the two opposing leaders. In proposing this new theoretical argument for risky choices, however, we must be careful to recognize that existing theories could also explain risky choices—a reckless gamble could be used to signal a leader’s resolve, for example. Therefore, we must demonstrate that our argument based on institutionally-induced risk preferences provides a better explanation for some important wars. For example, explanations based on informational and monitoring problems or commitment problems provide a very poor account of the process that brought the United States into the First (and perhaps also the Second) World War. Our theory of risk-taking however, explains when and why leaders manipulate risk to their advantage both before and during war. As our analysis of war-time bargaining during the First World War will show, by the end of 1916 the German political leadership was so desperate for a “sufficient peace” that they launched unrestricted submarine warfare in the full knowledge that this would probably bring the United States into the war. Moreover, fully aware of the danger of the German submarine weapon, in the last months of 1916 Great Britain was unwilling to offer Germany terms that might have ended the war and obviated the German need to gamble. Neither monitoring problems or private information about the leader’s adventurousness provide satisfactory explanations for this behavior, a fact which sets our model apart from
previous explanations.

We proceed in two sections. In the theoretical section, we present a model of crisis bargaining to examine how the manipulation of risk affects the strategic interaction between opposing leaders. In the empirical section we present a detailed case study of the German decision to launch unrestricted submarine warfare during the First World War. Our complete information model allows us to closely trace the proposed mechanism; thus we avoid the problem inherent in many incomplete information models which rely on unobservables.

The Model

In this section, we develop a model of crisis bargaining that incorporates institutionally-induced risk preferences of leaders and the ability to gamble. We begin by deriving from first principles the shape of the utility functions for the leaders involved in the dispute. Specifically, we will show how the interaction of the leader and his supporters creates institutionally-induced risk preferences. We then incorporate these induced utility functions into a bargaining game and characterize the equilibrium of this game.

We consider the leaders of two countries involved in a dispute. The set of possible outcomes of the dispute is represented by $O = [0, 1]$. Within this range, larger values correspond to outcomes that are more favorable to country 1 and smaller values correspond to outcomes that are more favorable to country 2. In each country, there is a set of $n$ individuals, which we call the selectorate as in Bueno de Mesquita et al. (2003). In country $i$, the probability that a member of the selectorate will support the current leader is given by a function $s_i(x) : O \rightarrow [0, 1]$. We suppose that the current leader will be retained if more than half of the selectorate supports her and will be removed if less than half of the selectorate supports her.\footnote{This majority requirement is an obvious, if arbitrary, choice. This choice is not critical, though, as it is easy to see that our findings would continue to hold with a different support threshold.} For simplicity, we suppose that the leaders of both countries care only about holding office and do not care directly about the outcome of the dispute.\footnote{Again, this assumption can be weakened to permit leaders to care about both office and outcome without significantly changing our results.} Thus, we normalize the value of retaining office at 1 and the value of removal at 0. It should be noted that because the end result that each leader cares about is dichotomous (retain office or removal), each leader is necessarily risk neutral over these end results.

Given our assumptions, then, the number of supporters in country $i$ will be distributed as a binomial random variable with mean $ns_i(x)$. We denote this random variable by $V_i$. The leader of country $i$ cares only about holding office and, as just discussed, the leader is
risk neutral over retaining office and removal. But because the probability that the leader is removed or retained depends on the probability of majority support which, in turn, depends on the outcome of the dispute, the leader indirectly cares about the outcome as well. In this way, leaders in our model have institutionally-induced risk preferences over the outcome of the dispute. As is standard, these risk preferences reflect whether or not the leader prefers a risky choice involving several possible outcomes to its mean value with certainty.\(^4\) Of particular importance is the fact that, in the model, these risk preferences are the result of the political behavior of the selectorate, rather than an assumption about the preferences of the leader. To derive the formal expression for these preferences, we use the well-known normal approximation to the binomial distribution.\(^5\) Specifically, the \textit{induced utility function} for the leader of country \(i\) is given by

\[
    u_i(x) = 1 \cdot P[V_i \geq n/2] + 0 \cdot P[V_i < n/2] \\
    = P[V_i \geq n/2] \\
    \simeq 1 - \Phi\left[\frac{n/2 - ns_i(x)}{\sqrt{n}}\right] \\
    \simeq 1 - \Phi\left[\sqrt{n}(1/2 - s_i(x))\right] \\
    \simeq \Phi\left[\sqrt{n}(s_i(x) - 1/2)\right],
\]

where \(\Phi[\cdot]\) is the cumulative distribution function of a standard normal.

In order to proceed, we need to consider the form of the functions \(s_i(x)\) for \(i = 1, 2\). For simplicity, we assume that these functions are linear; that is, \(s_1(x) = \beta_1 x\) and \(s_2(x) = \beta_2(1 - x)\), where \(\beta_1 > 0\) and \(\beta_2 > 0\). The parameter \(\beta_i\) measures how important the policy outcome \(x\) is to the selectorate of country \(i\). As only the ratio of these two parameters affects our results, we set \(\beta_1 = 1\) and write \(s_2(x) = \alpha(1 - x)\) with \(\alpha = \beta_2/\beta_1\) and \(\alpha > 1/2\).\(^6\) Thus, \(\alpha\) measures the \textit{relative importance} of the war outcome for country 2. The larger \(\alpha\) is, the less important the war outcome is for 2 relative to 1. To see this, note that \((1 - x)\) is the share for country 2 and thus, if \(\alpha\) is large, even a small share leads to a high probability of support for the leader of country 2. If \(\alpha\) is small, the selectorate requires a much larger share in order to give the same probability of support.

As illustrated in Figure 1, these assumptions imply that the induced utility functions

\(^4\)Note that this comparison involves the mean value of the uncertain choice and thus differs from models in which players mix over several strategies, as in models in which a weak type “bluffs” between two strategies.

\(^5\)Specifically, we will use the fact that \(V_i - ns_i(x)/\sqrt{n}\) converges to a unit normal, as \(n\) gets large.

\(^6\)Technically, when \(\alpha > 1\), we should adjust \(p_2(x)\) appropriately in order to insure that this function properly represents a probability. However, it turns out this adjustment is inconsequential to our results.
of both leaders have a logistic or “S-shape”; marginal utility is increasing at first and subsequently decreasing. Thus, the institutionally-induced risk preferences of the leaders are that they are risk-seeking for undesirable outcomes and risk-averse for desirable outcomes. Fearon (1995, 388) has argued that risk-acceptant leaders are rare. His argument, however, refers to leaders that are risk-acceptant over the entire range of outcomes. Yet individuals and leaders who are risk-acceptant under some, but not all, circumstances do not appear to be rare. Indeed, this assumption is supported by some of the early experimental work on the measurement of utility by Friedman and Savage (1948) and Mosteller and Nogee (1967). Moreover, Bueno de Mesquita, Siverson and Woller (1992); Bueno de Mesquita and Siverson (1995); and Chiozza and Goemans (2003, 2004) have recently shown that the outcome of the war can affect the tenure and political position of the leadership and ruling elite. If leaders and the ruling elite value being in power highly, as seems likely, then it seems likewise plausible that the leadership’s marginal utility increases as the outcome of the war improves and thereby increases their chances of staying in power. Only when the leadership’s tenure is safe will their marginal utility decrease with further increases in the outcome of the war.

In our model, we combine these institutionally-induced risk preferences with the ability
to rationally gamble. By “gambling” we mean the ability to introduce deliberate risk into the the outcome of an international crisis. We suppose that the mean outcome of the crisis (or war), $x_0 \in [0, 1]$, is given exogenously. In a crisis $x_0$ represents the status quo and therefore is common knowledge. Alternatively, from the perspective of intra-war bargaining, the assumption that $x_0$ is common knowledge can be interpreted to mean that the combatants have fought the war for a sufficient period to reveal to their opponents their private information about their strength, cost tolerance and that the combatants can now reliably estimate and agree on the expected costs of the war (Fearon, 1995; Goemans, 2000).

Formally, a gamble by country $i$ is any random variable $X_i$ on $[0, 1]$ with mean equal to $x_0$. Note that we include in this definition a degenerate gamble that puts probability 1 on the value $x_0$. For a given gamble $X_i$, it will be useful later to refer to the corresponding mean zero gamble $\tilde{X}_i = X_i - x_0$. Using this notation for a given gamble $X_i$, we say that leader $i$ has institutionally-induced risk-seeking preferences over $X_i$ if the leader prefers the gamble $X_i$ to the degenerate gamble that puts probability 1 on the value $x_0$ and we say that leader $i$ has institutionally-induced risk-averse preferences if the leader prefers the value $x_0$ with certainty to the gamble $X_i$.

The substantive meaning behind the notion that leaders can pick a rational gamble is that political and military leaders can generate risk by designing and choosing particular military (and political-diplomatic) strategies. The notion that leaders can deliberately generate risk can of course be traced back to Schelling and maybe even Sun-Tzu (Schelling, 1960, 1966). But beyond the possible introduction of nuclear weapons, there subsequently has been little work on how leaders can practically generate such risk during war. There would seem to be two general ways to generate risk. First, leaders can generate risk during war by making a known trade-off. An example would be to strip the front of soldiers in one area to mass them to break through elsewhere. This maneuver obviously generates an increased risk that the enemy breaks through one’s own lines along the demurred front. The risks of such a maneuver can be calculated with the help of well-known force-to-space or force-to-force models (Biddle et al., 1991; Taylor, 1980). Second, leaders can try something new on a continuum, such as the use of gas shells at sea, or, as Schelling suggested in situations of compellence or deterrence, start a (new) limited war to generate the risk of a larger war (Schelling, 1960, 1966).

We now present a simple bargaining game of an international crisis (or war) that incorporates institutionally-induced risk preferences and the ability to rationally gamble. The game consists of two stages. In the first stage, the leader of country 1 chooses between making an offer $z \in [0, 1]$ to country 2 or imposing a rational gamble $X_1$ with mean $x_0$. In
the second stage of the game, the choices available to the leader of country 2 depend on the action chosen in the first stage. If the leader of country 1 made an offer $z$, then the leader of country 2 can either accept this offer, resulting in utilities $u_1(z)$ and $u_2(z)$ for the two leaders, or reject the offer and impose a rational gamble $X_2$ with mean $x_0$. On the other hand, if the leader of country 1 chose a rational gamble $X_1$ in the first stage of the game, then the leader of country 2 chooses a mean zero gamble $\tilde{X}_2$ and the resulting outcome is drawn from the combined gamble $W = X_1 + \tilde{X}_2$. A key feature of this model is that both sides have the ability to rationally gamble—country 1 can choose to impose a gamble instead of making an offer and country 2 has the option to reject the offer and impose a gamble instead. Moreover, if both countries choose to gamble, their choices are combined in an overall gamble. In this way, each country always has the ability to make the outcome of the game more risky and therefore the outcome of the war is partly endogenously determined. Loosely speaking, the model basically interprets the question of war termination as a choice of variance: zero variance for offers of settlement that will be accepted, high variance otherwise.

As this is a game of complete information, we can solve for the set of subgame perfect
equilibria by backwards induction.\footnote{As is standard, we assume that if a leader is indifferent between accepting and rejecting an offer, then she accepts the offer. In addition, if a leader is indifferent over making several possible proposals, we assume she makes an offer that will be accepted.} To proceed with the solution, let $X^*_i$ be an optimal gamble with mean $x_0$ for country $i$ as described in Appendix A, keeping in mind that the optimal gamble may be the degenerate lottery with probability 1 on $x_0$.\footnote{With our assumptions about the induced utility functions of the leaders of both countries, it is clear that a unique optimal gamble exists.} Consider first the situation in which the leader of country 1 has made an offer $z$ and the leader of country 2 must decide whether to accept the offer or reject it and impose an optimal gamble $X^*_2$. For the leader of country 2, the expected utility for such a gamble is $Eu_2(X^*_2)$ and therefore the leader of country 2 will accept an offer of $z$ if $u_2(z) \geq Eu_2(X^*)$ and reject the offer otherwise. In other words, the largest offer that the leader of country 2 will accept is the value $z^*$ that satisfies $u_2(z^*) = Eu_2(X^*)$.\footnote{As the expected utility of the optimal gamble $Eu_2(X^*_2)$ is well-defined and $u_2$ is strictly decreasing, a unique solution $z^*$ exists.} These values are illustrated in Figure 2. The second possibility for stage two of the game is that the leader of country 1 has imposed a gamble $X_1$. In this case, the choice of the mean zero gamble $X_2$ depends on the values of certain parameters as we explain below.

Moving back to the first stage of the game, we consider the decision of country 1’s leader. We first examine the possibility that the leader makes an offer $z$ and calculate what such an offer would be. Clearly, country 1’s leader will never make an offer less than $z^*$ and any offer greater than $z^*$ will be rejected. The question is whether the leader of country 1 is better off by offering $z^*$ or forcing country 2 to rationally gamble. Clearly, the optimal action is to offer $z = z^*$ if $u_1(z^*) \geq Eu_1(X^*_2)$ and to offer some $z > z^*$ (which will be rejected) otherwise. The second possibility is that the leader of country 1 imposes a rational gamble $X_1$. As the optimal choice here depends on the anticipated reaction of the leader of country 2, the optimal choice for country 1’s leader depends on the values of the parameters of the model as we explain below.

To deal with this, we next consider several cases. The simplest case to consider is when $s_2(x_0) > 1/2$. In this situation, the utility function of the leader of country 2 is concave around $x_0$ and thus the optimal gamble $X^*_2$ is simply the degenerate lottery that puts probability one on $x_0$. Moreover, if country 1’s leader chooses a gamble $X_1$, the optimal mean zero gamble $X_2$ for country 2 is simply a degenerate lottery that puts probability one on 0. Given these choices in stage two of the game, the leader of country 1 will anticipate that any offer greater than $x_0$ will be rejected and so the resulting outcome of the optimal offer will be $x_0$. On the other hand, if the leader of country 1 imposes a gamble $X_1$,
the leader knows that $\tilde{X}_2 = 0$ and so the resulting outcome is $W = X_1$. Therefore, the equilibrium outcome in this case is the optimal gamble $X^*_1$, because either $Eu_1(X^*_1) > u_1(x_0)$ or $X^*_1 = x_0$. Intuitively, in this case, the current state of the war $x_0$ is favorable to country 2 and so there is no reason for the leader of country 2 to make the outcome more risky. Because $x_0$ is favorable to country 2, there is no offer that the leader of country 1 prefers to $x_0$ and that country 2 is willing to accept. In addition, the leader of country 1 prefers to make a rational gamble and the leader of country 2 will not add variance to this gamble. Therefore, the equilibrium outcome in this case is $X^*_1$.

![Figure 3: Equilibrium outcome is gamble](image)

The other possible case, namely $s_2(x_0) < 1/2$ is somewhat more complicated. In this situation, the optimal gamble $X^*_2$ for country 2 will be non-degenerate as $u_2$ is convex in a neighborhood of $x_0$. However, in this case, the utility function of the leader of country 1 is concave around $x_0$ and thus the leader of country 1 dislikes risk. In particular, it is never optimal for the leader of country 1 to choose a rational gamble because after the action of the leader of country 2 the outcome of the crisis will be, at best, the optimal gamble $X^*_2$. Therefore, the leader of country 1 will always choose to make an offer $z$, which could be accepted or rejected. In fact, as we illustrate next, both of these outcomes can be equilibrium outcomes. For the first case, suppose that $\alpha < 1$. This case is illustrated in
Figure 3. Here, the support of the optimal gamble for country 2 consists of two points, \( x_1^* \) and 1. As shown in the figure, the leader of player 1 receives relatively little value from \( z^* \), the largest offer that the leader of country 2 will accept, when compared to the expected utility of the optimal gamble, \( Eu_1(X_2^*) \). Therefore, in this example, the equilibrium outcome is for the leader of country 1 to make an unacceptable offer and the leader of country 2 will impose the optimal gamble \( X_2^* \).

Alternatively, suppose that \( \alpha > 1 \), as illustrated in Figure 4. In this case, the value of \( z^* \) is large enough that the leader of country 1 prefers to settle the matter on these terms instead of risking the gamble \( X_2^* \) that would otherwise result. Thus the equilibrium outcome is for the leader of country 1 to propose the offer \( z^* \) which will be accepted by the leader of country 2. Intuitively, for the case represented in Figure 4, the optimal gamble for country 2 occurs over the risk-averse region of 1’s utility function and thus 1 prefers to settle rather than face the uncertainty of the gamble.

Thus, it appears that the critical factor that determines whether the outcome of the game is the gamble \( X_2^* \) or the settlement \( z^* \) is whether \( \alpha \) is greater than or less than one. The following results establishes this claim formally.
Figure 5: Equilibrium outcome when $x_0 = .8$ ($n = 200$)

**Theorem 1.** For sufficiently large $n$, the model has a unique equilibrium outcome, as follows:

- If $s_2(x_0) > 1/2$, then the unique equilibrium outcome is the optimal gamble $X_1^*$.

- If $s_2(x_0) < 1/2$ and $\alpha < 1$, then the unique equilibrium outcome is the optimal gamble $X_2^*$ and if $s_2(x_0) < 1/2$ and $\alpha > 1$, then the unique equilibrium outcome is an accepted offer at $z^*$.

The proof of this theorem is found in Appendix B. A numerical analysis that illustrates the theorem is given in Figure 5. In this figure, the expected utility of each of the choices for the leader of country 1 is plotted as a function of $\alpha$ for the case $s_2(x_0) < 1/2$. Specifically, we graph $u_1(z^*)$ and $Eu_1(X_2^*)$ as $\alpha$ varies. As is clear from the figure, when $\alpha < 1$, $Eu_1(X_2^*) > u_1(z^*)$, resulting in an equilibrium choice of the optimal gamble and when $\alpha > 1$, $u_1(z^*) > Eu_1(X_2^*)$, resulting in an equilibrium choice of the settlement $z^*$.

In the context of international relations, we can give two complementary interpretations of this theorem. In the case of pre-war crisis bargaining, we view the gamble outcome as representing war initiation. This is the sense in which this a model in which war occurs
with complete information. Alternatively, in the case of intra-war bargaining, the gamble outcome reflects a continuation (with a high-variance military strategy) of the war even when the players agree on the expected outcome of the fighting under the existing terms.

In either case, Theorem 1 says that when leaders have institutionally-induced risk preferences, if either conditions favor country 2 or conditions favor country 1 but the war outcome is more important to country 2, then the end result is the risky option of war initiation or continuation with a military higher variance strategy. On the other hand, if conditions favor country 1 and the war outcome is more important to this country, then the end result is a peaceful settlement.

We conclude this section with four observations about our model. First, in this model we have assumed that war is costless. But given that, in equilibrium, the leader of the country imposing a gamble has a strict preference for this gamble, it is clear that our results would carry over to a model in which war is costly to both sides or a model in which the optimal gamble represented a tradeoff between mean and variance. In either case, the logic presented here would give war as a possible equilibrium outcome.

Second, in order to keep the model tractable, we have assumed that a leader may choose any rational gamble, as long as its mean is $x_0$. Clearly, in the real world there could be many constraints on the ability of leaders to formulate and implement a rational gamble. In other words, the precise optimal gamble that we have identified may not be feasible in practice. While we have abstracted away from these issues in this paper, we are confident that the qualitative features of our findings are robust and would extend to such situations. In particular, risk-taking as a result of domestic political institutions leading to war is, we argue, a general claim.

Third, the behavior of voters in our model is captured by the support function $s_i(x)$ and thus voters are not explicitly modeled as players in the game. It may seem that this rules out interesting strategic behavior on the part of voters as, indeed, voters would be better off if they could commit to not punish leaders who begin the game at a disadvantage. However, there is no way for voters to credibly make such a commitment; once a bad crisis outcome occurs, voters have no reason not to remove the leader. Therefore, abstracting away from the details of voter behavior does not significantly limit our findings.

Finally, it should be emphasized that these results assume that the size of the selectorate, $n_i$, is fixed and is the same in both countries. A natural question that arises is how our results differ if the size of the selectorates in the two countries differ. In other words, how do the incentives to gamble or settle differ across different regime types, from autocracy to democracy? We leave this interesting question to further research.
Tracing the Model

It is notoriously hard to subject these kinds of models to empirical tests, largely because they contain unobservable variables. A model such as offered above is difficult to test statistically because of the fine-grained information needed on expectations of the outcome of the war and the utility functions of the players. We therefore limit our aspirations here to a demonstration that the mechanism we identify in the theoretical model operates in one (very important) case.

To trace the model, we examine British (and Allied) and German bargaining over possible terms of peace between mid-December 1916 and 9 January 1917 and the connection with simultaneous internal German deliberations over unrestricted submarine warfare. We begin with the opening peace moves. On 18 December 1916, President Wilson sent all bellicent powers a note suggesting each state its terms of peace (Scott, 1921:12-15). The next day, in his first speech as British Prime Minister, Lloyd George replied and presented Britain’s terms. Aware of the threat of Germany’s potential gamble of unrestricted submarine warfare Lloyd George nevertheless laid down tough terms of settlement. He told the House of Commons: “The only terms on which it is possible for peace to be obtained and maintained in Europe ... [are] complete restitution, full reparation, effectual guarantees” (House of Commons, no date, col.1335). As in the previous year, the main effectual guarantee remained the destruction of Prussian militarism and the removal of “the Prussian military caste,” whose “arrogant spirit” according to Lloyd George, lay at the root of this war (House of Commons, no date, col.1337). British leaders knew that these terms would be unacceptable to Germany (French, 1995, 34-5).

It is hardly surprising that the German leadership would find such terms unacceptable, because they amounted to little less than asking them to sign their own death warrant. Indeed, these terms asked for the one thing that the leadership was fighting to preserve, their exclusionary hold over domestic politics. The Germans, in response, on 9 January 1917 decided to implement their gambling option: unrestricted submarine warfare.

To show that these events of December 1916–January 1917 correspond to the the logic and mechanism of our model, we must show that the leaders expected the outcome of the war to have a bigger impact on Germany’s leaders than on British leaders, e.g., $\alpha < 1$.

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10 On 12 December 1916, German Chancellor Bethmann Hollweg had announced a peace note by the Central Powers. The note drafted by Germany offered no conditions beyond the pious hope that settlement and peace negotiations “would aim to secure the existence, honour, and free development of their peoples,” and lead to a lasting peace (Scott, 1921, 1-3; Stevenson, 1991, 105).

11 Later that same day, Lord Curzon offered more specifics in the House of Lords (War Speeches by British Ministers 1914 - 1916, 1917, 373).
Second, we must show that the expected outcome of the war, $x_0$, would lead to the removal of Germany’s, but not Britain’s leaders. Third, we must show that Britain’s and Germany’s leaders were aware not only of Germany’s ability to gamble but also its ‘magnitude.’ Finally, we need to establish that Germany’s leaders chose to gamble because that was the only way they could achieve an outcome that would allow them to stay in office. Britain’s leaders, on the other hand, preferred to let the Germans gamble rather than pay the premium required to obviate Germany’s gamble. We conclude with a brief section to probe the empirical plausibility of alternative explanations that focus on private information and commitment problems.

**The Outcome of the War and the Fate of the Leaders**

The outcome of the war would have a bigger impact on the fate of Germany’s leaders than on Britain’s leaders ($\alpha < 1$ in the language of our formal model) because Germany’s leaders were unwilling to compromise with and offer political reforms to reward the rising working classes for their sacrifices during the war. Britain’s leaders, in contrast, were willing and indeed did offer and initiate political reforms to reward the rising working class for their sacrifices. As a result, to stay in office, Germany’s leaders needed an outcome of the war sufficient to buy off the working classes. Britain’s leaders, in contrast, could and did use another policy instrument: reform of the franchise.

Since at least the 1880s the old order in the German Empire representing the interests of the industrialists and the landed nobility worried about the rising Socialist threat. To prepare for and wage this coming battle all forces were asked to combine against Socialism. This common front was achieved in the 1890s under the so-called *Sammlungspolitik*, authored by Johannes von Miquel. But in the 1913 Reichstag elections the Socialists won 67 new seats to give them 110 deputies, and thereby became the largest party in the new Reichstag of 406 members. The representatives of the old order, mainly in the bureaucracy and the army, virtually panicked. The essential feature of the domestic political struggle before as well as during the war, is aptly summarized by Gordon Craig (1972, 260):

> There is no doubt that there were many who sincerely believed that no compromise with Social Democracy was possible, but there were more, in the councils of the Conservative, Centre, and National Liberal parties, and the Bund der Landwirte and the Centralverband deutscher Industrieller, who saw that the price of collaboration with the working class was social and political change of a kind that they thought they could not afford.
Unwilling to compromise and initiate any significant domestic political reform, the only way Germany’s leaders could reward the people for their sacrifices was by the outcome, specifically, the spoils, of the war. As all historians of the First World War agree, the German leadership was acutely aware of the need to score a profit on the war, or the old order would be irrevocably changed. Generals, industrialists and intellectuals (Gatzke, 1966, 120) alike warned of the dangers of an “insufficient peace,” which would reward the people for their sacrifices with nothing but increased taxes. Such a peace, it was widely argued, would bring the Monarchy down.

The historian of German war aims L.L. Farrar (1973, 44) argues that already in the first months of the war “German leaders increasingly perceived an inversely proportional relationship between war aims and reforms. A victory and achievement of war aims would obviate demands for reform, while defeat and failure to achieve war aims would result in reform, if not revolution.” (See also Zechlin (1961 & 1963, 1961:271-2).) The conservative forces hoped to buy off the populace through economic gains at the expense of their international enemies. These gains were to take two forms. First, much of the territory to be annexed was intended for colonization by “reliable elements.” In Germany’s demands in the east, as Fischer (1961, 341-51) and von Müller (1959, 122) note “besides annexation, settlement and resettlement [of the current population] stood in the center of all plans regarding the border strips.” In this regard, the Polish Frontier Strip deserves special attention, because it became official policy that this “Strip was to be settled with nationally reliable German elements” (Fischer, 1961, 271). In addition to this sizable strip, Courland was also to be re-settled and the German and Prussian Ministries of the Interior became officially involved with settlement plans in the summer of 1916. Similar plans for settlement and colonization were proposed for annexations in the west. In his famous memorandum of September 1914, Bethmann Hollweg noted with approval the Emperor’s plan to reward “deserving non-commissioned officers and troops” in the form of land grants and settlement of the territories to be annexed from Belgium and France (Basler, 1962, 382).

Second, the conservative forces worried about the post-war consequences of the enormous financial costs of the war. Higher taxes to pay for these costs would almost certainly lead to domestic unrest. These fears were echoed by Pan-Germans, National Liberals, Conservatives, Foreign Minister Jagow, and even the Emperor himself (Dahlin, 1933; Thieme, 1963, 54; Janssen, 1967, 26-8; Scherer and Grunewald, 1966, II:191). Higher taxes could only be avoided by shifting the costs of the war on Germany’s enemies. Germany’s demands therefore included war indemnities.

The hope of the conservative forces that the working class could be bought off by annex-
ations and war reparations was not unfounded. Significant parts of the working class agreed with war aims that included the acquisition of land for settlement and a war indemnity. For example, one association of workers organizations demanded “a peace which will guarantee an indemnity for the sacrifices imposed by the enemy and ... which will offer the working population the opportunity of a secure livelihood and unhindered development” (quoted in Gatzke (1966, 19, 202-3)).

In summary, Germany’s leaders perceived a clear and relatively well-defined threshold outcome of the war, beyond which lurked the abyss in the form of a social revolution. An outcome which failed to reward the people for their sacrifices would threaten not only their power and privileges but their very lives. The alternative, political reforms to reward the people was considered anathema. As Gatzke (1966, 273, emphasis added) notes, Germany’s leaders feared the loss of their privileges perhaps even more than defeat: “to most of the beneficiaries of the Hohenzollern regime, the loss of [their] privileges was at least as vital a threat as the military defeat of their country.”

In contrast, the British political leadership represented, albeit perhaps grudgingly, the interests of the British people to a large extent. While King George V nominally ruled Great Britain, the country was in fact governed by a Cabinet of ministers (although the title of minister was not used until 1916). This cabinet was shaped by the voice and opinions of the Prime Minister who could not serve against the wishes of the House of Commons. The House of Commons, thus, held a central place in the policy making process and every Cabinet and Prime Minister spend considerable amounts of time and effort defending their policies before the House.

It would be misleading, however, to classify pre-war Great Britain as a fully democratic country. While the House of Lords had been stripped of its veto power in 1911 and the House of Commons exercised a great deal of democratic control over the British policy making process, this House was not elected by the equal male franchise let alone universal suffrage. Britain was one of only two countries in Europe at the time with more or less representative governments that did not have manhood suffrage (Hungary was the other) and deliberately discriminated on a class base (Matthew, McKibbin and Kay, 1976, 735).

While Britain’s democracy was thus far from total, the party of the working classes and poor, Labour, had real political power and 42 seats in the 1910 Parliament. Labour was represented in the four standing committees in proportion to its numbers in the House and thereby had real influence. Most important, perhaps, for the representative nature of the British democracy was that the rise of the working class was not seen in zero-sum terms. While the German Oligarchs saw themselves locked in a life or death struggle with the rising
working classes, British policy makers saw that rise as a change in degree, not in kind. If necessary, the British leadership was willing to compromise and give up some of their power, to the smallest possible degree to be sure, to the working classes. One particularly cynical Lord of the Whig party pithily

compared Parliament to a traveler in a sledge, pursued by a band of famished wolves. From time to time, he throws them quarters of venison to distract their attention and keep them back so that, half-satiated, they may be less ferocious when they gain the horse’s head. Of course, it is necessary to husband the venison, and make it last as long as possible by cutting it up into little pieces. Part of our nobility ... meritoriously devote themselves to this ungrateful task (quoted in Le May (1979, 20)).

To summarize, the British political leadership was willing to share power and buy off the rising working classes with significant political concessions. The strongest evidence was provided during the war itself when during 1916 and later in 1917 the government prepared and introduced and passed in 1918 the “Representation of the People Act,” which more than doubled the franchise and extended the right to vote to women over thirty and abolished most of the plural votes. The British political leadership could thus wield two weapons in their struggle to stay in power, the outcome of the war and, perhaps more effectively, domestic political reform.

The Expected Outcome of War at the End of 1916

We now turn to the expectations of the outcome of the war at the end of 1916. Already in November 1914 the German leadership had come to the conclusion they could not win the war against the combined power of their enemies if the Entente remained intact (Scherer and Grunewald, 1962, I, #13: 15-19). However, because December 1916 and January 1917 saw the first public flurry of peace probes, e.g., President Wilson’s offer of mediation and

12Labour leader Henderson warned the Cabinet on the eve of the TUC conference in September 1915 that “if [conscription] was introduced it should be accompanied by universal suffrage, heavy taxes on the rich and the promise that it would be abandoned at the end of the war” (French, 1986, 129). A bill to introduce conscription was presented to the House of Commons in January 1916, the final bill was signed by the King on 25 May 1916. Work on the reform of the franchise started in the Speaker’s Conference of late September 1916 (Turner, 1992, 129). Apparently, Lloyd George had already concluded in early 1916 that a reform of the franchise was called for (Riddell, c1933, 193).

13If the German leadership was worried about its survival so early in the war, we would expect the German leadership to try a succession of bigger and bigger gambles—higher and higher variance strategies—in an attempt to gain the necessary terms of settlement. A good case can be made that German attempts at fomenting revolution in Russia, starting in 1915, the Verdun offensive of 1916, unrestricted submarine warfare
Lloyd George’s statement of Britain’s terms, while at the same time the German leadership weighed the decision whether or not to launch unrestricted U-boat warfare, December 1916–January 1917 is the best period to track the interaction between the (anticipated) outcome of the war and the choice of bargaining and military strategy.

The historical record is clear that at the end of 1916 Germany had become decidedly pessimistic about its chances of victory. Politicians and military leaders agreed that Germany could not hold out much longer. On 9 January, the day the decision in favor of unrestricted submarine warfare was finally made, Bethmann Hollweg explicitly warned that “[w]e should be perfectly certain that, so far as the military situation is concerned, great military strokes are insufficient as such to win the war” (Official German Documents Relating to the World War, 1923, II: 1320-1). Even Ludendorff admitted that “[i]f the war lasted our defeat seemed inevitable . . .” (Ludendorff, 1919, 307, Bethmann Hollweg, 1921, II:133-4). Getting ahead of ourselves just slightly, the pessimism of the Central Powers was clearly expressed by Baron Flotow in his report to the Austro-Hungarian Foreign Minister summarizing “the reasons put forward by the Germans for the justification of the unrestricted U-boat warfare.”

Time is against us and favours the Entente; if, therefore, the Entente can keep up the desire for war there will be still less prospect of our obtaining a peace on our own terms. ... It will be impossible for the Central Powers to continue the war after 1917 with any prospect of success. Peace must, therefore, unless it finally has to be proposed by the enemy, be secured in the course of this year, which means that we must enforce it (Czernin, 1919, 118-124).

While the German leadership was justifiably pessimistic about the outcome of the war, British leaders were confident about eventual victory. While not as optimistic as the French, Lloyd George was hopeful about the upcoming Nivelle offensive (Hankey, 1961, 2: 613). The British Chief of the Imperial General Staff, General Robertson was similarly optimistic about an eventual victory, but warned that victory should not be expected in 1917. When Lloyd George became Prime Minister in December 1916, Robertson (1926, 286-7) wrote him a long memorandum which expressed his confidence in ultimate and decisive victory. “I have no hesitation in saying that we can win if we will only do the right thing. If I

in 1917 and the stormtrooper offensives and the attempted sortie of the High Seas fleet in 1918 are indeed such a succession of high variance strategies. The stubborn persistence after the failure of the first stormtrooper offensive of March 21st, to launch 4 more stormtrooper offensives, fits clearly within a gambling—high variance—strategy. The subsequent offensives simply threw away well-trained and irreplaceable German stormtroopers while the allies were waiting, in Pétain’s words, for “the tanks and the Americans to arrive.”
thought otherwise I would tell you so. ... I have never yet hear any military officer of standing express any other opinion.” Fresh in office, and assured of ultimate victory, Lloyd George could be confident that the outcome of the war as projected at the end of 1916 would allow him to stay in office.

**Germany’s Ability to Gamble**

In their expectations about the outcome of the war, British leaders were well aware of the threat posed by the German submarines. Restricted submarine warfare had recommenced in mid-October 1916 and British leaders were worried about its successes and took the possibility of further U-boat action clearly into account. The Cabinet had been informed on 28 December 1916 that Germany would probably resume unrestricted submarine warfare the next year from March on (Fest, 1972, 291). Prime Minister Lloyd George, Chief of the Imperial General Staff General Robertson, First Sea Lord Admiral Jellicoe and the President of the Board of Trade, Walter Runciman (Guinn, 1965, 166; Robertson, 1926, I: 289; French, 1986, 224; Bourne, 1989, 68) were all well aware of the potential implications. Thus, by December 1916 British leaders were aware of the potential gamble of unrestricted submarine warfare by the Germans (French, 1995, 5).

We now establish that decision to launch unrestricted submarine warfare was a rational gamble for Germany’s leaders. We show that unrestricted submarine warfare was not a superior strategy which would result in a higher expected (mean) outcome of the war. Instead, unrestricted submarine warfare was an *optimal gamble* which would increase the variance of the outcome of the war and thereby make it possible to achieve the required “reasonable,” “sensible” or “sufficient” terms which would keep the German leadership in power. Unrestricted U-boat warfare increased the variance of the outcome of the war because it offered the only chance to win the war and obtain “sufficient” terms, while at the same time it increased the likelihood of American intervention and thereby an even worse defeat.

When Quartermaster-General Ludendorff estimated Germany’s chances for 1917 he compared “the problem of obtaining peace, the chance of defeat without unrestricted submarine warfare and the possibility of victory by means of such a campaign, accompanied by an attack by our surface fleet and a defensive war on land.” As a result of his deliberations, he concluded that “unrestricted submarine warfare was now the only means left to secure a victorious end to the war within a reasonable time” (Ludendorff, 1919, I: 308, 312). In the third week of December 1916, Ludendorff and Hindenburg told Bethmann Hollweg that unrestricted submarine warfare was “the only means to bring the war to a rapid end” (Beth-
mann Hollweg, 1921, II: 130, 133-4). Chancellor Bethmann Hollweg had steadfastly opposed unrestricted submarine warfare as a “Vabanque-spiel”—an all-or-nothing gamble—in which the stakes would be Germany’s existence as a major power and her national future (Birnbaum, 1958, 59-60, see also 346-7). However, Ludendorff and Hindenburg’s conclusion that Germany would probably lose the war without unrestricted submarine warfare now forced the Chancellor’s hand. As Reichstag representative Hanssen (1955, 156) clearly foresaw in his diary entry of 20 December 1916:

> When the Junkers asked Bethmann last spring how he would conclude an acceptable peace for Germany, since he had rejected submarine warfare, he answered by referring to Verdun. But he made no reply when they confronted him with the same question this fall, after the failure of the Verdun offensive. His stand was untenable. In his extremity he thereafter held to the peace offering of December 16th, but on its rejection he was forced to recognize the Kaiser and the generals so completely especially Hindenburg and Ludendorff, that a battle was bound to result. And thus his opposition to submarine warfare has been broken. Unless he yields, his days as Chancellor are numbered.

The connection between “an acceptable peace” or “sufficient” peace terms and U-boat warfare was explicitly made again by Secretary of State for Foreign Affairs Arthur Zimmermann known for his infamous telegram, who remarked in early January 1917: “Show me a way to obtain a reasonable peace and I would be the first to reject the idea of U-boat warfare” (Czernin, 1919, 120). Thus, at the end of December 1916, as Gatzke (1966) notes, “Germany’s ruling class saw in the unrestricted use of the submarine the only means to gain a victorious peace, which in turn, they felt, was necessary to maintain the existing political and social order.”

While unrestricted submarine warfare increased the probability of a “sufficient” peace by forcing the British to the peace table, at the same time it increased the probability of a disaster for Germany by increasing the likelihood of American intervention. The German Ambassador to the United States, Count Bernstorff, warned repeatedly that the United States would enter the war if Germany returned to unrestricted submarine warfare (Fischer, 1961, 368). Not only the Chancellor, but, as has gone largely unnoticed so far, also the military seems to have believed him.14 While the military seems to have underplayed the

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14It might be argued that even without unrestricted submarine warfare, the US would have entered the war—perhaps for real-politik reasons—and hence that unrestricted submarine warfare represents much less of a gamble than we claim it to be. We do not find this argument plausible. First, President Wilson had
likelihood of American intervention in their discussions with political leaders such as Bethmann Hollweg and Minister of the Interior Helfferich, in discussions among themselves they were more frank. On 8 January 1917, the day before the fateful decisions was finally made, Admiral von Holtzendorf, Chief of the General Staff of the Admiralty, Quartermaster General Ludendorff and Marshall von Hindenburg held a meeting to come to an understanding and agreement regarding the inauguration of unrestricted submarine warfare. In his summary of the results of their negotiations, Hindenburg noted: “So we hold together. It must be. We count upon war with America and have taken all precautions” (Birnbaum, 1958, 315). But as all were well aware, America’s entry would drastically change the balance of power between the Entente and the Central Powers. Moreover, Germany’s leaders were fully aware that Britain could take effective countermeasures, more aware and sanguine perhaps than many prominent officers of the British Navy (Official German Documents Relating to the World War, 1923, II: 1154-63).

Unrestricted submarine warfare thus clearly increased the variance of the outcome of the war. On the one hand, it would increase the likelihood of a victory over Britain and “sufficient” peace terms. On the other hand, it would increase the likelihood of a total defeat if Britain could not be defeated before America would mobilize her potential against Germany. The naval authorities told Bethmann Hollweg in early January: “that they may be relied on, even though they are not considered capable of crushing England, at least to be able, before America can come in, so to weaken the British Island Empire that only one desire will be left to English politicians, that of seating themselves with us at the Conference table” (Czernin, 1919, 116; Ludendorff, 1919, I: 249). When the Chancellor objected that success and failure of unrestricted submarine warfare were equally impossible to prove, and asked “in what position should [Germany] find [itself] in case the admirals were mistaken,” the Admiralty gave a revealing reply. Rather than argue for the prospects of success, the Admiralty pointed out the poverty of alternatives when it “promptly asked just been re-elected in November 1916 on a platform of continued armed neutrality and his commitment to keep the US out of the war. It was only unrestricted submarine warfare that forced his hand, and made it possible to deviate from his campaign promises. (See his second inaugural address of 4 March 1917.)

Second, to justify Wilson’s change in policy and assure continued public support for the war, the Committee for Public Information listed six “Official Reasons” (Harding, 1918, 70–71) why the America entered World War I. First and foremost, America entered the war “I. Because of the renewal by Germany of her submarine warfare in a more violent form that ever before, contrary to the assurances given to our Government in the spring of 1916. …” The second reason also invokes unrestricted submarine warfare with its violation of the rights of neutrals. It stated that Germany’s leaders “had repudiated wholesale the commonly accepted principles of law and humanity, and was “running amok” as an international desperado ….” Traditional considerations of the balance of power came only third. The Germans, moreover, certainly did not seem to think that America would enter the war in any case (Stevenson, 1991, 72–73).
what sort of position the Chancellor expected to find when autumn arrived without having
made a proper use of the U-boats and we found ourselves, through exhaustion, compelled to
beg for peace” (Czernin, 1919, 137, emphasis in original). While the Chancellor thus asked
for an explicit evaluation of how unrestricted submarine warfare would affect the terms of
peace in case of success as well as in case of failure, the Admiralty’s evasion reveals they
recognized it constituted a last-gasp gamble.

Others were less reticent in their assessment of the consequences of failure. Bethmann
Hollweg concluded: “should success not materialize, then the worst end awaits us” (Beth-
mann Hollweg, 1921, II: 137). He characterized unrestricted submarine warfare as a “throw
of the dice, for which stakes truly are the existence of Germany” (Birnbaum, 1958, 346-7).15
Other German leaders expressed similar fears (Czernin, 1919, 116; Fischer, 1961, 362, 267).
Ludendorff calculated that “in case U-boat warfare would not work, [America’s entrance
would mean] a serious increase in the enemy’s power and a significant shift in the balance of
forces. It could not be doubted that America, if she entered the war, would arm herself in
the same way that England had done, and that the Entente would call forth from the United
States ever more armaments by virtue of its view and energy” (Ludendorff, 1919, I: 247).
The Secretary of State for the Interior, Karl Helfferich, also expressed that unrestricted
submarine warfare would not improve but rather could worsen the outcome of the war. In
late August 1916 he warned: “I see nothing but catastrophe following the application of
the U-boat weapon at this time. A method which will lead us out of one serious situation
into the toils of another more serious, is not practical if we are not able to adopt counter-
measures for the purpose of rendering the other disadvantageous result [US intervention]
ineffectual (Official German Documents Relating to the World War, 1923, II: 1154-63, em-
phasis added).”16 Even Admiral Holtzendorf expressed similar fears (Birnbaum, 1958, 321,
209, 210, 307). Although Germany’s leaders were thus well aware of the potential dangers
of failure, Ludendorff pointed to the crux of the matter in early 1917, when he warned:
“Don’t think I approach the matter in some sort of jubilant mood. In spite of all promises
by the Navy, I do not indulge in the hope to bring England to her knees by U-boat warfare.
But I certainly hope that the lack of shipping space in England will be so large that it will
be ready for a sensible peace” (Direnberger, 1936, 24). Unrestricted submarine warfare thus
offered the only way to obtain a “sensible peace” and maintain Germany’s elite in power.

15Earlier in 1916 he warned that if unrestricted submarine warfare would bring America and other neutrals
into the war, this would create “a state of affairs, which we will have created ourselves, where the war will
have to be fought until the most bitter end under all circumstances” (Birnbaum, 1958, 60).
16It is likely that the infamous Zimmermann telegram was an attempt to “render the other disadvantageous
result ineffectual.”
On 9 January 1917, in a Crown Council, the Emperor, the military and political leaders crossed the Rubicon and decided to launch unrestricted submarine warfare on 1 February 1917. The German government announced on 31 January it would once again adopt unrestricted submarine warfare. In the month that followed, German submarines sank several American ships and killed American passengers on other ships. On 1 March the infamous Zimmermann telegram was published in the United States and, to the astonishment of the world, acknowledged as genuine by Zimmermann himself two days later. These events left President Wilson with no other option but to ask Congress to declare war on Germany on 2 April 1917 (Lansing, 1917; Shaw, 1918, 372–383).

Competing Explanations

As a final justification of our theoretical argument, we must show that alternative explanations based on private information or commitment problems (Fearon, 1995; Powell, 2006) fail to explain unrestricted submarine warfare and German and British behavior. First, it might be argued that Germany launched unrestricted submarine warfare to signal its strength or resolve.

This competing explanation fails because Germany had launched unrestricted submarine warfare twice before, in February 1915 and March 1916. Both times Germany backed down because of protests and threats of retaliation by the Neutral powers, in particular the United States. Thus, as this signal had already been sent twice, it is extremely unlikely that unrestricted submarine warfare would have provided the British with any new information about Germany’s strength or resolve.

As for the second competing explanation, it could be argued that Germany’s leaders feared that Britain and its other enemies would not abide by the terms of any currently available settlement. Anticipating commitment problems, then, and as insurance, it could be argued that German leaders required significantly better terms; the only way to obtain them was by the high variance strategy of unrestricted submarine warfare. This potential competing explanation fails for two reasons. First, there is no evidence that Germany had suddenly become more pessimistic about the enforceability of any terms of peace. Second, the increased German demands of December 1916 focused mainly on paying off the German people for their sacrifices, and only marginally on protections for a future war in case any of Germany’s enemies reneged. To be sure, at the end of 1916 Germany demanded a “good strategic border” with Russia (Scherer and Grunewald, 1962; Fischer, 1961). But this good strategic border served two purposes; on the one hand, it served to better contain

17 We thank an anonymous reviewer for suggestions about potential competing explanations.
Russia. On the other hand, the additional territory was explicitly intended to reward “loyal” Germans for their sacrifices (Fischer, 1961). The increase in war aims in the west was also largely economic in nature. Belgium would have to pay a higher monthly tribute and hand over the raw materials in the Campine. Crucially, and counter to the alternative explanation centered on commitment problems, at the end of 1916 Germany was willing to guarantee the conditional restoration of Belgium, against opposition from the Army and even though this would make defense against any potential French and British attack much more difficult (Scherer and Grunewald, 1962). Germany even dropped its demand for German “protection” of the Flemish coast (Scherer and Grunewald, 1962; Fischer, 1961). Bethmann Hollweg was willing to trade the conditional restoration of Belgium in return for the return of Germany’s colonies. These colonies contained important economic resources and could strengthen the German economy and provide jobs and better wages to German workers but would be of little help to Germany in a future war. Therefore, the commitment problem explanation also fails to explain Germany’s behavior. In contrast, the consensus among historians is overwhelming that Germany’s leaders needed extensive gains from the war to buy off the people and prevent an overthrow of the old order.

**Gambling in War**

The events of December 1916–January 1917 show that the intuitions behind our model play out as expected. Britain was aware of the danger of a German gamble but nonetheless preferred to let Germany gamble rather than settle on terms most likely far beyond Germany’s ability to obtain on the battlefield. Germany’s leaders, meanwhile, were desperate to secure an outcome of the war which would enable them to stay in power. Almost seventy years ago, Arthur Rosenberg arrived at the same conclusion that the persistent demand of German military and naval leaders for unrestricted submarine warfare and the final consent by politicians is comprehensible only in the light of their need for a peace with victory (Rosenberg, 1928; Birnbaum, 1958).

Of course this example can by no means substitute for a rigorous and preferably large-$N$ test. However, as we have endeavored to show, these instances of rational gambles tend to be very important and have posed puzzles for conventional historical analyses. We have deliberately picked a case where such a gamble can be argued to have had a decisive influence on the outcome of history. It is likely that a better understanding of the ability to manipulate risk can throw light on several other important historical battles and hotly disputed military strategies. In the case of Pearl Harbor, the high variance mechanism would go far to explain what could otherwise be seen as the Machiavellian behavior of
Roosevelt and his alleged foreknowledge of Japanese plans to attack. In his article on the Japanese decision to attack Pearl Harbor, Sagan (1988, 351-2) argues that the United States unintentionally provoked Japan into a war and thus attributes the war to miscalculation. Our model here offers an alternative rationalist framework to understand and analyze the nature of the bargaining process between Japan and the United States. We suggest the same causal mechanism that brought the United States into the First World War may also have brought her into the Second World War.

It is not only during war that the ability to generate and manipulate risk can dramatically affect the outcome of bargaining. Almost a quarter century after Schelling, international relations theorists can fruitfully revisit the important topic of risk; new methods and ideas may offer rich rewards.

**Conclusion**

Risk-taking is an intrinsic part of international conflict. In this article we have offered an explanation of risk-taking grounded in domestic political institutions and shown that completely informed leaders sometimes go to war in response to their institutionally-induced risk preferences. Specifically, we have shown that the need to retain the loyalty of a segment of the selectorate generates risk-seeking behavior over some range of crisis outcomes. This in turn results in the leaders’ choice of risky options in some cases; war emerges as a rational gamble. We have shown how the crisis is resolved can depend on the nature of the status quo and on which selectorate cares more about the outcome of the conflict. Finally, we have applied the model to a particularly important historic setting: the German decision to launch unrestricted submarine warfare which brought the United States into World War I. As we have described in detail, this case nicely illustrates the mechanism at work in our formal model.

The model we presented is deliberately simple to most clearly illustrate our insight about institutionally-induced risk preferences. Clearly, further work could explore several interesting avenues. First, it would be interesting to investigate the effect of regime type on the risk preferences of leaders. Intuitively, it seems reasonable to suggest that as the size of the selectorate that a leader must satisfy grows, the more extreme the risk preferences of the leader; both more risk seeking over bad outcomes and more risk averse over good outcomes. Similarly, the size of the selectorate will affect the size of the gamble. This suggests a systematic link between regime type and risk preferences worthy of further exploration.

A second useful extension to our model would be to link the crisis outcome that the
selectorate cares about to the leader’s value of holding or losing office. In particular, it seems natural to suppose that leaders who are removed from office after a total defeat could potentially face a dramatically worse fate than those who lose office after a more moderate outcome. Once again, the nature of this link could depend on the regime type, with leaders of democratic regimes having brighter prospects than leaders of authoritarian regimes. In this case, our model would generate useful comparative statics about the tradeoff leaders face between the probability of staying in power versus the benefits or costs of holding or losing power.

A third issue that deserves further attention is the structure and timing of our bargaining model. In the present version, even though both sides have the ability to gamble, only one side can make an offer. If our model were generalized to allow a richer bargaining structure with, for example, alternating offers, we anticipate that our main findings would still hold, although the exact form of the equilibrium would change somewhat. Thus, we are confident that our results are general, and not dependent on the precise details of the extensive form presented here. Future work which incorporates a richer framework of gambling and bargaining offers prospects for a better understanding not only of war but generally any bargaining situation where actors have the ability to manipulate risk.
Appendix A: Optimal Gambles

In this section we lay out the structure and assumptions of a general analysis of optimal gambles. Consider a given player $i$ and a real variable $x$, conventionally between zero and one, representing the relative success of the decision-maker. This variable could, for example, represent the share of assets that results from a bargain or the fraction of disputed territory controlled by the player. Player $i$‘s utility for the outcome $x$ is given by $u_i(x)$, a (strictly) increasing and continuously differentiable function. Likewise, player $i$‘s utility for a random variable $X$ is simply $E u_i(X)$, the expectation of $u_i(x)$ under the random variable $X$.

A central focus of this paper is the players’ ability to gamble. By “gambling” we mean the ability to enforce any random variable $X$, provided its mean is an exogenously given number $x_0$. Thus player $i$ can decide to enforce the deterministic outcome $x_0$ (the random variable equal to $x_0$ with probability 1); alternatively he can choose any two points, $x_1$ and $x_2$, on either side of $x_0$, say $x_1 < x_0 < x_2$, and enforce the random variable $X$ that equals $x_1$ with probability $\frac{x_2-x_0}{x_2-x_1}$ and equals $x_2$ with probability $\frac{x_0-x_1}{x_2-x_1}$ (so that the mean of $X$ is indeed $x_0$, as the reader may easily check); and finally he can enforce any other random variable (perhaps with support on interval around $x_0$) provided its mean is $x_0$. Figure 6 illustrates this construction by showing the (expected) utility of a player for the binary random variable $X$ supported by $x_1$ and $x_2$. The key question then is: given $u_2$ and $x_0$, what is the optimal gamble, the optimal $X$?

Before we answer this question formally, it may be useful to present this characterization in graphical terms. Specifically, the optimal gamble is determined by the following operation. First we construct $U^*_i$, the “concave hull” of $u_i$, which is the smallest concave function above $u_i$. Looking at Figure 6 and put simply, $U^*_i$ is the function that “connects the bumps” in $u_i$ from above. Then the optimal gamble for a player gives him an expected payoff of $U^*_i(x_0)$. Moreover, the optimal gamble by a player can be either the deterministic outcome $x_0$ or a (binary) probability distribution with weight at exactly two points which are the closest points in both directions where $U^*_i$ and $u_i$ coincide. The former case is when the optimal gamble is no gamble and it occurs when $x_0$ lies in a region where $U^*_i$ and $u_i$ coincide. The latter case occurs when $u_i(x_0)$ differs from $U^*_i(x_0)$. Then the optimal gamble is the binary probability distribution supported by the points $x^*_1$ and $x^*_2$. (See Figure 6)

The example in Figure 6 illustrates the difference between non-optimal and optimal

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18This general analysis should be applicable to a whole range of phenomena where the manipulation of risk potentially offers benefits, including but not limited to crisis bargaining, civil litigation (Mnookin, Ross and Arrow, 1995, 21), and war termination.
gambles. The figure illustrates how $U^*_i$ – the “concave hull” as described above – generates the optimal gamble with the highest expected utility. If a player chooses a probability distribution with a carrier of the two points $x_1$ and $x_2$, for example, he has picked a non-optimal gamble. This gamble is non-optimal because it has a lower expected utility than the gamble with the probability distribution with the two carrier points at $x_1^*$ and $x_2^*$. As the figure illustrates, any random variable $X$ constructed from two carrier points other than $x_1^*$ and $x_2^*$ or from any carrier, binary or not, generates a sub-optimal gamble. We call the difference between the two carrier points, $|x_2^* - x_1^*|$, the magnitude of the gamble. The farther apart the two carrier points, the ‘bigger’ the gamble because the bigger the difference between the best and the worst outcomes.

We now present the analytical solution justifying this discussion. In order to identify an optimal gamble, we consider the following constrained optimization problem:

$$\max_{x_i, p_i} \sum_{i=1}^k p_i u(x_i),$$

subject to $\sum_{i=1}^k p_i x_i = x_0$,
\[
\sum_{i=1}^{k} p_i = 1, \\
p_i \geq 0 \quad i = 1, \ldots, k, \\
x_i \in [0,1] \quad i = 1, \ldots, k.
\]

The objective function of this maximization problem is simply the expected utility of the gamble, while the constraints are that the mean of the gamble must equal \( x_0 \), the probabilities are correctly defined, and the support of the gamble lies in the unit interval. Given the mix of equality and inequality constraints, the Lagrangian for this problem is\(^{19}\)

\[
L = \sum_{i=1}^{k} p_i u(x_i) - \mu \left[ \sum_{i=1}^{k} p_i x_i - x_0 \right] - \gamma \left[ \sum_{i=1}^{k} p_i = 1 \right] - \lambda_i[-p_i] - \nu_i[-x_i] - \tau_i[x_i - 1].
\]

The Kuhn-Tucker conditions for this problem are:

\[
\begin{align*}
    u(x_i) - \mu x_i - \gamma + \lambda_i &= 0 \\
p_i u'(x_i) - \mu p_i + \nu_i - \tau_i &= 0 \\
    \sum_{i=1}^{k} p_i x_i &= x_0 \\
    \sum_{i=1}^{k} p_i &= 1 \\
p_i \geq 0, \quad \lambda_i \geq 0, \quad \lambda_i p_i &= 0 \\
x_i \geq 0, \quad \nu_i \geq 0, \quad \nu_i x_i &= 0 \\
x_i \leq 1, \quad \tau_i \geq 0, \quad \tau_i(x_i - 1) &= 0
\end{align*}
\]

To begin with, if \( p_i > 0 \), then \( \lambda_i = 0 \), so

\[
\begin{align*}
u(x_i) - \mu x_i &= \gamma \\
    u'(x_i) - \mu + \frac{\nu_i - \tau_i}{p_i} &= 0.
\end{align*}
\]

In addition, if \( x_i \) is interior, \( \nu_i = \tau_i = 0 \). Therefore, \( u'(x_i) = \mu \). This implies that any interior point in the support of an optimal lottery must have the same utility slope. This is not surprising, because if it were not true, we could move two interior points in the support,

\(^{19}\)As the objective is a continuous function defined on a compact constraint set, this problem has a solution. As all constraints are linear, the constraint qualification holds, as well.
say \( x_i \) and \( x_j \), in appropriate directions to increase the total utility, given fixed probabilities.

On the other hand, if \( x_i = 0 \) is in the support of an optimal lottery, then \( \tau_i = 0 \) and so we have \( u'(0) = \mu - \nu_i p_i \). As \( \nu_i \) and \( p_i \) are greater than or equal to zero, we can simplify this to \( u'(0) \leq \mu \). Likewise, if \( x_i = 1 \) receives positive probability, a similar argument gives \( u'(1) \geq \mu \).

Now we examine the first equation of (1). This says that for all \( x_i \) in the support of an optimal lottery, \( u(x_i) - \mu x_i = \gamma \). For \( x_i \in (0,1) \), we know \( \mu = u'(x_i) \), and therefore \( u(x_i) = u'(x_i)x_i + \gamma \) for all interior points in the support of an optimal lottery. But this is the equation for a straight line with \( y \)-intercept \( \gamma \). Thus, not only must each interior point with positive probability have the same slope, but they must all have the same \( y \)-intercept. Another way to express this is that the tangent lines to \( u \) at each interior point with positive probability must all be the same. This is illustrated in Figure 6 in which the points \( u_i(x_i^*) \) and \( u_i(x_i^2) \) both lie on the tangent line that defines an optimal gamble.

In addition, if \( x_i = 0 \) is in the support of an optimal lottery, the first equation of (1) becomes \( u(0) = \gamma \). As we just identified, \( \gamma \) is the \( y \)-intercept of the line tangent to the utility function at the interior points of an optimal lottery. Thus, if \( x_i = 0 \) is in the support of an optimal lottery, the point \( u(0) \) must lie on the optimal lottery tangent line. Similarly, if \( x_i = 1 \) is in the support of an optimal lottery, \( u(1) \) must lie on the optimal lottery tangent line.

As a final observation, multiply both sides of the first equation of (1) by \( p_i \) to get \( p_iu(x_i) - \mu p_ix_i = p_i\gamma \). Now summing these equations over all \( x_i \) yields

\[
\sum_i p_iu(x_i) - \mu \sum_i p_ix_i = \gamma \sum_i p_i \\
\sum_i p_iu(x_i) - \mu x_0 = \gamma \\
\sum_i p_iu(x_i) = \mu x_0 + \gamma.
\]

This implies that the value of an optimal lottery with mean \( x_o \) is purely a function of \( \mu \), the slope of the tangent line at \( u(x_i) \) for an interior \( x_i \), and \( \gamma \), the \( y \)-intercept of this tangent line. In particular, the value of an optimal lottery does not depend on the probabilities, \((p_1, \ldots, p_n)\). Thus, given a choice of \( x \) values (determined by \( \mu \) and \( \gamma \)), any \( p \) values that satisfy

\[
\sum_{i=1}^k p_i x_i = x_0 \quad \text{and} \quad \sum_{i=1}^k p_i = 1
\]

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will give the same value $\sum p_i u(x_i)$. This result is important if there are several optimal lotteries. By definition, any two optimal lotteries must have the same expected utilities and therefore this result implies that if there are multiple optimal lotteries, there will exist an optimal lottery with only two values. This justifies our focus on two value optimal lotteries.

**Appendix B: Proof of Theorem 1**

The case of $s_2(x_0) > 1/2$ is dealt with in the text. So we will focus on the case $s_2(x_0) < 1/2$. We will show that for sufficiently large $n$, if $\alpha < 1$, then the unique equilibrium outcome is the optimal gamble $X^*_2$ and if $\alpha > 1$, then the unique equilibrium outcome is an accepted offer at $z^*$.

Recall that $u_1(x) = \Phi[\sqrt{n}(x - 1/2)]$ and $u_2(x) = \Phi[\sqrt{n}((\alpha(1-x)) - 1/2)]$, with $\alpha > 1/2$. For later use, define

$$v_1(x) = 1 - u_1(x) = \Phi[\sqrt{n}(1/2 - x)].$$

It is easy to verify that $u_2(x) = v_1(1 - \alpha(1 - x))$.

Given the shape of $u_2$ and the condition that $s_2(x_0) < 1/2$, it follows from the results in Appendix A that the optimal gamble $X^*_2$ with mean $x_0$ has support $x^*_1$ and 1, where $x^*_1$ satisfies

$$u'_2(x^*_1) = \frac{u_2(1) - u_2(x^*_1)}{1 - x^*_1}.$$ 

Clearly, for large enough $n$, $u_2(1) \sim 0$, so we can write this condition as

$$u'_2(x^*_1) = \frac{-u_2(x^*_1)}{1 - x^*_1}. \quad (2)$$

Differentiating $u_2$ with respect to $x$, we have

$$u'_2(x) = \phi[\sqrt{n}((\alpha(1-x)) - 1/2)] \cdot \sqrt{n}\alpha(-1), \quad (3)$$

where $\phi[\cdot]$ is the density function of a standard normal. Substituting equation (3) into (2), we have

$$\phi[\sqrt{n}((\alpha(1-x)) - 1/2)] = \frac{\Phi[\sqrt{n}((\alpha(1-x^*_1)) - 1/2)]}{\sqrt{n}\alpha(1 - x^*_1)}. \quad (4)$$

Now if we let $\bar{x}$ solve

$$\phi[\sqrt{n}(\bar{x} - 1/2)] = \frac{\Phi[\sqrt{n}(\bar{x}) - 1/2]}{\sqrt{n}\bar{x}}, \quad (5)$$

then we have $\bar{x} = \alpha(1 - x^*_1)$, where $\bar{x}$ is a constant that does not depend on $\alpha$. From this,
it is easy to see that \( u_2(x_1^*) = \Phi[\sqrt{n}(\bar{x} - 1/2)] \) is a constant that does not depend on \( \alpha \).

The other value we must define is \( z^* \). As discussed in the text, \( z^* \) must satisfy \( u_2(z^*) = Eu_2(X_2^*) \). This is equivalent to \( u_2(z^*) = u_2(1) - (1 - x_0)u_2'(x_1^*) \sim -(1-x_0)u_2'(x_1^*) \), as above. Combining this with equation (2) gives

\[
\begin{align*}
    u_2(z^*) &= -(1-x_0) \left[ \frac{-u_2(x_1^*)}{1-x_1^*} \right] \\
    &= \frac{1-x_0}{1-x_1^*} u_2(x_1^*). 
\end{align*}
\]

Therefore, we have

\[
\frac{u_2(z^*)}{u_2(x_1^*)} = \frac{1-x_0}{1-x_1^*}. \tag{6}
\]

From this and the relationship \( \bar{x} = \alpha(1-x_1^*) \), we can see that \( u_2(z^*) \) is linear.

With these preliminaries taken care of, we can now turn to the main analysis. As discussed in the text, it is never optimal for the leader of country 1 to choose to gamble and therefore the equilibrium outcome is determined by comparing the value of \( u_1(z^*) \) and \( Eu_1(X_2^*) \). The latter value is given by

\[
Eu_1(X_2^*) = u_1(x_1^*) + (x_0 - x_1^*) \frac{u_1(1) - u_1(x_1^*)}{1-x_1^*}. 
\]

As \( u_1(1) \sim 1 \), this expression is equivalent to

\[
Eu_1(X_2^*) = \frac{1-x_0}{1-x_1^*} u_1(x_1^*) + \frac{x_0 - x_1^*}{1-x_1^*}. 
\]

So the equilibrium condition is

\[
Eu_1(X_2^*) \leq u_1(z^*) \\
\frac{1-x_0}{1-x_1^*} u_1(x_1^*) + \frac{x_0 - x_1^*}{1-x_1^*} \leq u_1(z^*) \\
(1-x_0)u_1(x_1^*) + (x_0 - 1 + 1 - x_1^*) \leq (1-x_1^*)u_1(z^*) \\
(1-x_0)(u_1(x_1^*) - 1) \leq (1-x_1^*)(u_1(z^*) - 1) \\
\frac{1-x_0}{1-x_1^*} \geq \frac{u_1(z^*) - 1}{u_1(x_1^*) - 1} \\
\frac{1-x_0}{1-x_1^*} \geq \frac{v_1(z^*)}{v_1(x_1^*)}.
\]
We will use this last condition in two ways. Substituting into equation (6) gives

\[ Eu_1(X_2) \leq u_1(z^*) \iff \frac{u_2(z^*)}{u_2(x_1^*)} \geq \frac{v_1(z^*)}{v_1(x_1^*)}. \]

From the definitions of \( u_2 \) and \( v_1 \), it is easy to see that this condition implies that if \( \alpha = 1 \), then \( Eu_1(X_2) = u_1(z^*) \). The second way we use this condition is to use the fact that \( \bar{x} = \alpha(1-x_1^*) \). Doing so yields

\[ Eu_1(X_2) \leq u_1(z^*) \iff \alpha \frac{1-x_0}{\bar{x}} \geq \frac{v_1(z^*)}{v_1(x_1^*)}. \]  

(7)

From this condition, in order to establish our result, it suffices to show that

\[ \alpha < 1 \implies \alpha k < \frac{v_1(z^*)}{v_1(x_1^*)} \quad \text{and} \quad \alpha > 1 \implies \alpha k > \frac{v_1(z^*)}{v_1(x_1^*)}, \]

where \( k = \frac{1-x_0}{\bar{x}} \). Because \( Eu_1(X_2) = u_1(z^*) \) when \( \alpha = 1 \), we know that the graph of \( \alpha k \) intersects the graph of \( v_1(z^*)/v_1(x_1^*) \) at \( \alpha = 1 \). As the slope of \( \alpha k \) is just \( k \), it suffices to show that the derivative of \( v_1(z^*)/v_1(x_1^*) \) is less than \( k \). We now show that this is indeed the case.

The derivative of \( v_1(z^*)/v_1(x_1^*) \) with respect to \( \alpha \) is given by

\[ \frac{d}{d\alpha} \frac{v_1(z^*)}{v_1(x_1^*)} = \frac{v_1(x_1^*)v_1'(z^*) \frac{dx^*}{d\alpha} - v_1(z^*)v_1'(x_1^*) \frac{dx^*_1}{d\alpha}}{(v_1(x_1^*))^2}. \]

The value of \( \frac{dx^*_1}{d\alpha} \) is obtained from the fact that \( \bar{x} = \alpha(1-x_1^*) \) and that value of \( \frac{dx^*}{d\alpha} \) is obtained by differentiating equation (6) (after making the substitution \( u_2(x) = v_1(1-\alpha(1-x)) \)) and solving for \( \frac{dx^*}{d\alpha} \). Omitting a significant amount of algebra, we find that

\[ \frac{d}{d\alpha} \frac{v_1(z^*)}{v_1(x_1^*)} = \frac{k v_1'(z^*)}{\alpha v_1'(1-\alpha(1-z^*))} + \frac{(1-z^*)v_1'(z^*)v_1(x_1^*) - (1-x_1^*)v_1'(x_1^*)v_1(z^*)}{\alpha v_1(x_1^*)v_1(z^*)}. \]

(8)

Using a standard approximation to the ratio \( \phi(x)/\Phi(x) \) (Feller, 1968), it can be shown that \( (1-y)v_1'(y)/v_1(y) \) is decreasing in \( y \in [0,1] \). As \( x_1^* < z^* \), this implies that the second term in equation 7 is negative. For \( \alpha \neq 1 \), it is clear that as \( n \to \infty \), \( z^* \) converges to a value such that \( v_1'(z^*) \to 0 \) and \( v_1'(1-\alpha(1-z^*)) \) converges to a positive value. Therefore, the first term in equation 7 goes to zero as \( n \to \infty \). Therefore the derivative of \( v_1(z^*)/v_1(x_1^*) \) is less than \( k > 0 \), so the theorem is established.
References


